WMS26

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PRE-LEAVING CERTIFICATE EXAMINATION, 2011

MARKING SCHEME

MATHEMATICS
(Project Maths – Phase 2)

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GENERAL GUIDELINES FOR EXAMINERS

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders – mathematical errors/omissions (–3)
   - Slips – numerical errors (–1)
   - Misreadings (provided task is not oversimplified) (–1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3, …, S1, S2, …, M2, M2, … etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that:
   - Any *correct, relevant* step in a part of a question merits at least the attempt mark for that part.
   - If deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded.
   - A mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled W1, W2, … etc.

4. The phrase ''hit or miss'' means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase ''and stops'' means that no more work is shown by the candidate.

6. Special notes relating to the marking of the particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

APPLYING THE GUIDELINES

Examples of the different types of error:

Blunders (i.e. mathematical errors) (-3)

- Algebraic errors: $8x+9x=17x^2$ or $5p\times 4p=20p$ or $(-3)^2=6$
- Sign error $-3(-4)=-12$
- Decimal errors
- Fraction error (incorrect fraction, inversion etc); apply once
- Cross-multiplication error
- Operation chosen is incorrect. (e.g., multiplication instead of division)
- Transposition error: e.g. $-2x-k+3\Rightarrow -2x=3+k$ or $-3x=6\Rightarrow x=2$ or $4x=12\Rightarrow x=8$; each time
- Distribution error (once per term, unless directed otherwise) e.g. $3(2x+4)=6x+4$ or $\frac{1}{2}(3-x)=5\Rightarrow 6-x=5$
- Expanding brackets incorrectly: e.g. $(2x-3)(x+4)=8x^2-12$
- Omission, if not oversimplified
- Index error, each time unless directed otherwise
- Factorisation: error in one or both factors of a quadratic: apply once $2x^2-2x-3=(2x-1)(x+3)$
- Root errors from candidate's factors: error in one or both roots: apply once
- Error(s) in transcribing formulae from tables (assuming it generates mathematical acceptable answer(s). Serious errors or over simplification will merit attempt marks at most
- Central sign error in *uv* or *u*/*v* formulae
- Omission of $\div v^2$ or division not done in *u*/*v* formula (apply once)
- Vice-versa substitution in *uv* or *u*/*v* formulae (apply once)
- Quadratic formula (*acceptable*) and its application apply a maximum of two blunders

Slips (-1)

- Numerical slips: $4+7=10$ or $3\times 6=24$, but $5+3=15$ is a blunder
- An omitted round-off or incorrect round off to a required degree of accuracy, or an early round off, is penalised as a slip each time.
- However an early round-off which has the effect of simplifying the work is at least a blunder
- Omission of units of measurement or giving the incorrect units of measurement in an answer is treated as a slip, once per part (a), (b) and (c) of each question. Only applies where a candidate would otherwise have achieved full marks in each subpart

Misreadings (-1)

- Writing 2436 for 2346 will not alter the nature of the question so $\mathrm{M}(-1)$

  However, writing 5000 for 5026 will simplify the work and is penalised as at least a blunder.

Note: Correct relevant formula *isolated* and stops: if formula is *not* in Tables, award attempt mark.

OVERVIEW OF MARKING SCHEME

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scale	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

All questions marking category are shown throughout the solutions.

SOLUTIONS TO PAPER 1

QUESTION 1

Part (a)	**5C marks**
Part (b)	**10D marks**
Part (c)	**10D marks**

(a) Write $-3+\sqrt{3}i$ in Polar Form.

$r=\sqrt{(-3)^2+(\sqrt{3})^2}=\sqrt{12}$ **2 marks**

$\tan a=\frac{\sqrt{3}}{3} \quad \therefore a=\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)=30°$

$\theta=180-30=150°$ or $\frac{5\pi}{3}$ **4 marks**

$\sqrt{12}\left(\cos(150)+i\sin(150)\right)$ **5 marks**

(b) Given that $z^3-3z^2+7z-5=0$ has one integer root, find all three roots of the cubic equation

$(1)^3-3(1)^2+7(1)-5=0 \quad \therefore (z-1)$ is a factor **2 marks**

$$\begin{array}{r} z^2-2z+5 \\ z-1\overline{)z^3-3z^2+7z-5} \\ \underline{z^3-z^2} \\ -2z^2+7z \\ \underline{2z^2-2z} \\ 5z-5 \\ \underline{5z-5} \\ 0 \end{array}$$

5 marks

$$\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}$$

8 marks

$$\frac{2\pm\sqrt{-16}}{2}=\frac{2\pm 4i}{2}$$

$1+2i$ and $1-2i$ **10 marks**

(c) Use De Moivre's theorem to solve the equation $z^3 - 1 = 0$.

$z^3 = 1 \quad \therefore \theta = 0° \quad$ and $\quad r = 1$

Angle 2 marks
Modulus 5 marks

$$z^3 = \left[\cos(2n\pi) + i\sin(2n\pi)\right]$$

$$z = \left[\cos(2n\pi) + i\sin(2n\pi)\right]^{\frac{1}{3}} \quad \rightarrow \quad z = \left[\cos\left(\frac{2n\pi}{3}\right) + i\sin\left(\frac{2n\pi}{3}\right)\right]$$

8 marks

n = 0

$$z = \cos(0) + i\sin(0) = 1$$

n = 1

$$z = \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

n = 2

$$z = \cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

10 marks

QUESTION 2

Part (a)	**10D marks**
Part (b)	**5C marks**
Part (c)	**10D marks**

(a) If $x^3 + bx^2 + cx + d = 0$ and $f(-1) = 0$, $f(-2) = -20$ and $f(-3) = 3f(-2)$ find the values of b, c and d where $b, c, d \in \mathbb{Z}$.

