PM50

Pre-Leaving Certificate Examination
Triailscrúdú na hArdteistiméireachta

PRE-LEAVING CERTIFICATE EXAMINATION, 2011

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

2½ hours

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 question
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer questions as follows:

In Section A, answer **all four** questions.

In Section B, answer **both** Question 5 **and** Question 6.

In Section C, answer **any two** of the three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Section A	Concepts and Skills	100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

(a) Write $-3+\sqrt{3}i$ in Polar Form.

(b) Given that $z^3-3z^2+7z-5=0$ has one integer root, find all three roots of the cubic equation.

(c) Use De Moivre's theorem to solve the equation $z^3 - 1 = 0$.

Question 2 **(25 marks)**

(a) If $x^3 + bx^2 + cx + d = 0$ and $f(-1) = 0$, $f(-2) = -20$ and $f(-3) = 3f(-2)$ find the values of b, c and d where $b, c, d \in \mathbb{Z}$.

(b) Solve the equation $\log_2(x^2) = (\log_2(x))^2$.

(c) Prove by induction that $n\left(n^2-1\right)$ is divisible by 3, for all $n \geq 2, n \in \mathbb{N}$.

Question 3 **(25 marks)**

(a) Find the value of x for which $f(x) = x^2 + bx + 3b = 0$ has exactly one real root where $b \in \mathbb{Z}, b > 0$.

(b) Show that the sequence $T_n = 2\left(3^{n+1}\right)$ is geometric.

(c) Evaluate $\sum_{n=1}^{\infty} \frac{1}{(n+4)(n+5)}$.

Question 4 **(25 marks)**

(a) Solve the simultaneous equations,

$$4x + 3y + 2x = 15$$
$$x + 2y - z = 9$$
$$3x + y + z = 8$$

(b) Solve the equation $3^{2x+1} - 28(3^x) + 9 = 0$.

Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6.

Question 5 **(50 marks)**

A newly married couple have decided to purchase their first house. They have decided they need to borrow an amount A from their bank. Interest of r% APR will be applied to the mortgage. The couple borrow the money over n years and make an annual repayment of €m.

(a) Write an equation to show the amount owing at the end of the first year, *D1* in terms of A, r and m after year 1.

(b) Write an equation to show the amount owing at the end of the second year, *D2* and the third year, *D3* in terms of A, r and m after year 1.

(c) Use the above results to devise a formula for the amount owed D_n for the n^{th} year.

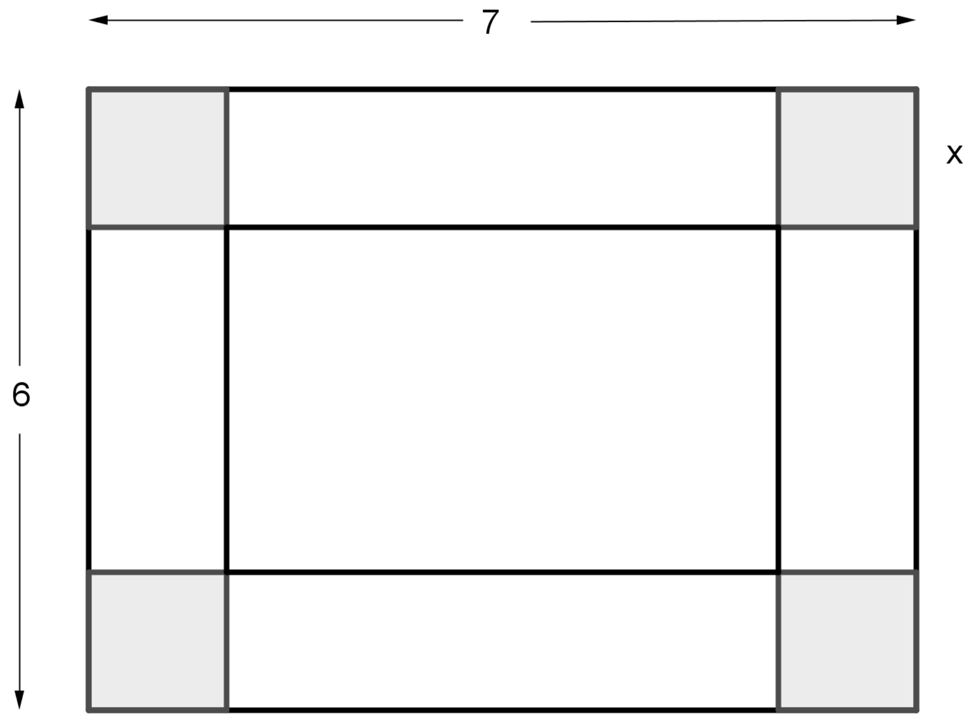
(d) If $D_n = 0$ after the final year of the mortgage show that the annual repayment m can be written as $m = \dfrac{Ar(1+r)^n}{(1+r)^n - 1}$.

