P 6

Pre-Leaving Certificate Examination, 2013

Triailscrúdú na hArdteistiméireachta, 2013

Mathematics
(Project Maths – Phase 2)

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Instructions

There are **three** sections in this examination paper:

Section A	Concepts and Skills	100 marks	4 questions
Section B	Contexts and Applications	100 marks	2 questions
Section C	Functions and Calculus (old syllabus)	100 marks	3 questions

Answer all eight questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

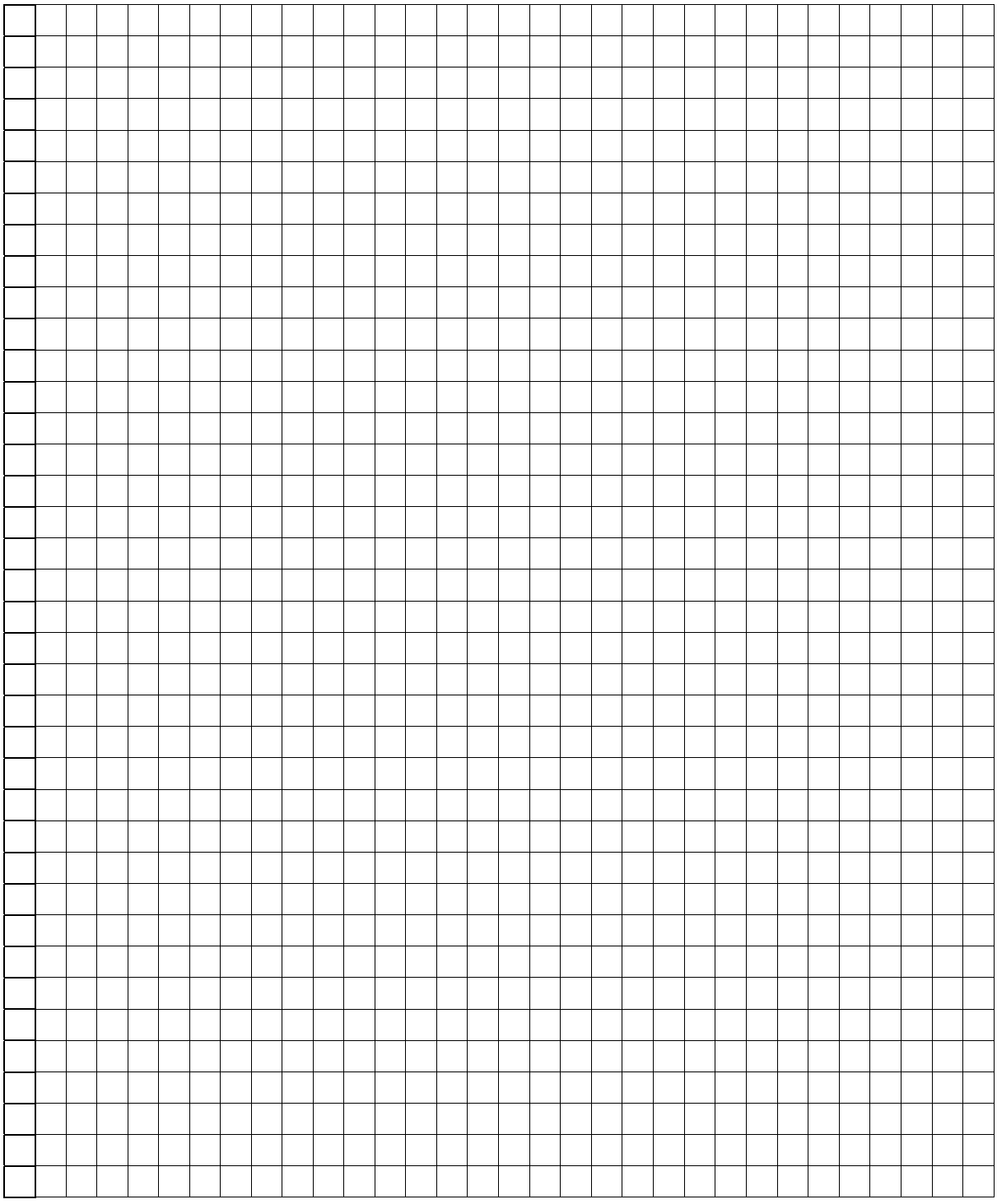
Answers should be given in simplest form, where relevant.

Section A — Concepts and Skills — 100 marks

Answer **all four** questions from this section.

Question 1 **(25 marks)**

(a) Solve: $2x+3y+z=2$
$x-y+z=-4$
$x-2y-2z=2$



(b) Let $f(x) = 2x^3 + ax^2 - 17x + b$ where a and b are constants.

Given that $(x-1)$ and $(x+4)$ are factors of $f(x)$, find the value of a and the value of b.

(c) Simplify: $\dfrac{x^2-x-2}{x^2-3x} \div \dfrac{x^2+5x+4}{x^2-x-6}$

Question 2 **(25 marks)**

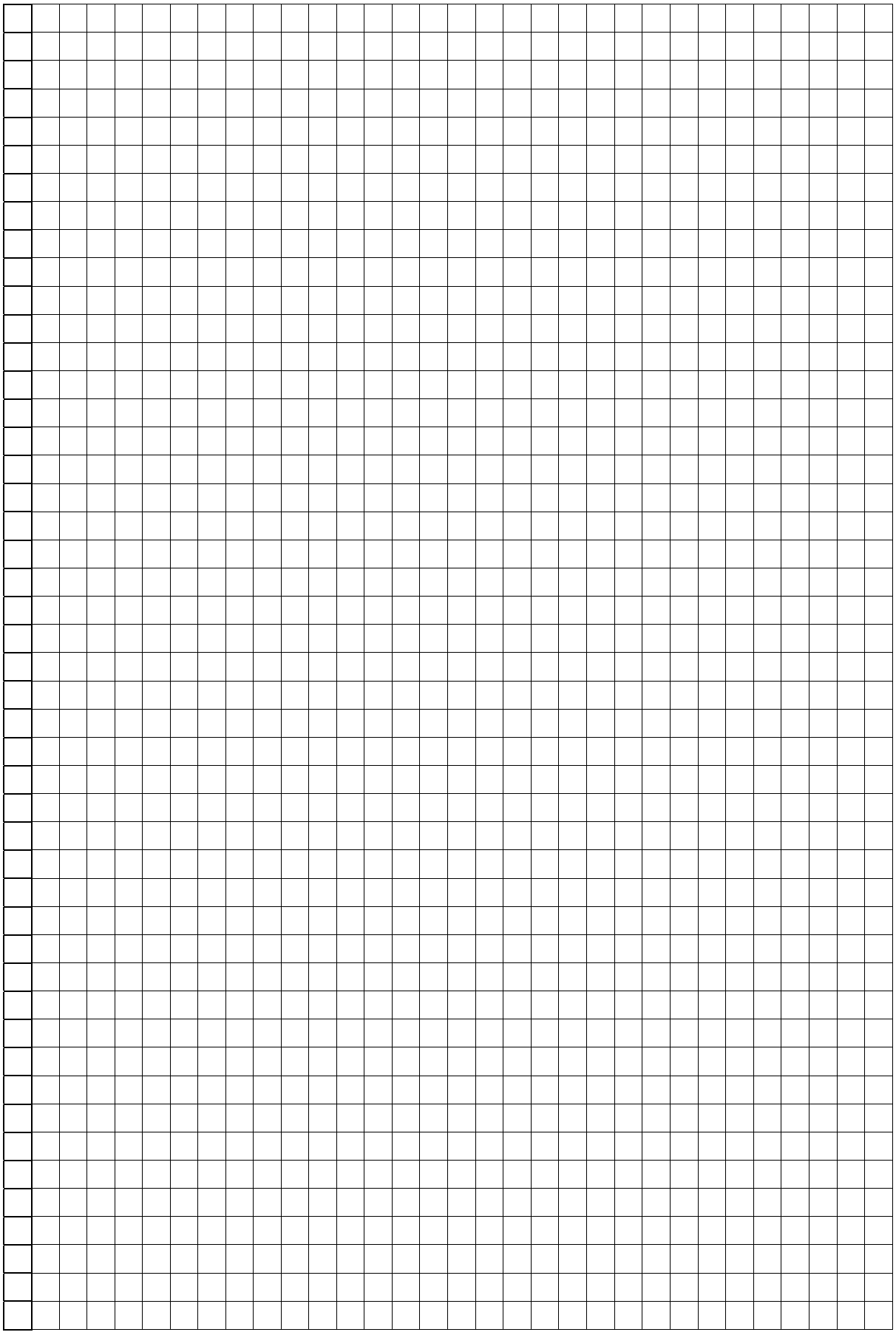
(a) $4x-4$, $x+4$ and $2x-7$ are three consecutive terms in a sequence where $x \in \mathbb{Z}$. Investigate if the sequence is arithmetic or geometric. Explain your answer fully.