$f(-1) \;\rightarrow$	$(-1)^3 + b(-1)^2 + c(-1) + d = 0$	$\therefore b - c + d = 1$	Equation 1
$f(-2) \;\rightarrow$	$(-2)^3 + b(-2)^2 + c(-2) + d = -20$	$\therefore 4b - 2c + d = -12$	Equation 2
$f(-3) \;\rightarrow$	$(-3)^3 + b(-3)^2 + c(-3) + d = -60$	$\therefore 9b - 3c + d = -33$	Equation 3

Eq2 - Eq1
$3b - c = -13$ Equation 5

Eq3 - Eq1
$8b - 2c = -34$ Equation 5

$8b - 2c - 2(3b - c) = -34 + 26$

$2b = -8$ $\quad \therefore b = -4$
$3(-4) - c = -13$ $\quad \therefore c = 1$
$(-4) - (1) + d = 1$ $\quad \therefore d = 6$

Correct Equations	**2 marks**
One correct variable	**5 marks**
Two correct variables	**8 marks**
Three correct variables	**10 marks**

(b) Solve the equation $\log_2(x^2) = (\log_2(x))^2$.

$\log_2(x^2) = (\log_2(x))^2$

$2\log_2(x) = (\log_2(x))^2$ $\quad$ let $y = \log_2 x$ $\quad$ **2 marks**

$2y = y^2$ $\quad \therefore y^2 - 2y = 0$
$y = 0$ or $y = 2$ $\quad$ **4 marks**

$\therefore \log_2 x = 0 \rightarrow x = 1$ and $\log_2 x = 2 \rightarrow x = 4$ $\quad$ **5 marks**

(c) Prove by induction that $n\left(n^2-1\right)$ is divisible by 3, for all $n \geq 2, n \in \mathbb{N}$.

Prove for $n=2\ldots\ldots 2\left(2^2-1\right)=6$ which is divisible by 3	**2 marks**
Assume true for $n=k$ $\quad \therefore \left(k^3-k\right)=3A$	**5 marks**
Prove true for $n=(k+1)$	
$\left((k+1)^3-(k+1)\right)$	**8 marks**
$k^3+k^2+2k^2+2k+k+1-k-1$	
$\left(k^3-k\right)+3k^2+3k$	
$3A+3k^2+3k$	
$3\left(A+k^2+k\right)$	
$3B$ *for some* $B \in \mathbb{Z}$	**10 marks**
Thus $P(k+1)$ is true assuming $P(k)$ is true	

QUESTION 3

Part (a)	**5C marks**
Part (b)	**10C marks**
Part (c)	**10D marks**

(a) Find the value of x for which $f(x) = x^2 + bx + 3b = 0$ has exactly one real root where $b \in \mathbb{Z}, b > 0$.

$b^2 - 4(1)(3b) = 0$ **2 marks**

$b^2 - 12b = 0$

$b(b-12) = 0$

$b = 0$ or $b = 12$ **4 marks**

But $b \in \mathbb{Z}, \quad b > 0$

$\therefore b = 12$ **5 marks**

(b) Show that the sequence $T_n = 2\left(3^{n+1}\right)$ is geometric.

$T_n = 2\left(3^{n+1}\right)$

$T_{n+1} = 2\left(3^{n+2}\right)$ **3 marks**

$\frac{T_{n+1}}{T_n} = \frac{2.3^n.3}{2.3^n.3^2} = \frac{5}{18} = \frac{1}{3}$, which is a constant $\therefore$ series is geometric.

Correct Fraction **7 marks**

Correct Solution **10 marks**

(c) Evaluate $\sum_{n=1}^{\infty} \frac{1}{(n+4)(n+5)}$.

$\frac{1}{(n+4)(n+5)} = \frac{1}{n+4} - \frac{1}{n+5}$ **2 marks**

$n = 1$ $\quad \frac{1}{5} - \frac{1}{6}$

$n = 2$ $\quad \frac{1}{6} - \frac{1}{7}$

$n = k$ $\quad \frac{1}{k+4} - \frac{1}{k+5}$ **5 marks**

S_k $\quad \frac{1}{5} - \frac{1}{k+5}$ **8 marks**

$S_\infty = \frac{1}{5} - \frac{1}{\infty} = \frac{1}{5}$ **10 marks**

QUESTION 4

Part (a)	**10D marks**
Part (b)	**15D marks**

(a) Solve the simultaneous equations,

$$4x+3y+2x=15$$
$$x+2y-z=9$$
$$3x+y+z=8$$

$4x+3y+2z=15$
$2x+4y-2x=18$
$6x+7y=33$

$x+2y-z=9$
$3x+y+z=8$
$4x+3y=17$

$18x+21y=99$
$-28x-21y=-119$
$-10x=-20 \qquad \therefore x=2$

$8+3y=17 \qquad \therefore y=3$

$8+9+2x=15 \qquad \therefore z=-1$

Correct elimination	**2 marks**
One variable found	**5 marks**
Two variables found	**8 marks**
Three variables found	**10 marks**

(b) Solve the equation $3^{2x+1}-28\left(3^x\right)+9=0$.

$3^{2x+1}-28\left(3^x\right)+9=0$

$3^1 3^{2x}-28\left(3^x\right)+9=0 \quad \rightarrow \quad 3\left(3^x\right)^2-28\left(3^x\right)+9=0$ **4 marks**

Let $y=3^x$

$3y^2-28y+9=0$ **7 marks**

$(3y-1)(y-9)=0$

$y=\frac{1}{3}$ or $y=9$ **11 marks**

$3^x=\frac{1}{3} \quad \therefore x=-1$ and $3^x=9 \quad \therefore x=2$ **15 marks**

QUESTION 5

Part (a)	**5 marks**
Part (b)	**5 marks**
Part (c)	**10D marks**

A newly married couple have decided to purchase their first house. They have decided they need to borrow an amount *A* from their bank. Interest of *r*% APR will be applied to the mortgage. The couple borrow the money over *n* years and make an annual repayment of €m.