(e) If the couple borrow €200,000 over 25 years at 3% interest per annum, how much will they repay annually?

(f) Calculate the interest paid on the mortgage as a percentage of the original borrowings.

Question 6 **(50 marks)**

An open top plastic carton is manufactured from a sheet as shown.



(a) Use the dimensions shown to write an expression for the capacity of the box.

(b) Use the dimension shown to write an expression for the total surface area of the plastic container.

(c) Draw a suitable graph to represent the volume of the box as a function of x.

(d) From your graph determine the area of the box that will give a maximum capacity.

Section C	Functions and Calculus (old syllabus)	100 marks

Answer **any two** of the three questions from this section.

Question 7 **(50 marks)**

(a) Given $f\left(x\right)=\left(\sin x+1\right)^2$, find the value of $f'\left(\dfrac{\pi}{6}\right)$.

(b) **(i)** Show that the curve $y=x^3-6x^2+18x+5$ is increasing for all values of x.

(ii) If $y=\dfrac{x^2}{\sqrt{x-1}}$, find the values of a and b, a and $b\in\mathbb{R}$, if $\dfrac{dy}{dx}=\dfrac{ax^2+bx}{2(x-1)^{\frac{3}{2}}}$.

(c) A curve has equation: $y = \left(x^2 - 3\right) \cdot e^x$.

(i) Find $\dfrac{dy}{dx}$.

(ii) Find $\dfrac{d^2y}{dx^2}$.

(iii) Find the x-ordinate of each of the stationary points of the curve.

(iv) Determine the nature of each of the stationary points.

Question 8 **(50 marks)**

(a) Differentiate $\cos x$ with respect to x from first principles.

(b) **(i)** Given the curve: $3xy-2y^2=4$. Find the gradient of the tangent to the curve at the point $(2,1)$.

(ii) A curve is defined as follows:

$y = 2(\theta - \sin\theta), \quad x = 2(1 - \cos\theta)$.

Find the slope of the tangent to the curve at the point where $\theta = \dfrac{\pi}{2}$.

(c) Let $f(x)=\frac{1}{2}\left(e^{x}+e^{-x}\right)$.

(i) Find $f'(x)$, the derivative of $f(x)$.

(ii) Find $f''(x)$ and show that $f''(x)=f(x)$ where $f''(x)$ is the second derivative of $f(x)$.

(iii) Show that $\dfrac{f'(2x)}{f'(x)} = 2f(x)$, when $x \neq 0$.

Question 9 **(50 marks)**

(a) Find $\int (2\sin 2x - 1)\, dx$.

(b) Evaluate:

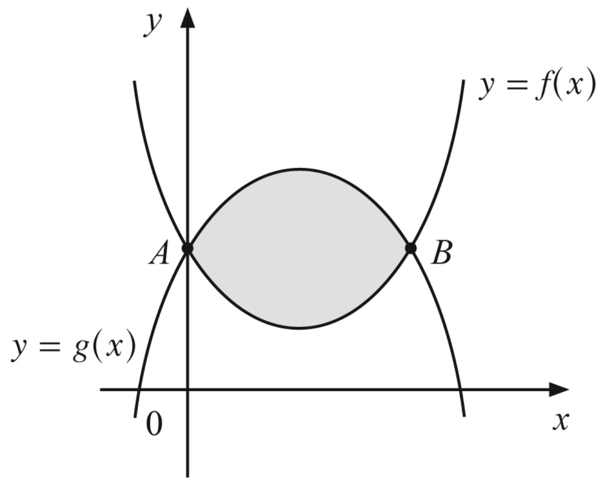
(i) $\int_2^4 \frac{dx}{\sqrt{16 - x^2}}$

(ii) $\int_0^1 \left(4xe^{x^2}+1\right)dx$

(c) The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = x^2 - 4x + 5$ and
$g(x) = 5 + 4x - x^2$.

$f(x)$ and $g(x)$ intersect at A and B.



(i) Find the x-ordinates of points A and B.

(ii) Find the area enclosed by the two curves.