(b) If $x^2 + 7x + 12$ is a factor of $x^3 + px^2 - 2x + r$, find the value of p and r.

(c) Solve the following inequality: $2x^2 + 7x - 4 \leq 0$

Question 3 **(25 marks)**

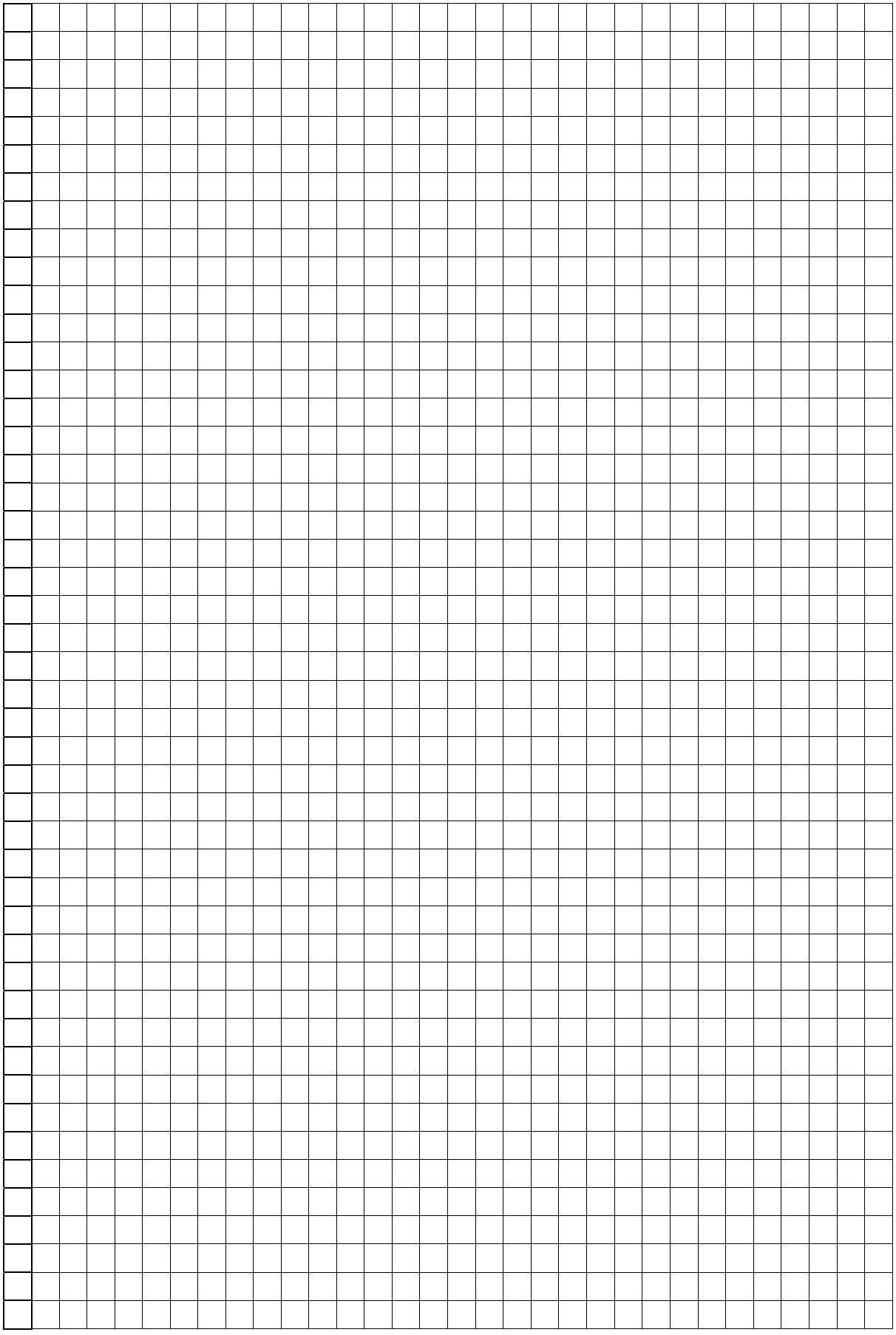
(a) Given that $z = a + bi$, where $a, b \in \mathbb{R}$, find the values of z if $z\bar{z} - 4i\bar{z} = 4 - 8i$.



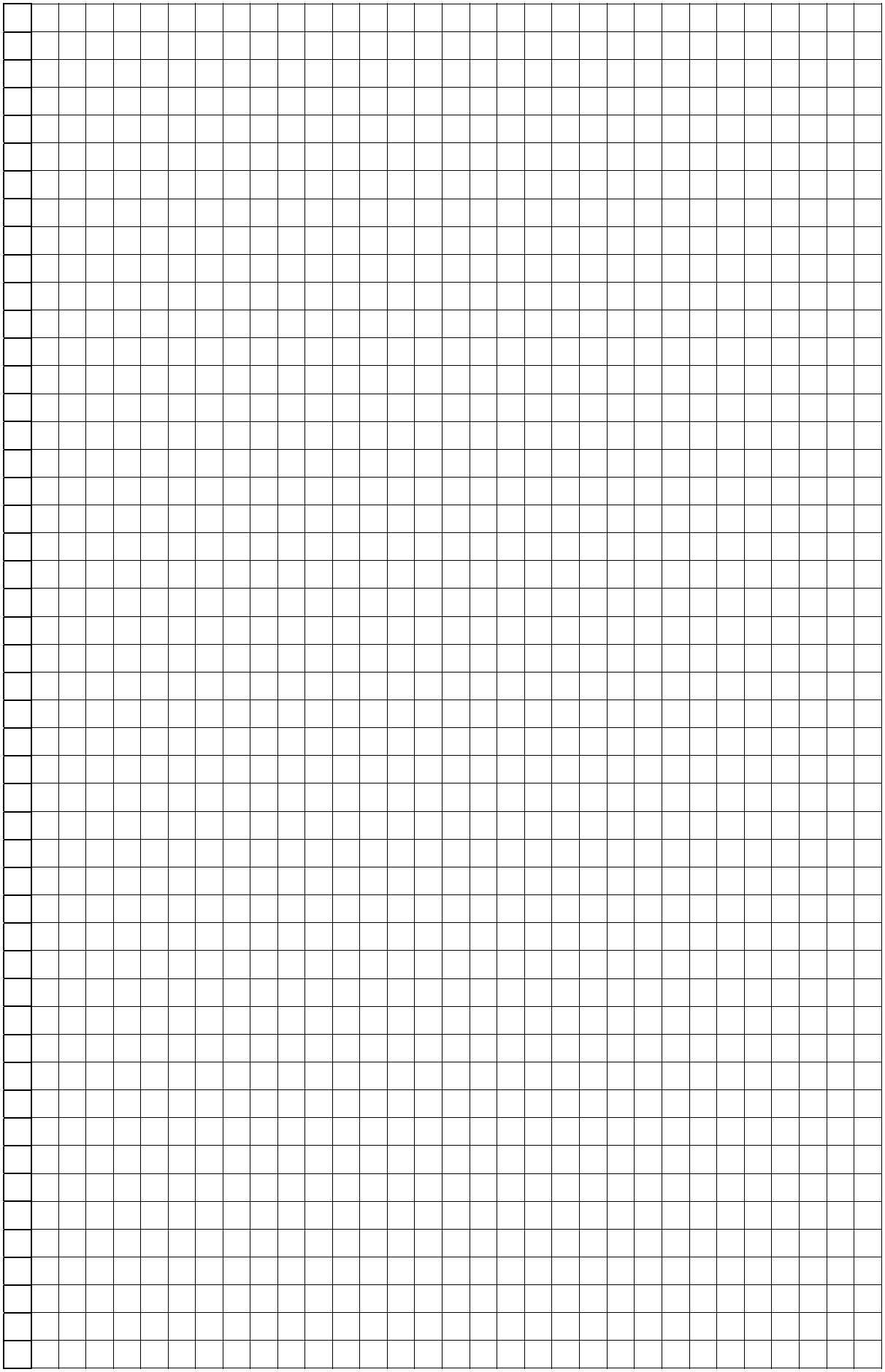
(b) Use De Moivre's Theorem to solve the equation $z^3 = 1$.