(a) Write an equation to show the amount owing at the end of the first year, *D1* in terms of *A*, *r* and m after year 1.

Any correct substitution	**2 marks**
$D_1 = A(1+r) - m$	**5 marks**

(b) Write an equation to show the amount owing at the end of the second year, *D2* and the third year, *D3* in terms of *A*, *r* and *m* after year 1.

$D_2 = \left[A(1+r) - m\right](1+r) - m$	**5 marks**
$D_2 = A(1+r)^2 - m(1+r) - m$	
$D_3 = \left[A(1+r)^2 - m(1+r) - m\right](1+r) - m$	**10 marks**
$D_3 = A(1+r)^3 - m(1+r)^2 - m(1+r) - m$	

(c) Use the above results to devise a formula for the amount owed D_n for the n^{th} year.

Accept correct part	**5 marks**
$D_n = A(1+r)^n - m(1+r)^{n-1} - m(1+r)^{n-2} - \ldots\ldots - m(1+r) - m$	**10 marks**

(d) If $D_n = 0$ after the final year of the mortgage show that the annual repayment *m* can be written as
$$m = \frac{Ar(1+r)^n}{(1+r)^n - 1}.$$

$D_n = A(1+r)^n - m(1+r)^{n-1} - m(1+r)^{n-2} - \ldots\ldots - m(1+r) - m$	
$D_n = 0$	
$\therefore 0 = A(1+r)^n - m(1+r)^{n-1} - m(1+r)^{n-2} - \ldots\ldots - m(1+r) - m$	**2 marks**
$m + m(1+r) + \ldots\ldots + m(1+r)^{n-2} + m(1+r)^{n-1} = A(1+r)^n$	**5 marks**
Geometric Progression on LHS with $a = m$ and $r = (1+r)$	
$\frac{m\left(1-(1+r)^n\right)}{-r} = A(1+r)^n$	**8 marks**
$m = \frac{Ar(1+r)^n}{(1+r)^n - 1}$	**10 marks**

(e) If the couple borrow €200,000 over 25 years at 3% interest per annum, how much will they repay annually?

$m = \frac{Ar(1+r)^n}{(1+r)^n - 1}$	
$m = \frac{(200000)(0.03)(1+(0.03))^{25}}{(1+0.03)^{25} - 1}$	**5 marks**
$m = €11,485.57$	**10 marks**

(f) Calculate the interest paid on the mortgage as a percentage of the original borrowings.

$€11,485.57 \times 25 = €287,139.25$	
Interest = $287,139.25 - 200,000 = €87,139.25$	**2 marks**
$\frac{87,139.25}{200,000} \times 100 = 43.57\%$	**5 marks**

QUESTION 6

Part (a)	**10 marks**
Part (b)	**5 marks**
Part (c)	**10 marks**

An open top plastic carton is manufactured from a sheet as shown.

(a) Use the dimensions shown to write an expression for the capacity of the box.

Capacity $=(7-2x)(6-2x)(x)$ **10 marks**

Capacity $=4x^3-26x^2+42x$

(b) Use the dimension shown to write an expression for the total surface area of the plastic container.

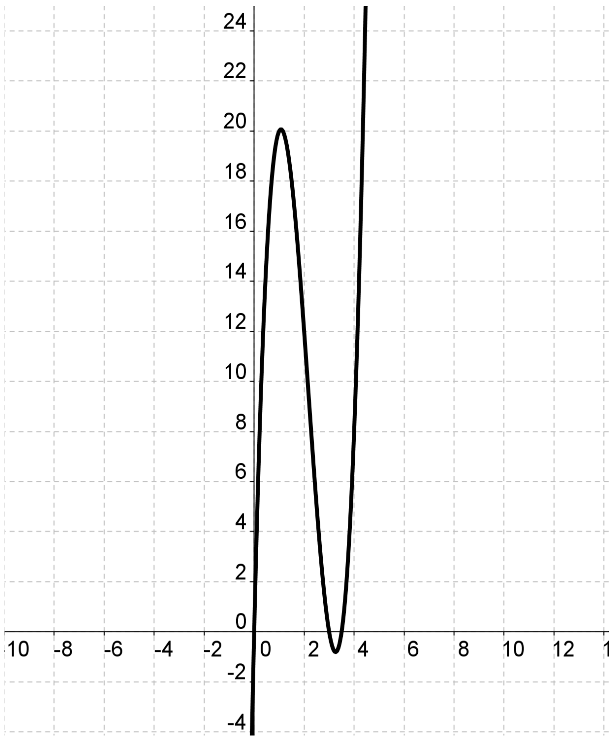
Surface Area $=(7-2x)(x)(2)+(6-2x)(x)(2)+(7-2x)(6-2x)$ **5 marks**

Surface Area $=14x-4x^2+12x-4x^2+42-14x-12x+4x^2$

Surface Area $=42-4x^2$ **10 marks**

(c) Draw a suitable graph to represent the volume of the box as a function of x.

Suitable Domain	**7 marks**
Max and Min clearly visible	**13 marks**
Curve	**20 marks**



(d) From your graph determine the area of the box that will give a maximum capacity.

20.5 ± 0.5 **10 marks**

QUESTION 7

Part (a)	**10 (5,5) marks**	**Att (2,2)**
Part (b)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**
Part (c)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**

Part (a)	**10 (5,5) marks**	**Att (2,2)**

Differentiation	**5 marks**	**Att 2**
Evaluation	**5 marks**	**Att 2**

$f'(x) = 2(\sin x + 1)\cos x$ **5m, A2**

$f'\left(\frac{\pi}{6}\right) = 2\left(\sin\frac{\pi}{6} + 1\right)\cos\frac{\pi}{6}$

$= 2\left(\frac{3}{2}\right)\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$ **5m, A2**

Blunders (–3)
B1: Differentiation error.
B2: Indices.

Slips (–1)
S1: Numerical.

