PM50

Pre-Leaving Certificate Examination, 2012

Triailscrúdú na hArdteistiméireachta, 2012

Mathematics
(Project Maths – Phase 3)

Paper 1

Higher Level

2½ hours

300 marks

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) If $p(2-4i)-q(-5-3i)=3(12+2i)$, find the value of $\boldsymbol{p}$ and $\boldsymbol{q}$ if $p,q\in\mathbb{Z}$.

(b) Express $z = -1 + \sqrt{3}$ in polar form and hence find the value of z^7 in rectangular form.

Question 2 **(25 marks)**

(a) If $S_1 = 2$ and $S_3 = 12$ for an arithmetic sequence, find the value of ***a***, the first term of the sequence and ***d***, the common difference.

(b) Prove that the sum of the first ***n*** even numbers is $n^2 + n$.

(c) Find the sum of the even numbers from 101 to 200 inclusive.

(d) Prove by induction that $3^n + 1$ is divisible by 2, for all $n \in \mathbb{N}$.

Question 3 **(25 marks)**

The function $f : x \to x^3 + ax^2 + bx + c$ crosses the x-axis at $x = 1$ and $x = 4$ where $a, b, c \in \mathbb{Z}$.

(a) Give three possible values for ***c***, explaining your choices fully.

(b) The general term of a sequence is given by $T_n = an^2 + bn + c$. If the first four terms are 1, 10, 23, 40 find the values of ***a***, ***b*** and ***c***.

Question 4 **(25 marks)**

(a) Solve the simultaneous equations:

$$3x + 2y = 1$$
$$x^2 + 2xy + 15 = 0$$

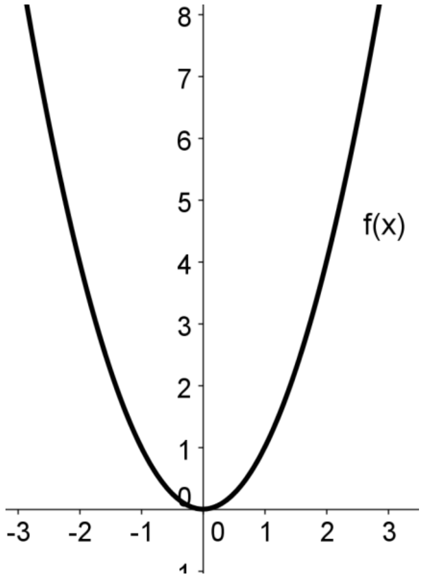
(b) Draw a sketch of the function $(x+2)(x-1)^2(x+2)$.

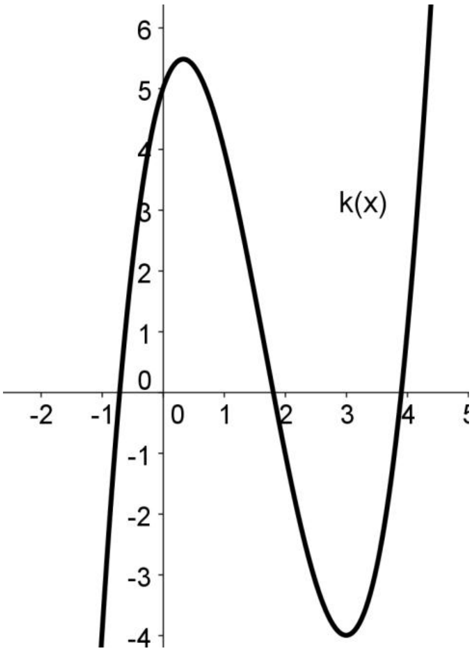
(c) Estimate the values of ***x*** where the slopes of the tangents to the graph would be equal to zero.