Question 4 **(25 marks)**

(a) If $T_{n+1} = (T_n)^2 - 3n$, find T_5, given that $T_1 = 1$.



(b) Prove, by induction, that for any natural number $n^3 + 2n$ is divisible by 3.

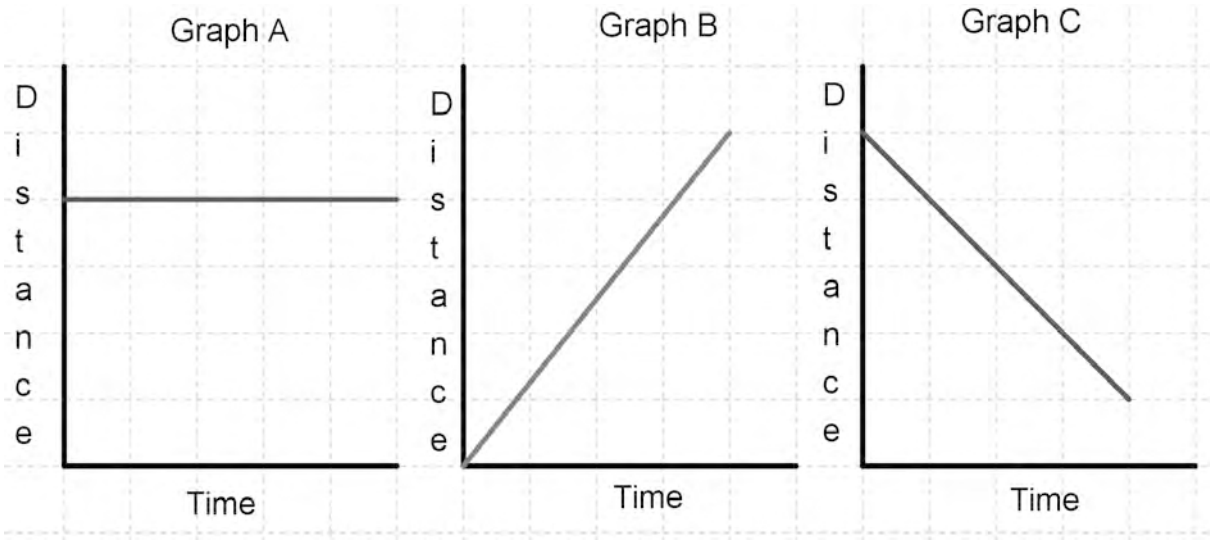


Section B	Contexts and Applications	100 marks

Answer **both** Question 5 **and** Question 6.

Question 5 **(50 marks)**

(a) The following graphs show the distance a cyclist is from a marker in a race. Examine the graphs and describe the motion of the cyclist relative to the marker for each.



Graph A

Graph B

Graph C

(b) The formula $g = \frac{GM}{d^2}$ can be used to determine the gravitational force on an object where $G = 6.7 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}$, M is the mass of the object, and d is the distance from the object to the centre of the earth.

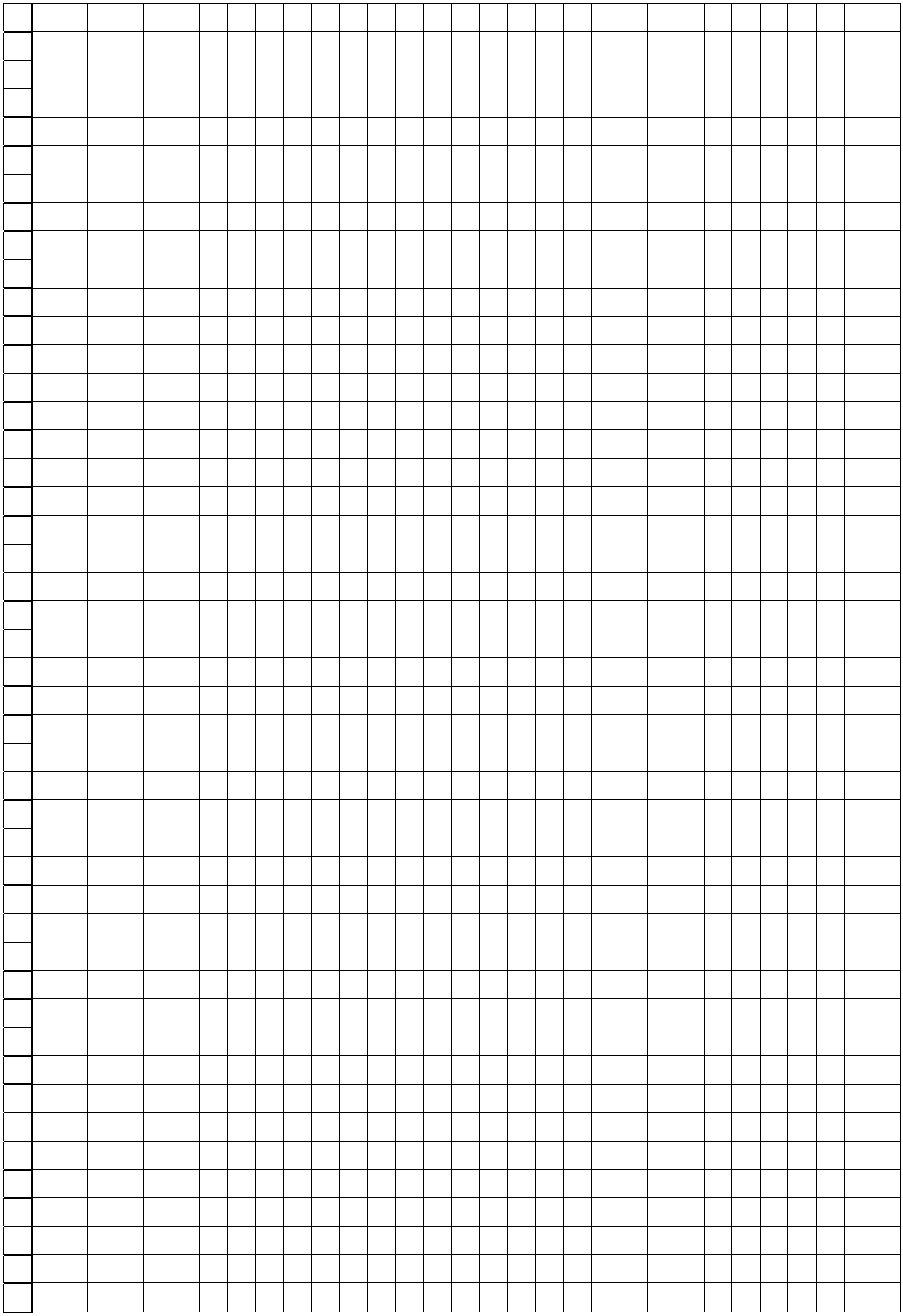
(i) Given that on the surface of the earth $g = 9.81\ \text{ms}^{-2}$, calculate the radius of the earth correct to two significant figures.

(ii) Find the acceleration due to gravity 500 km above the earth's surface correct to two decimal places.

(iii) The distance between the centre of the earth and the centre of the moon is 3.8×10^{8} m and the mass of the moon is 7.41×10^{22} kg. At what point between the earth and the moon is the force of gravity on an object zero?