Attempts (2)
A1: Any correct differential.

Part (b) (i)	**10 (5,5) marks**	**Att (2,2)**

Differentiation	**5 marks**	**Att 2**
Increasing function	**5 marks**	**Att 2**

$y = x^3 - 6x^2 + 18x + 5$

$y' = 3x^2 - 12x + 18$ **5m, A2**

$y' = 3(x^2 - 4x + 6)$ $\quad\quad y' = 3\left[(x^2 - 4x + 4) + 2\right]$

$y' = 3(x-2)^2 + 2$

since $(x-2)^2 \geq 0$, for all x, $y' > 0$ for all x

$\Rightarrow$ curve is increasing for all x. **5m, A2**

Blunders (–3)
B1: Differentiation error.
B2: Indices.

Slips (–1)
S1: Numerical.

Worthless (0)
W1: No differentiation.

Part (b) (ii) **10 (5,5) marks** **Att (2,2)**

Differentiation **5 marks** **Att 2**
Finish **5 marks** **Att 2**

$$y = x^2 (x-1)^{-\frac{1}{2}}$$

$$y' = x^2 \left(-\frac{1}{2}\right)(x-1)^{-\frac{3}{2}} + (x-1)^{-\frac{1}{2}}(2x) \qquad \textbf{5m, A2}$$

$$= x(x-1)^{-\frac{3}{2}} \left[-\frac{x}{2} + (x-1)^1 \cdot 2 \right]$$

$$= x(x-1)^{-\frac{3}{2}} \left[\frac{-x+4x-4}{2} \right]$$

$$= x(x-1)^{-\frac{3}{2}} \left[\frac{3x-4}{2} \right]$$

$$= \frac{3x^2-4x}{2(x-1)^{\frac{3}{2}}} \qquad \Rightarrow \quad a = 3, b = -4 \qquad \textbf{5m, A2}$$

Blunders (–3)
B1: Differentiation error.
B2: Indices.
B3: Factors.
B4: Mathematical blunder.

Slips (–1)
S1: Numerical.

Attempts (2)
A1: Any correct differentiation.

Worthless (0)
W1: No differentiation.

Note:
If answer is written as $y' = \dfrac{3x^2-4x}{2(x-1)^{\frac{3}{2}}}$, award full marks.

Part (c) | **20 (5,5,5,5) marks** | **Att (2,2,2,3)**

First Derivative	**5 marks**	**Att 2**
Second Derivative	**5 marks**	**Att 2**
$y' = 0$	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

(i) $y = (x^2 - 3)e^x \Rightarrow y' = (x^2 - 3)e^x + e^x(2x)$

$= e^x(x^2 + 2x - 3)$ **5m, A2**

(ii) $y'' = e^x(2x + 2) + (x^2 + 2x - 3)e^x$

$= e^x(x^2 + 4x - 1)$ **5m, A2**

(iii) $y' = 0 \Rightarrow e^x(x^2 + 2x - 3) = 0$

$\Rightarrow (x + 3)(x - 1) = 0$

$x = -3$ and 1 **5m, A2**

(iv) $y'' = e^x(x^2 + 4x - 1)$

$x = -3,\quad y'' = e^{-3}(9 - 12 - 1)$ i.e. negative

$\Rightarrow$ max. for $x = -3$

$x = 1,\quad y'' = e(1 + 4 - 1)$ i.e. positive

$\Rightarrow$ min. for $x = 1$ **5m, A2**

Blunders (–3)

B1: Differentiation error.

B2: $f'(x) \neq 0$.

B3: Indices.

B4: Factors.

B5: Error in deciding max/min value.

Slips (–1)

S1: Numerical.

Worthless (0)

W1: No differentiation.

W2: Integration.

QUESTION 8

Part (a)	**15 (5,5,5) marks**	**Att (2,2,2)**
Part (b)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**
Part (c)	**15 (5,5,5) marks**	**Att (2,2,2)**

Part (a)	**15 (5,5,5) marks**	**Att (2,2,2)**

$f(x+h)-f(x)$	**5 marks**	**Att 2**
Product form (R.H.S.)	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

$f(x)=\cos x$

$f(x+h)=\cos(x+h)$

$f(x+h)-f(x)=\cos(x+h)-\cos x$ **5m, A2**

$=-2\sin\left(\frac{2x+h}{2}\right)\sin\frac{h}{2}$

$\frac{f(x+h)-f(x)}{h}=-2\sin\frac{(2x+h)}{2}\cdot\frac{\sin\frac{h}{2}}{h}$

$=-\sin\frac{2x+h}{2}\cdot\frac{\sin\frac{h}{2}}{\frac{h}{2}}$ **5m, A2**

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=-\sin x(1)= \quad -\sin x$ **5m, A2**

OR

$y=\cos x$

$y+\Delta y=\cos(x+\Delta x)$

$\Delta y=\cos(x+\Delta x)-\cos x$ **5m, A2**

$=-2\sin\left(\frac{2x+\Delta x}{2}\right)\cdot\sin\frac{\Delta x}{2}$

$\frac{\Delta y}{\Delta x}=-\sin\left(\frac{2x+\Delta x}{2}\right)\cdot\frac{\sin\frac{\Delta x}{2}}{\Delta x}$

$=-2\sin\left(\frac{2x+\Delta x}{2}\right)\cdot\frac{\sin\frac{\Delta x}{2}}{\frac{\Delta x}{2}}$ **5m, A2**

$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=-\sin x(1)$

$\Delta x\to 0=-\sin x$ **5m, A2**

Blunders (–3)

B1: Error: $f(x+h)$ or $(x+\Delta x)$.

B2: No limits shown or implied or no indication $h \to 0$.

B3: $h \to \infty$.

Worthless (0)

W1: Not first principles.