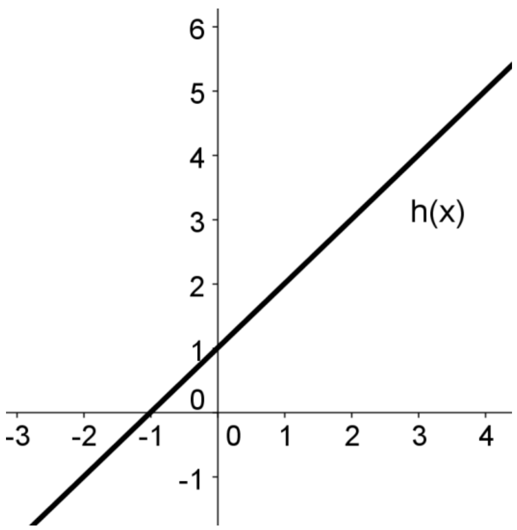
Question 5 **(25 marks)**

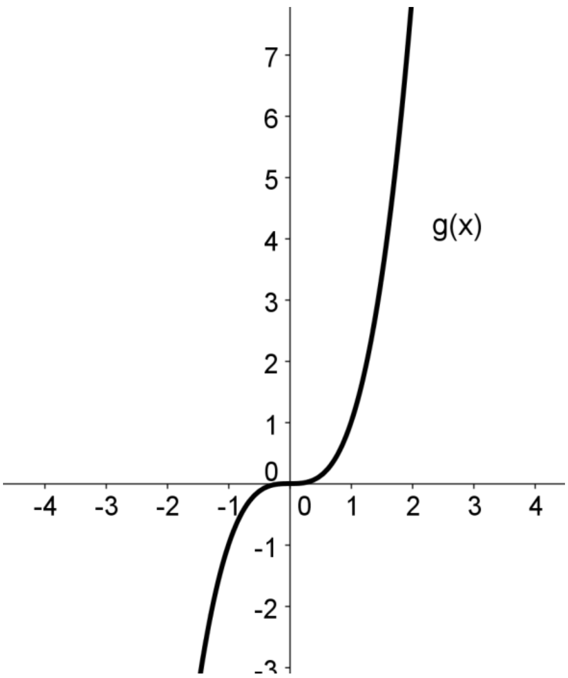
(a) Explain the terms *injective* and *surjective*.

(b) Examine the following functions and state whether each function is injective or surjective.









$f(x)$ ______________________

$g(x)$ ______________________

$h(x)$ ______________________

$k(x)$ ______________________

Question 6 **(25 marks)**

(a) Explain the difference between a definite integral and an indefinite integral.

(b) Give an example of a definite integral and an indefinite integral.

(c) Find $\int_1^4 \ln x$.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

The profits of a company, in 000's can be modelled by the function, $P(m) = -m^3 + 5m^2 + 6m$ where $\boldsymbol{m}$ is the amount spent on advertising.

(a) Draw a suitable graph of the function to show the profit of the company.

(b) From your graph estimate the maximum profit of the company.

(c) Verify your answer from part (b) using differential calculus.

(d) A sample of radioactive material decay can be modelled by the function

$$D(t) = ae^{bt}$$

where ***a*** and ***b*** are constants and ***t*** is the time passed in weeks.
50 g of material is purchased and in 5 days it has decayed to 25 g.
Calculate the value of ***a*** and ***b***.

Question 8 **(50 marks)**

Financial institutions use the formula:

$$B = P(1+i)^n - \frac{m\left[(1+i)^n - 1\right]}{i}$$

to calculate the balance ***B***, on a borrowed principal, ***P***, at interest rate, ***i*** , for term ***n*** after ***m*** equal monthly repayments.

(a) If the balance on the loan ***B*** = 0 after the $\boldsymbol{n}^{\text{th}}$ payment, show that the monthly payment can be written as:

$$m = \frac{iP}{1-(1+i)^{-n}}$$

(b) John borrows €15,000 at a monthly rate of 0.65% for 36 months. Calculate his monthly repayment.

(c) By how much would John's monthly repayment decrease if he borrows the money at the same rate but over a 5 year period?

(d) John decides to borrow the money over a three year period. Calculate the balance of the loan as a percentage of the original borrowing after the 24th payment is made.

(e) A bank offers a short term loan at a rate of 14.25% APR. Calculate the equivalent rate if compounded monthly.

Question 9 **(50 marks)**

(a) Show that the $f(x) = \dfrac{-2}{x+3}$ has no turning points.

(b) Find the value $f'(0.5)$.

(c) A cylinder has a height h which is 5 times its radius r. Express the volume of the cylinder in terms of ***r***.

(d) Express the rate of change in the volume of the cylinder in terms of ***r***.

(e) Determine the equation of the tangent to the curve $f\left(x\right)=x^3+3x^2-10x-24$ at the point (0, -24).