Question 6 **(50 marks)**

(a) If you borrow €1,000 for one year at a nominal interest rate of 12% per year compounded quarterly, how much do you owe at the end of the year?



(b) An individual who plans to retire in 20 years' time has decided to put an amount A, in the bank at the beginning of each month for the next 20 years at an interest rate r. After this time the individual plans to withdraw €1,000 at the beginning of each month for 30 years.

(i) Show that the present value of the deposits can be expressed in the form

$$A\left(\frac{1\left(1-\left(\frac{1}{1+r}\right)^{240}\right)}{1-\left(\frac{1}{1+r}\right)}\right)$$

(ii) If W is the amount withdrawn per month in the following 360 months, express the present value of the deposits in terms of W and r.

(iii) Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be to sustain the withdrawals in the future?

Section C Functions and Calculus (old syllabus) 100 marks

Answer **both** Question 7 **and** Question 8.

Question 7 **(50 marks)**

(a) Differentiate $\sqrt{3x^2+5x-2}$ with respect to x.

(b) Differentiate, from first principles, $\sin x$ with respect to x.

(c) The equation $x^3 - 4x + 1 = 0$ has three real roots that lie in the domain $-4 \le x \le 4$.

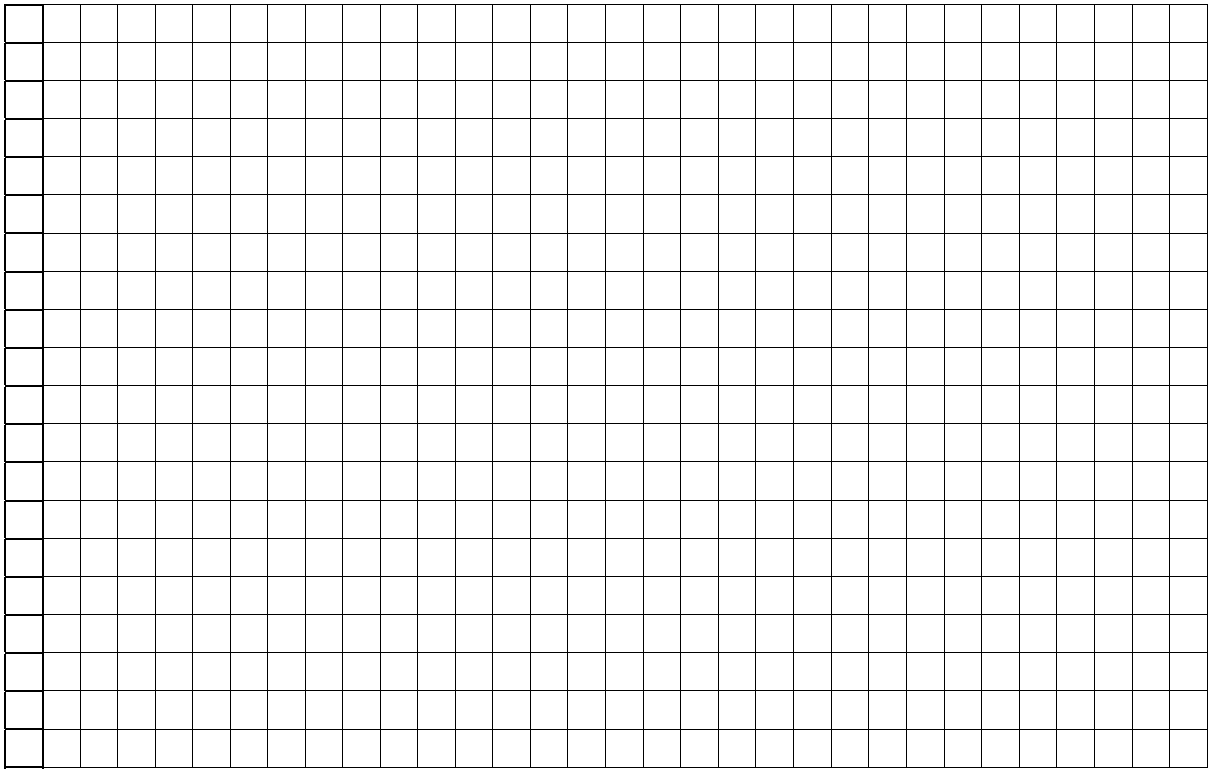
(i) Find two consecutive integers in the domain, between which a root exists.

(ii) Taking one of the integers from part (i) as the first approximation to the real root of the equation $x^3 - 4x + 1 = 0$, use the Newton-Raphson method to find x_2, the second approximation.

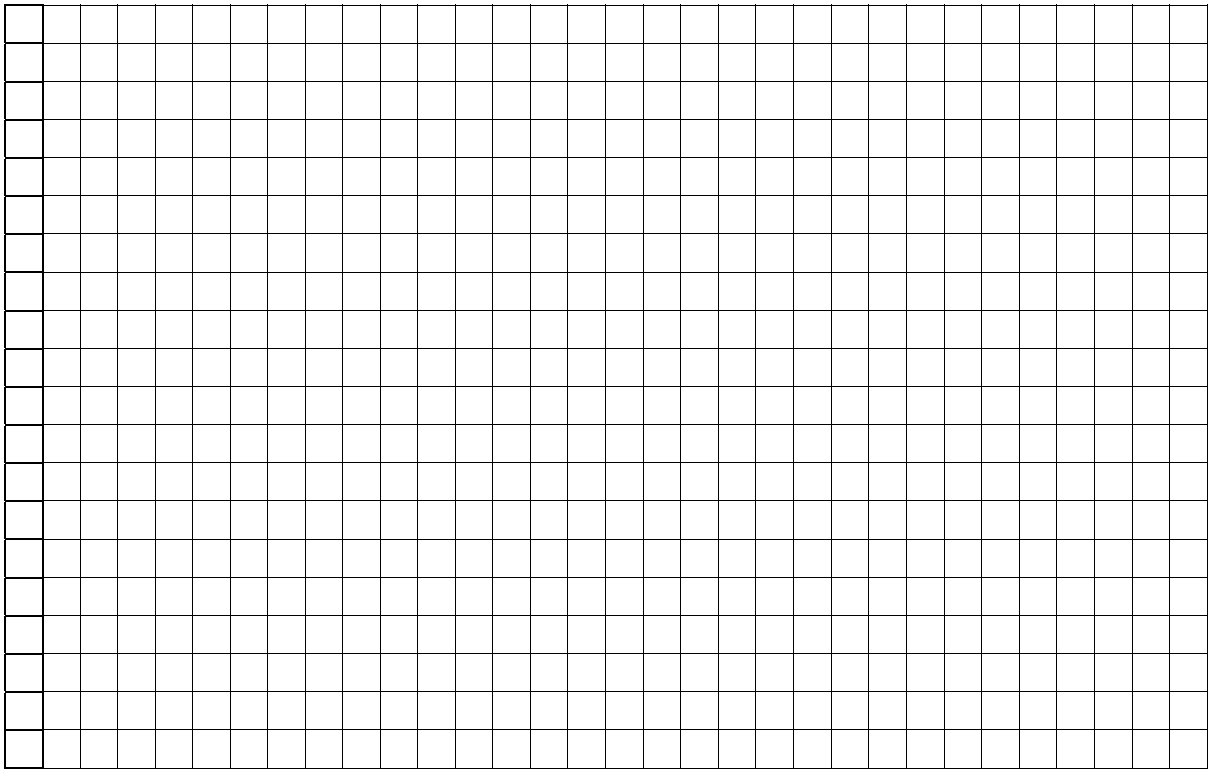
Question 8 **(50 marks)**

(a) Find:

(i) $\int 3x^2 dx$



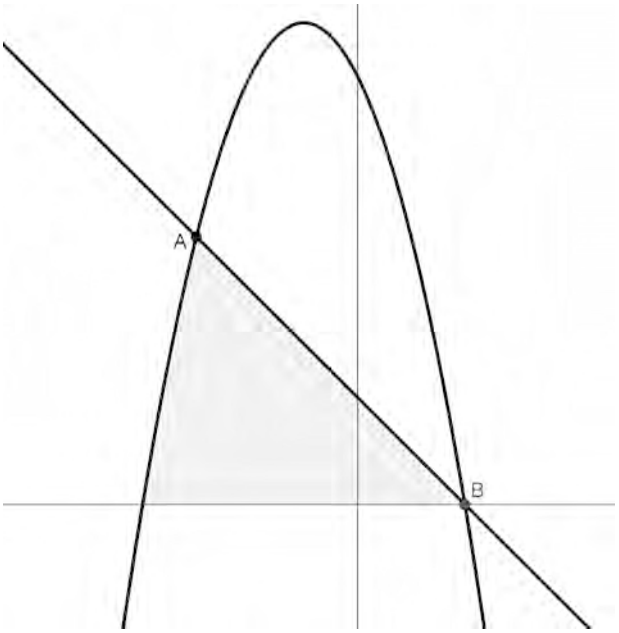
(ii) $\int \frac{x^4 - x^7}{x^3} dx$



(b) **(i)** Evaluate $\int_0^3 x\sqrt{3x^2+7}$.