Part (b) (i)	**10 (5,5) marks**	**Att (2,2)**
Differentiate	**5 marks**	**Att 2**
Isolate $\frac{dy}{dx}$	**5 marks**	**Att 2**

$3xy - 2y^2 = 4$ Differentiate $w.r.t.x$:

$$3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx} = 0$$ **5m, A2**

$$\frac{dy}{dx}(3x - 4y) = -3y$$

$$\frac{dy}{dx} = \frac{-3y}{3x - 4y}$$

At $(2,1)$ $\frac{dy}{dx} = \frac{-3}{6-4} = -\frac{3}{2}$ **5m, A2**

Blunders (–3)

B1: Differentiation.

B2: Indices.

Slips (–1)

S1: Numerical.

Attempts (2)

A1: Error in differentiation method.

A2: Any correct differential shown.

A3: $\frac{dy}{dx} = 3x\frac{dy}{dx} + 3y - 4y\frac{dy}{dx}$ and uses the three $\left(\frac{dy}{dx}\right)$ terms.

Part (b) (ii) **10 (5,5) marks** **Att (2,2)**

$\frac{dy}{d\theta}, \frac{dx}{d\theta}$ **5 marks** **Att 2**

$\frac{dy}{dx}$ **5 marks** **Att 2**

	$y = 2(\theta - \sin\theta)$	$x = 2(1 - \cos\theta)$	
	$\frac{dy}{d\theta} = 2(1 - \cos\theta)$	$\frac{dx}{d\theta} = 2(\sin\theta)$	**5m, A2**
	$\frac{dy}{dx} = \frac{2(1-\cos\theta)}{2(\sin\theta)}$		
$\theta = \frac{\pi}{2},$	$\frac{dy}{dx} = \frac{2\left(1-\cos\frac{\pi}{2}\right)}{2\left(\sin\frac{\pi}{2}\right)} = \frac{2(1)}{2} = 1$		**5m, A2**

Blunders (–3)
B1: Differentiation.
B2: Error in getting $\frac{dy}{dx}$.

Slips (–1)
S1: Numerical.

Attempts (2)
A1: Error in differentiation formula.

Part (c)	15(5,5,5) marks	Att (2,2,2)
First Derivative	**5 marks**	**Att 2**
Second Derivative	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

(i) $f(x)=\frac{1}{2}\left(e^{x}+e^{-x}\right)$

$f'(x)=\frac{1}{2}\left(e^{x}+e^{-x}(-1)\right)$

$=\frac{1}{2}\left(e^{x}-e^{-x}\right)$ **5m, A2**

$f''(x)=\frac{1}{2}\left(e^{x}-e^{-x}(-1)\right)$

$=\frac{1}{2}\left(e^{x}+e^{-x}\right)$

$\Rightarrow f''(x)=f(x)$ **5m, A2**

(ii) $f'(x)=\frac{1}{2}\left(e^{x}-e^{-x}\right)$

$f'(2x)=\frac{1}{2}\left(e^{2x}-e^{-2x}\right)$

$$\frac{f'(2x)}{f'(x)}=\frac{\frac{1}{2}\left(e^{2x}-e^{-2x}\right)}{\frac{1}{2}\left(e^{x}-e^{-x}\right)}$$

$$=\frac{\left(e^{x}-e^{-x}\right)\left(e^{x}+e^{-x}\right)}{\left(e^{x}-e^{-x}\right)}$$

$=e^{x}+e^{-x}$

$=2\left[\frac{1}{2}\left(e^{x}+e^{-x}\right)\right]$

$=2f(x)$ **5m, A2**

Note:
Oversimplified differentiation in first 5 marks leads to Att2 at most in second and third 5 marks.

Blunders (–3)
B1: Differentiation.
B2: Indices.

Slips (–1)
S1: Numerical.

Worthless (0)
W1: No differentiation.
W2: Integration.

QUESTION 9

Part (a)	**10 marks**	**Att (3)**
Part (b)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**
Part (c)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**

Part (a) **10 marks** **Att (3)**

$$\int = 2\int \sin 2x dx - \int dx = 2\left(-\frac{1}{2}\right)\cos 2x - x + C$$

$$= -\cos 2x - x + C$$

10m, A3

<u>Blunders (–3)</u>

B1: Integration.

B2: '*C*' omitted.

<u>Attempts (2)</u>

A1: Only '*C*' correct.

A2: One integral correct.

<u>Worthless (0)</u>

W1: Differentiation for integration.

Part (b) (i)	10 (5,5) marks	Att (2,2)
Integration	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

$\int_2^4 \frac{dx}{\sqrt{4^2 - x^2}} = \sin^{-1} \frac{x}{4}\Big\vert_2^4$	**5m, A2**
$= \sin^{-1} 1 - \sin^{-1} \frac{1}{2}$	
$= \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$	**5m, A2**

Blunders (–3)

B1: Integration.
B2: Limits.
B3: '$\sin^{-1}$' function.
B4: Incorrect order in applying limits.

Slips (–1)

S1: Numerical.
S2: Answer not tidied up.

Attempts (2)

A1: Any correct step shown.

Worthless (0)

W1: Differentiation instead of integration.

Part (b) (ii) **10 (5,5) marks** **Att (2,2)**

Integration **5 marks** **Att 2**
Value **5 marks** **Att 2**

$\int =4\int xe^{x^2}dx+\int dx$	Let $u=x^2$	$x=0, u=0$	
$\int_0^1 =2\int_0^1 e^u du + x\Big\|_0^1$	$du=2xdx$	$x=1, u=1$	
	$2du=4xdx$		
			5m, A2
$=2e^u\Big\|_0^1+1$			
$=2e-2e^0+1$			
$=2e-2+1$			
$=2e-1$			**5m, A2**

* Incorrect substitution and unable to finish yields attempt at most.

<u>Blunders (–3)</u>
B1: Integration.
B2: Differentiation.
B3: Limits.
B4: Incorrect order in applying limits.
B5: Not calculating substituted limits.
B6: Not changing limits.
B7: Indices.

<u>Slips (–1)</u>
S1: Numerical.
S2: Answer not tidied up.

<u>Worthless (0)</u>
W1: Differentiation instead of integration except where other work merits attempts.

Part (c)	**20 (5,5,5,5) marks**	**Att (2,2,2,2)**
Points of intersection	**5 marks**	**Att 2**
First Integrand	**5 marks**	**Att 2**
Second Integrand	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

(i) $f(x) \cap g(x) \Rightarrow x^2 - 4x + 5 = 5 + 4x - x^2$

$$2x^2 - 8x = 0$$
$$x^2 - 4x = 0$$
$$x = 0\,,\,4$$
5m, A2

(ii) Area $= \int_0^4 \left[g(x) - f(x) \right] dx$ **5m, A2**

$$= \int_0^4 \left[5 + 4x - x^2 - x^2 + 4x - 5 \right] dx$$

$$= \int_0^4 \left(8x - 2x^2 \right) dx$$ **5m, A2**

$$= \frac{8x^2}{2} - \frac{2x^3}{3} \Big|_0^4$$

$$= 64 - \frac{128}{3}$$

$$= \frac{192 - 128}{3} = \frac{64}{3} \text{ units}^2$$ **5m, A2**

Blunders (–3)
B1: Integration.
B2: Indices.
B3: Error in calculating points of intersection.
B4: Error in area 'formula'.
B5: Incorrect order in applying limits.
B6: Uses $\pi \int y dx$ for area formula.

Attempts (2)
A1: Uses volume 'formula'.
A2: Uses y^2 in 'formula'.

Slips (–1)
S1: Numerical.

Worthless (0)
W1: Differentiation instead of integration except where other work merits attempt.
W2: Wrong area 'formula' and no work.

SOLUTIONS TO PAPER 2

QUESTION 1

Part (a)	**5C marks**
Part (b)	**10D marks**
Part (c)	**10D marks**

(a) About 2% of people possess a gene that is capable of fighting off a certain disease. If 25 people are selected at random find the probability that less than 2 people will possess the gene.

Posses Gene, $p = 0.02 \quad \therefore q = 0.98$

P (Less than 2 people) = P (none) + P (1 person) **3 marks**

P (Less than 2 people) $= \binom{25}{0}(0.02)^0(0.98)^{25} + \binom{25}{1}(0.02)^1(0.98)^{24}$ **7 marks**

P (Less than 2 people) $= 0.911 = 91.1\%$ **10 marks**

(b) Explain *Quota Sampling*. Give one advantage and one disadvantage of this type of sampling.

In **quota sampling**, the population is first segmented into sub-groups. Then judgment is used to select the subjects or units from each segment based on a specified proportion. For example, an interviewer may be told to sample 200 females and 300 males between the age of 45 and 60.
3 Marks

Advantage: This means that individuals can put a demand on who they want to sample (targeting)
7 Marks

Disadvantage: The selection of the sample is non-random unlike random sampling and can often be found unreliable. **10 Marks**

(c) Samples of size 10 are taken from a production line producing jars of drinking chocolate. The target weight of the jars is 200g. The means of the samples are normally distributed with a standard deviation of 2g. Calculate the limits between which you would expect 95% of the sample means to lie.

$95\% = \pm 2$ Standard Deviations

Limits $= 200g \pm 4g$ **2 marks**

Lie between $196g$ and $204g$ **5 marks**

QUESTION 2

Part (a)	**5A marks**
Part (b)	**15B marks**
Part (c)	**5A marks**

(a) Explain the significance of the *correlation coefficient* in statistics.

The correlation coefficient is a measure of the strength of the linear relationship between two variables.
Or similar **5 Marks**

(b) Draw a scatter diagram that shows a strong positive correlation.

Any set of correct points to show a strong positive correlation with a least 6 points.
6 Points **7 Marks**
Correct Graph **15 Marks**

(c) Calculate the correlation coefficient for the following set of bivariate data. Comment on the meaning of the coefficient in relation to the data.

Height (cm)	165	166	176	180	183	188
Foot Size (cm)	26	27	28	35	28	25

Calculator $= 0.144093616$ **5 marks**

QUESTION 3

Part (a)	**5C marks**
Part (b)	**10C marks**
Part (c)	**10D marks**

(a) The distance from $A(3,3)$ to $B(12,k)$ is $3\sqrt{10}$. Find two possible values for *k*.

$|AB| = \sqrt{(12-3)^2 + (k-3)^2}$

$\left(\sqrt{(12-3)^2 + (k-3)^2}\right)^2 = \left(3\sqrt{10}\right)^2$ **2 marks**

$81 + k^2 - 6k + 9 = 90$

$k^2 - 6k = 0$ **4 marks**

$k(k-6) = 0$ $\quad\therefore k = 0$ and $k = 6$ **5 marks**

(b) Find the measure of the acute angles between the lines $l: 5x + 4y = 13$ and $k: -2x + 3y - 4 = 0$.

$\tan\theta = \pm\dfrac{m_1 - m_2}{1 + m_1 m_2}$

$m_1 = -\dfrac{5}{4}, \quad m_2 = \dfrac{2}{3}$ **3 marks**

$\tan\theta = \pm\dfrac{-\frac{5}{4} - \frac{2}{3}}{1 + \left(-\frac{5}{4}\right)\left(\frac{2}{3}\right)} = \pm(-11.5)$ **7 marks**

$\tan^{-1}(11.5) = 85°$ **10 marks**

(c) Find the area enclosed of the triangle enclosed by the lines *l* and *k* and the y-axis.