(ii) Show that $\cos^3 x = \cos x\left(1-\sin^2 x\right)$. Hence, otherwise evaluate $\int_0^{\frac{\pi}{3}} \cos^3 x dx$.

(c) The line $y = 2 - x$ intersects the curve $y = (-x + 2)(x + 4)$ at the points A and B as shown. The shaded region is bounded by the curve, the line and the x-axis. Calculate the area of the shaded region.



P 7

Pre-Leaving Certificate Examination, 2013

Triailscrúdú na hArdteistiméireachta, 2013

Mathematics
(Project Maths – Phase 2)

Paper 2

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions, as follows:

In Section A, answer:

Questions 1 to 5 and

either Question 6A **or** Question 6B.

In Section B, answer Question 7, 8 and 9.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

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Answers should be given in simplest form, where relevant.

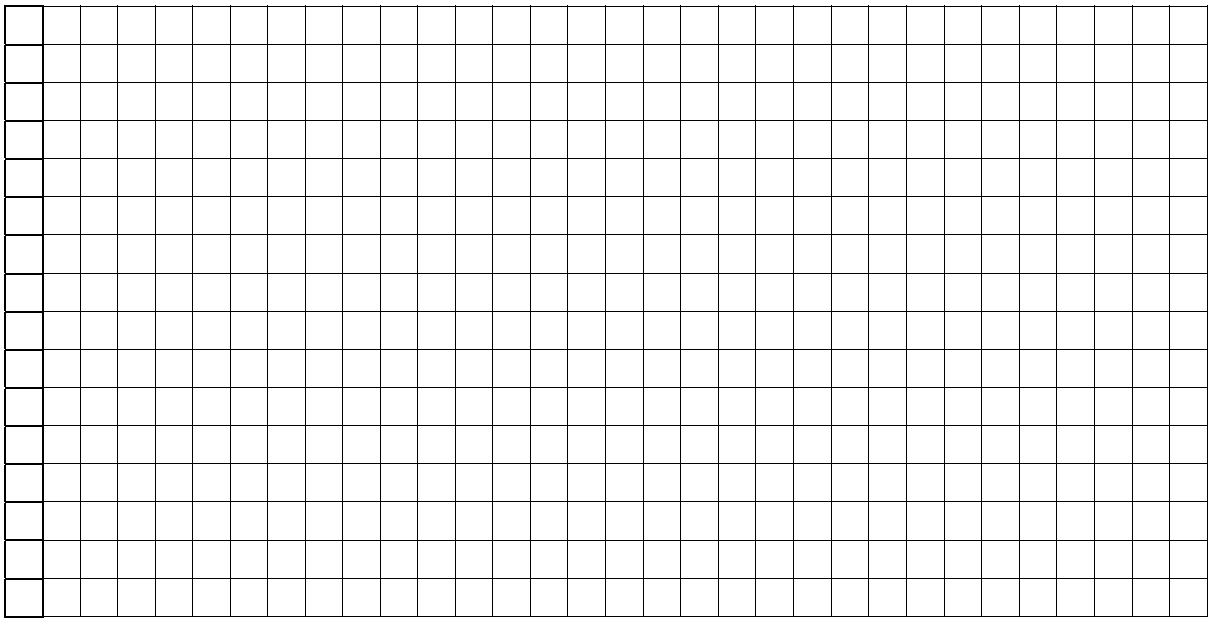
Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

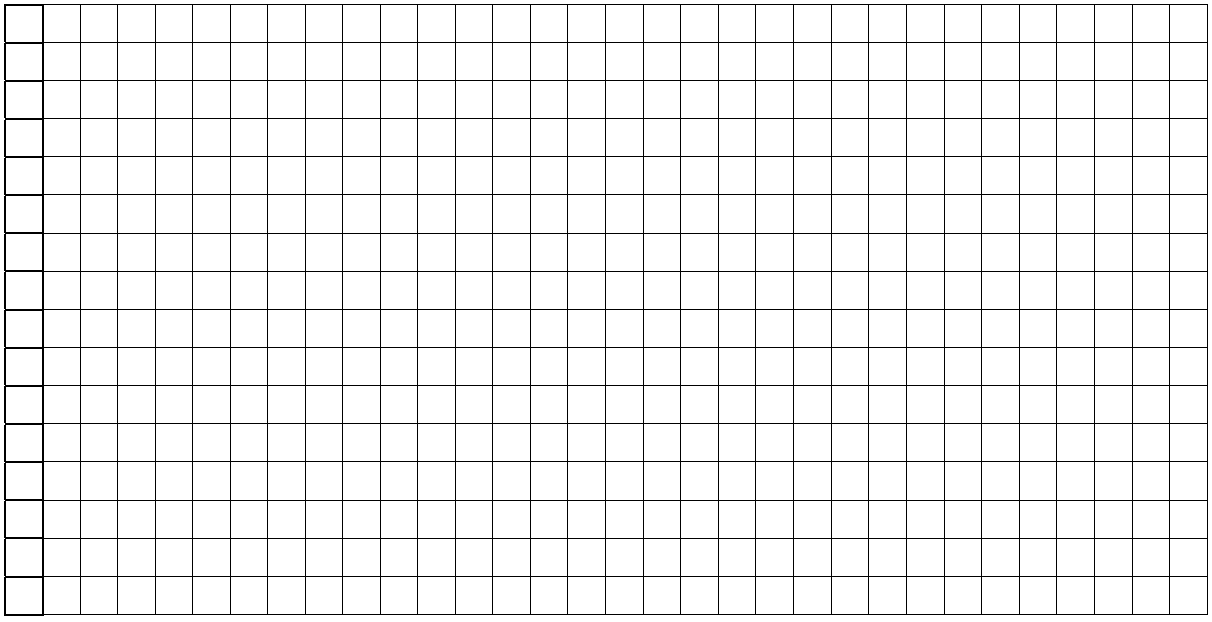
Question 1 **(25 marks)**

(a) 20% of a consignment of oranges are known to be bad. If five oranges are selected at random, find the probability that:

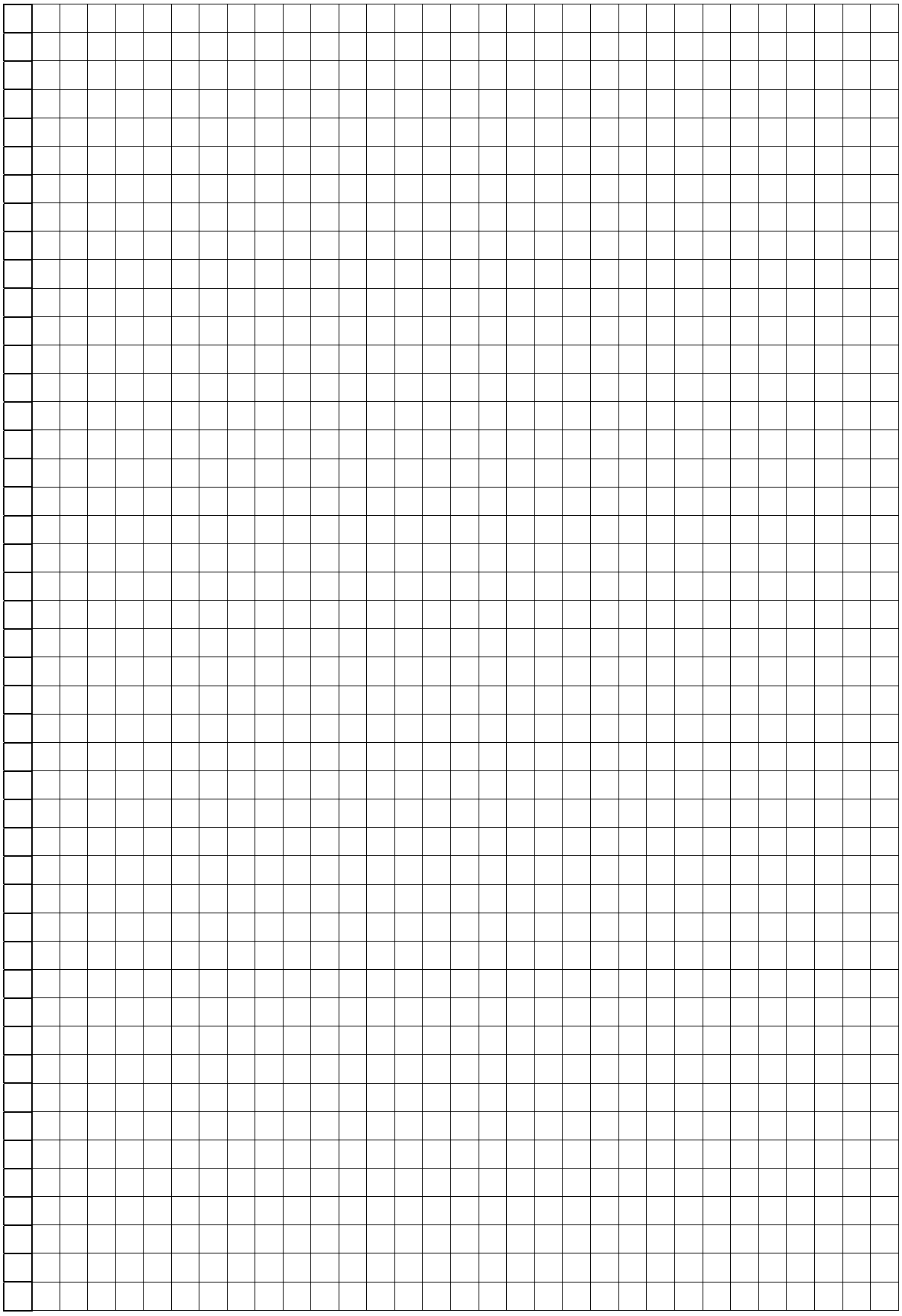
(i) all five are bad.



(ii) at least one is bad.

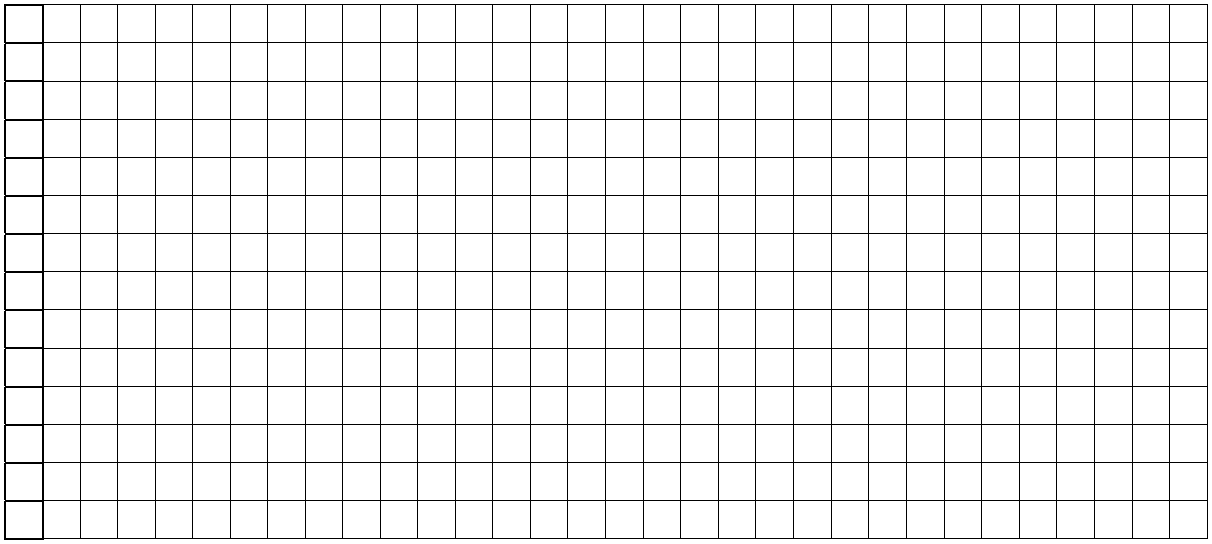


(b) A bag contains 8 blue marbles, 4 red marble and x white marbles. A marble is drawn at random and not replaced. A second marbles is drawn at random. If the probability that both marbles are white is $\frac{5}{51}$, how many white marbles are in the bag?

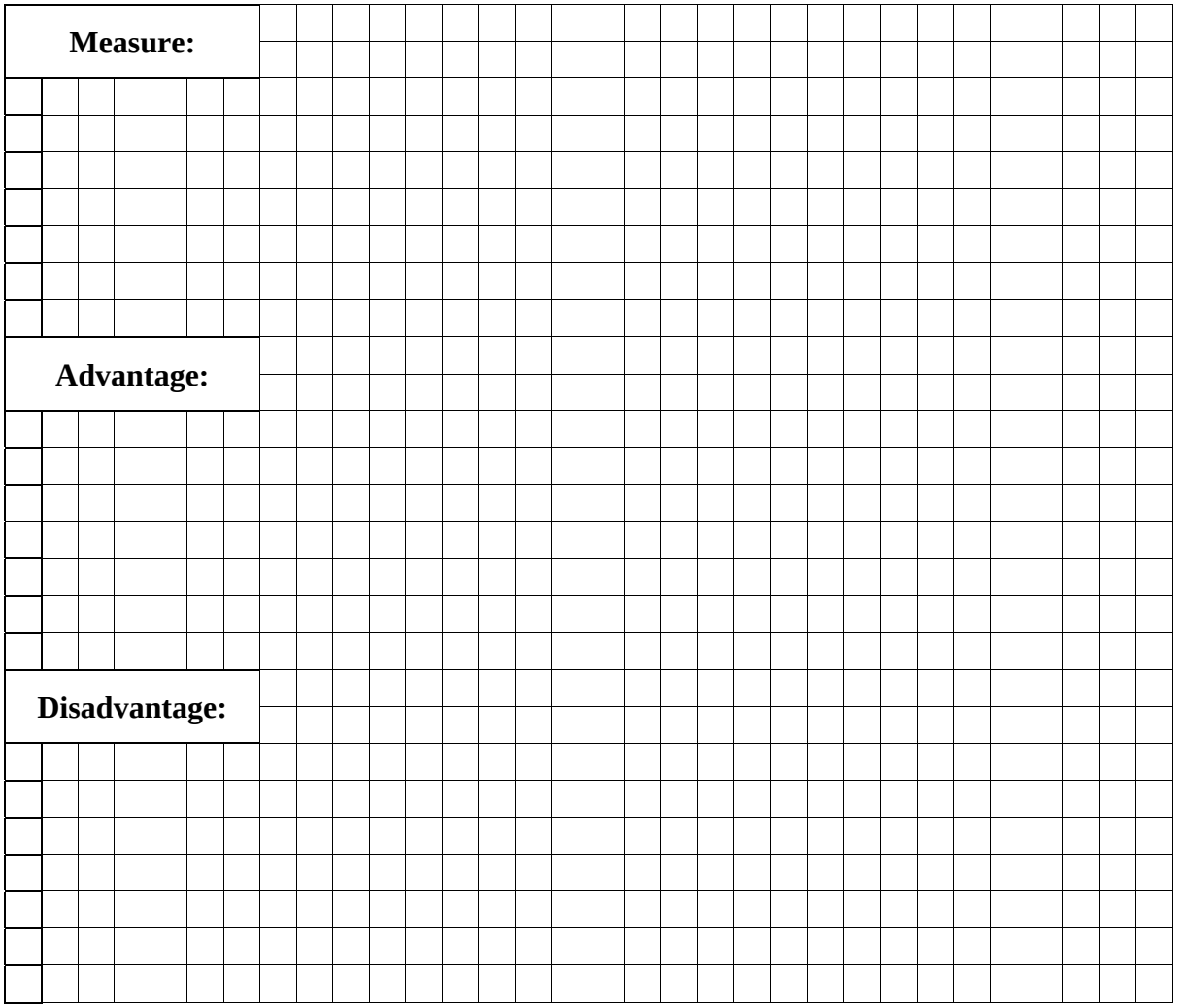


Question 2 **(25 marks)**

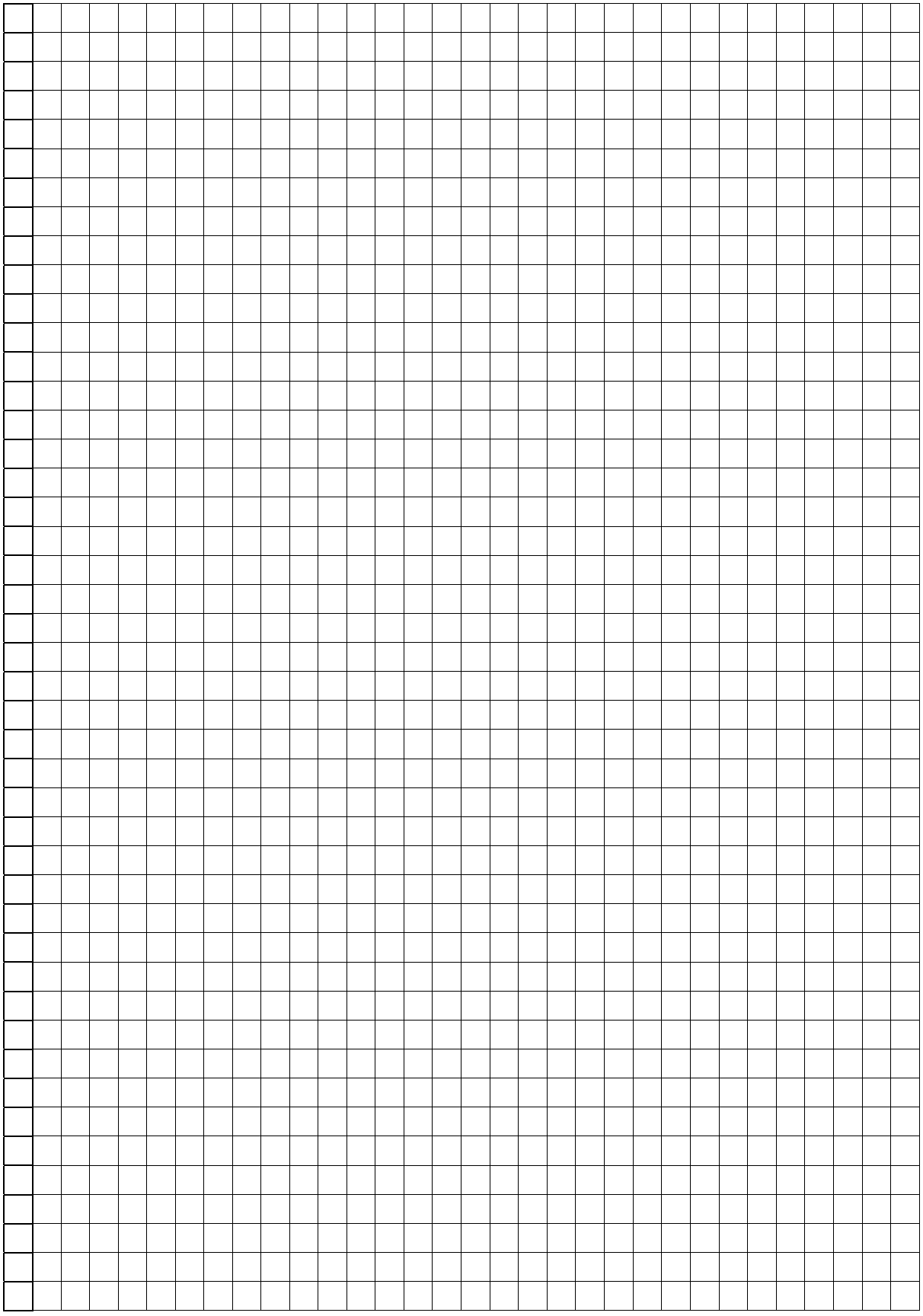
(a) Explain what is meant by ‘stratified sampling’ and give an example of this type of sampling.



(b) Name one measure of central tendency and give one advantage and one disadvantage of the named measure.

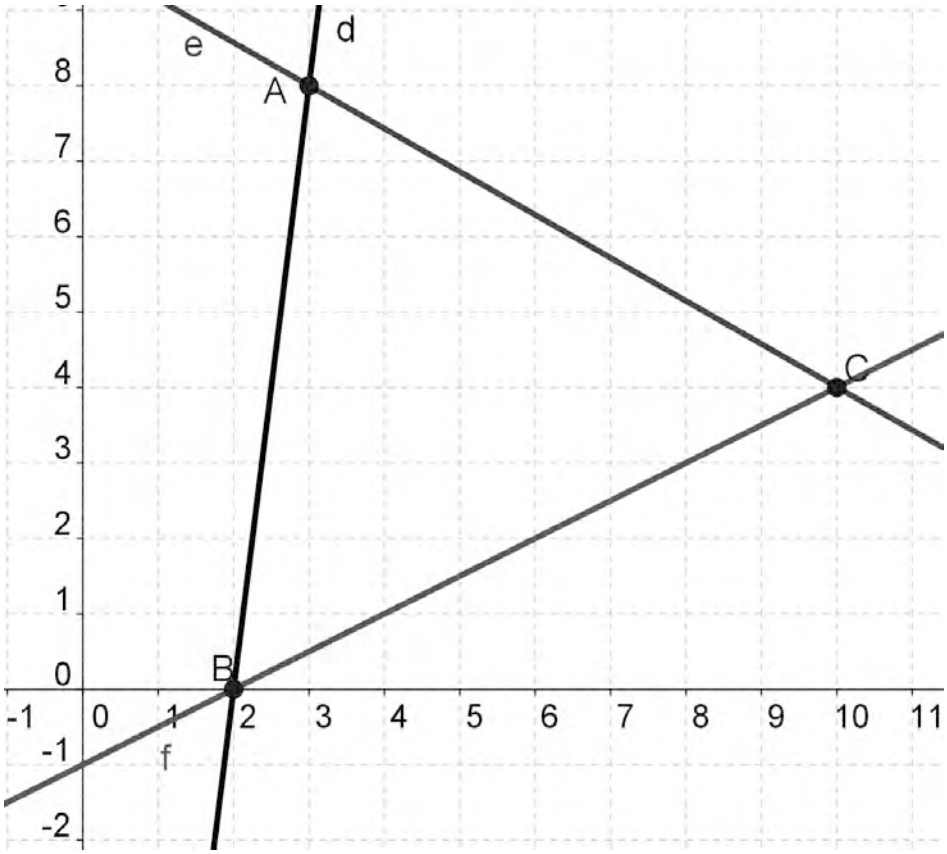


(c) A market research company is carrying out a national poll to find out people's opinion on the involvement of the E.U. in Ireland's banking crisis. The company picks 15 towns at random from a map of Ireland. They then choose 50 phone numbers from each town at random. These people will form the sample. Discuss the validity of choosing the sample in this fashion.