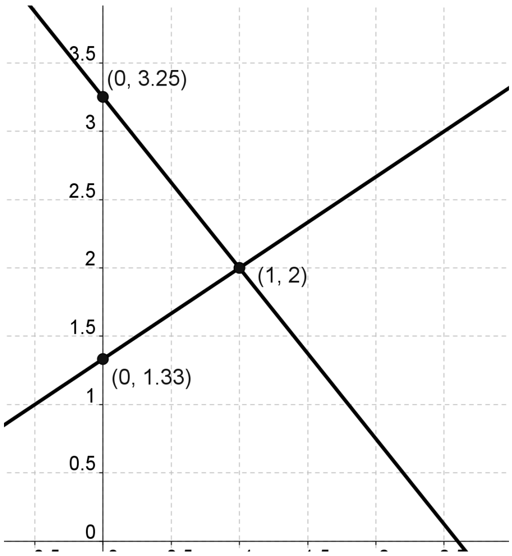
l: $5x + 4y = 13$ cuts the y-axis at point $\left(0, \dfrac{13}{4}\right)$

k: $-2x + 3y - 4 = 0$ cuts the y-axis at point $\left(0, \dfrac{2}{3}\right)$ **2 marks**

Lines intersect at the point $(1, 2)$ **5 marks**

$\text{Area} = \dfrac{1}{2} \times b \times \perp h = \dfrac{1}{2} \times \left(\dfrac{13}{4} - \dfrac{2}{3}\right) \times (1)$ **8 marks**

$\text{Area} = \dfrac{31}{24}$ or 1.29 sq.units **10 marks**



QUESTION 4

Part (a)	**5B marks**
Part (b)	**10B marks**
Part (c)	**10D marks**

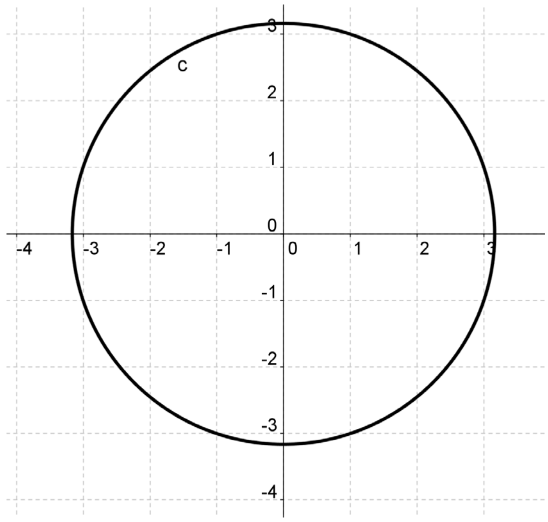
(a) The circle *c* has equation $x^2 + y^2 = 10$. Find three points that lie on circle c.

$(0, \sqrt{10})$ or $(0, -\sqrt{10})$ or $(\sqrt{10}, 0)$ or any three correct points.

One correct point	**2 marks**
Three correct points	**5 marks**

(b) Draw the circle *c*.

Circle with Incorrect radius	**5 Marks**
Circle with correct radius	**10 Marks**



(c) The centre of the circle *s* lies on the line $3x + 4y = 31$. The x-axis and the line $y = 8$ are tangents to the circle *s*. Find the equation of the circle.

Centre lies on line $y = 4$	**2 marks**
$3x + 4(4) = 31 \quad \rightarrow \quad x = 5$	**5 marks**
Center $(5, 4)$, radius = 4	**8 marks**
$(x-5)^2 + (y-4)^2 = 16$	**10 marks**

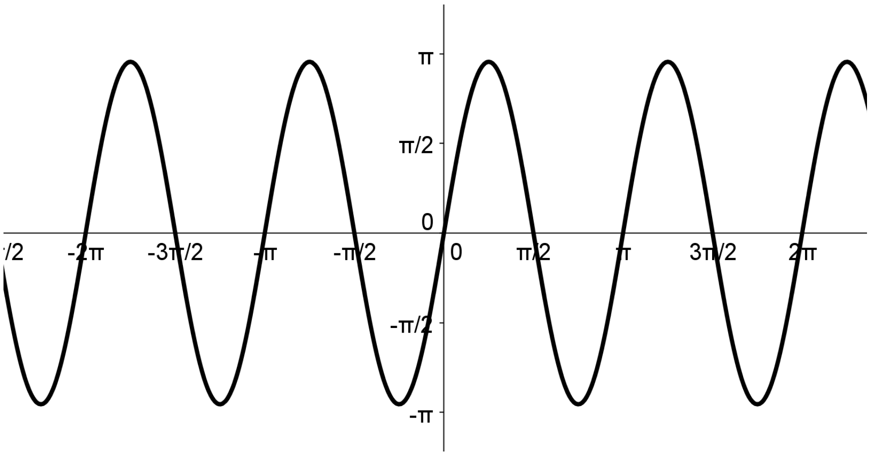
QUESTION 5

Part (a)	**10C marks**
Part (b)	**10 marks**
Part (c)	**5B marks**

(a) Match each function to its corresponding graph.

$g(x) = \sin(2x)$	**3 marks**
$h(x) = \sin(x)$	**7 marks**
$f(x) = 2\sin(x)$	**10 marks**

(b) A body oscillates under a force with a motion described by the function $3\sin(2\theta)$. Draw a graph of the function in the domain $-2\pi \le \theta \le 2\pi$.



Correct graph Apply blunders and slips as normal.	**10 marks**

(c) State in radians the period of the function. State the range of the function.

Period $= \pi$	**2 marks**
Range $= [-\pi, \pi]$	**5 marks**

QUESTION 6A

Part (a)	**5B marks**
Part (b)	**5B marks**
Part (c)	**15 marks**

(a) What is the *incentre* of a triangle?

The significance of the incentre is a point where the radius must be drawn from to have the biggest possible circle which touches all of the sides of the triangle. The incentre always remains inside the triangle as the name suggests because the circle it is the centre of must be located inside the triangle.

Attempt	**2 marks**
Correct Explanation	**5 marks**

(b) What is the *incircle* of a triangle?

The **incircle** or **inscribed circle** of a triangle is the largest circle contained in the triangle; it touches (is tangent to) the three sides

Attempt	**2 marks**
Correct Explanation	**5 marks**

(c) Construct the *incentre* of the following triangle.

Bisect 2 angles	**4 marks**
Marks Centre	**7 marks**
Perpendicular to one side	**11 marks**
Circle	**15 marks**

QUESTION 6B

Part (a)	**5A marks**
Part (b)	**5A marks**
Part (c)	**15 marks**

(a) Explain what is meant by the term *corollary*.

A **corollary** is a statement that follows readily from a previous statement. In mathematics a corollary typically follows a theorem. **5 marks**

(b) Give an example of one *corollary* you have encountered in your course.

Any corollary on course stated or explained **5 marks**

(c) Prove that if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.

Correct Construction	**4 marks**
Correct Statement	**7 marks**
Correct Statement	**11 marks**
Correct Statement	**15 marks**

QUESTION 7

Part (a)	
Part (i)	**5B marks**
Part (ii)	**10C marks**
Part (b)	
Part (i)	**10B marks**
Part (ii)	**10A marks**
Part (iii)	**10A marks**
Part (c)	
Part (i)	**5A marks**
Part (ii)	**5A marks**
Part (iii)	**5B marks**
Part (iv)	**15D marks**

(a) A football manufacturing company tests its product by dropping it from a given height and measuring the rebound height. Balls that rebound less than 130cm are rejected. Assume that the rebound height can be modelled by a normal distribution with a mean of 134cm and a standard deviation of 3cm.

(i) Construct a 95% confidence integral for this test.

95% between $\left(\bar{x} \pm 2\sigma\right)$	**2 marks**
$134 \pm 2(3) = 128cm$ and $140cm$	**5 marks**

(ii) 10,000 balls are tested in a single day. How many balls will have a rebound height between 125cm and 143cm?

$134 - 125 = 9cm = 3\sigma$	**3 marks**
$143 - 134 = 9cm = 3\sigma$	**7 marks**
99% lie with $+3\sigma \therefore$ $\quad 10000 \times 99\% = 9900$ balls	**10 marks**

(b) A study was carried out to investigate the relationship between the length of main roads (in 1,000000km) and the number of road accidents (in 100000) for a number of industrialised countries. The results were placed on the following scatter graph.

(i) Describe the correlation of the figures.

Weak	**5 marks**
Negative correlation	**10 marks**

(ii) Assign a value to the correlation coefficient for these figures.

-0.1 to -0.2	**10 marks**

(iii) Do you think the figures show that the more main roads in a country the more accidents will occur? Explain your answer fully.

These figures do not support the idea that more main roads will lead to more accidents. **10 marks**

(c) A poll carried out by a national newspaper early in the year indicated that 39% of voters would support the current government in a general election. A recent poll, taken by 1024 people, showed that only 252 people will support the current coalition government in the upcoming elections in 2011. Has support for the current government changed?

(i) State clearly the null hypothesis.

$H_0 =$ Support for the government has not changed **5 marks**

(ii) State clearly the alternative hypothesis.

$H_1 =$ Support for the government has changed **5 marks**

(iii) Calculate the margin of error.

Correct substitution **2 marks**

$$E = \frac{1}{\sqrt{1024}} = 0.03125$$ **5 marks**

(iv) Use the above information to either accept or reject the null hypothesis.

$$\hat{p} = \frac{252}{1024} = 0.246$$ **4 marks**

Confidence Interval $(CI) = (0.246 - 0.03125) < p < (0.246 + 0.03125)$ **7 marks**

$CI = 0.21475 < p < 0.27729$ **11 marks**

The population proportion 39% does not lie within the confidence interval so reject the null Hypothesis and accept the alternative that support for the government has changed.

Statement **15 marks**

QUESTION 8

Part (a)		
	Part (i)	**5B marks**
	Part (ii)	**10C marks**
	Part (iii)	**10C marks**
Part (b)		
	Part (i)	**5A marks**
	Part (ii)	**5A marks**
	Part (iii)	**15D marks**

(a) The Luxor Hotel in Las Vegas is modelled after the Great Pyramids of Egypt and boasts many world record attributes.
To begin the building's square base is 196.9m wide and is a total height of 106.7m to its tip.
Its spot light is claimed to be the brightest beam in the world at over 42 billion candle power.
It is visible from anywhere in the Las Vegas valley at night, and can be seen at flight level from above Los Angeles, California, over 275 miles (443 km) away

(i) Draw a diagram to show the measurements in the above article.

Correct diagram	**10 marks**

(ii) Calculate the length of the slant side of the Luxor correct to one decimal place.

Centre of base to corner $=\sqrt{(196.9)^2+(196.9)^2}=278.5m \div 2 = 139.3m$	**7 marks**
Slant Side $=\sqrt{(106.7)^2+(139.3)^2}=175.5m$	**15 marks**

(iii) Calculate the area of the four sides of the Luxor correct to the nearest square metre.

Angle at top: $(196.9)^2=(175.5)^2+(175.5)^2-2(175.5)(175.5)\cos\theta$	**7 marks**
$\theta = 68°$	**13 marks**
Area of 4 sides $=4\times\frac{1}{2}(175.5)(175.5)\sin 68 = 57{,}115m^2$	**20 marks**

(b) A bridge is being constructed of total length 40m and height 12m.

(i) Calculate the length of one of the slanted steel components.

$l^2 = 12^2 + (40 \div 8)^2 = 169$	**5 marks**
$l = 13m$	**10 marks**

(ii) Calculate the angle the slanted girders make with the ground.

$\sin\theta = \frac{12}{13}$	**5 marks**
$\theta = \sin^{-1}\frac{12}{13} = 67.38°$	**10 marks**

(iii) Calculate the total length of steel required to make this bridge.

$(40\times2)+(30\times2)+(16\times13)+(14\times12)=516m$

2 marks **5 marks** **8 marks** **10 marks**