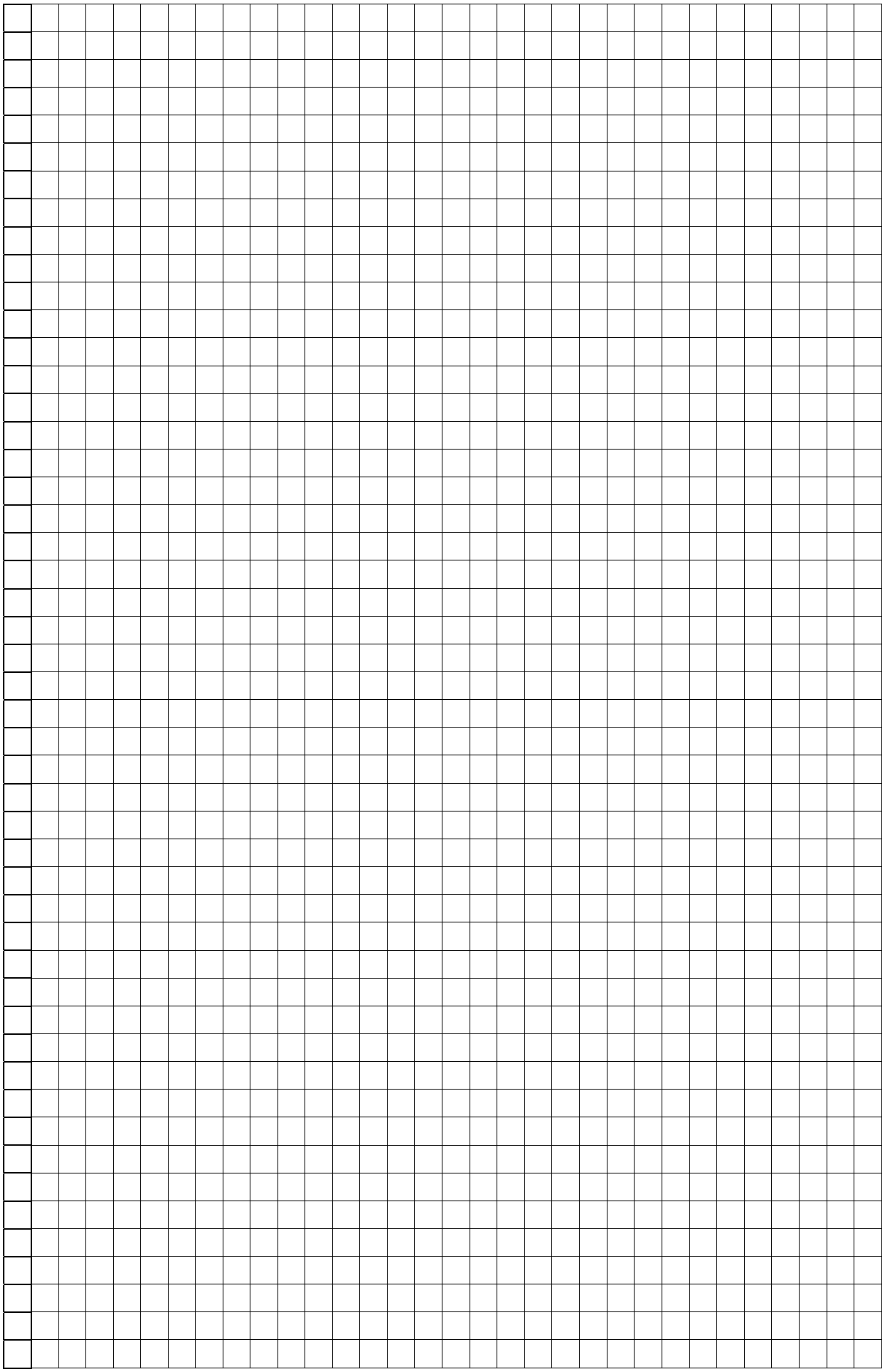


Question 3 **(25 marks)**

(a) The lines ***d***, ***e*** and ***f*** intersect as shown. By using the formula $\tan\theta = \pm\dfrac{m_1 - m_2}{1 + m_1 m_2}$ investigate the precise nature of the polygon formed by the lines ***d***, ***e*** and ***f***.

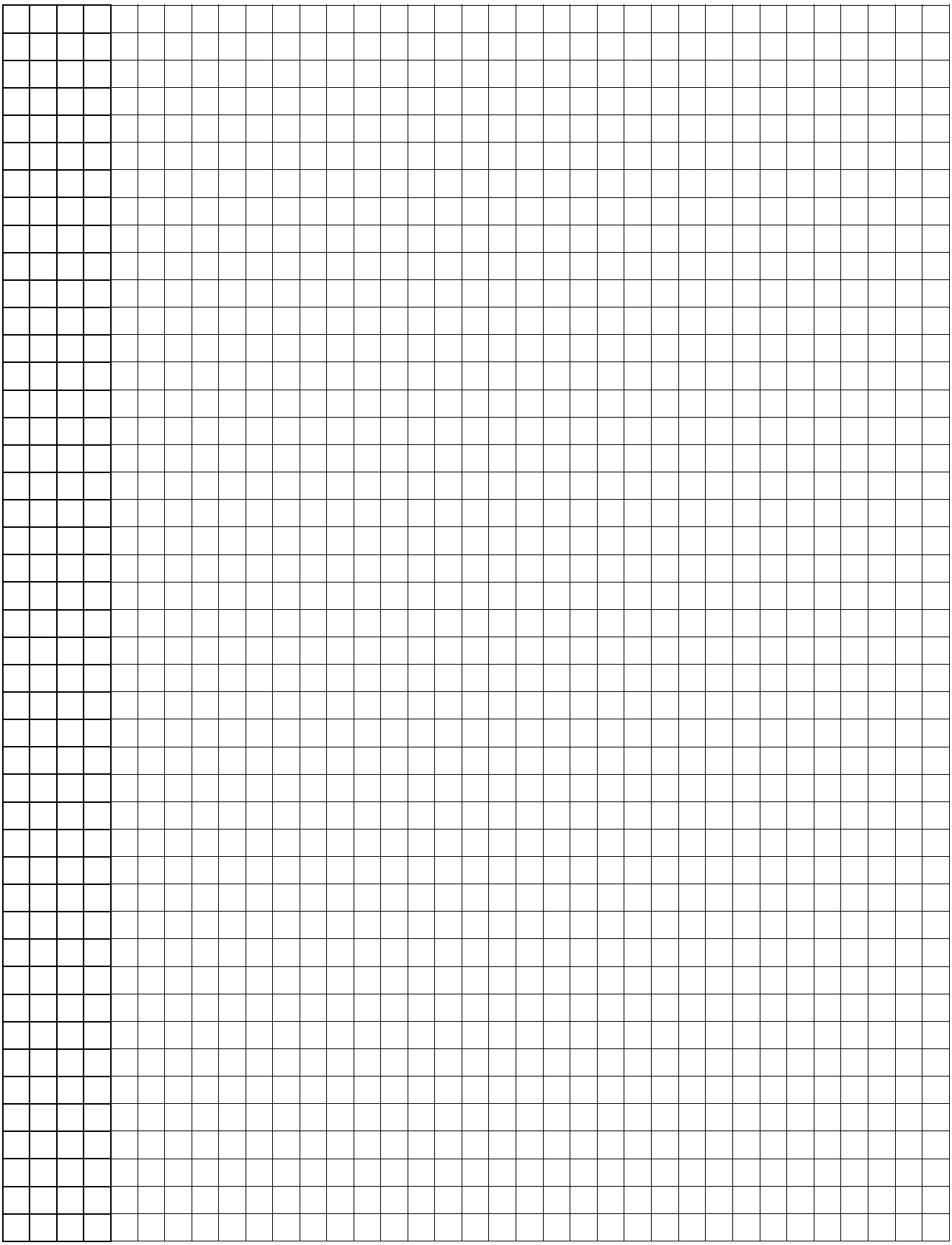


(b) Calculate the area of the polygon formed.



Question 4 **(25 marks)**

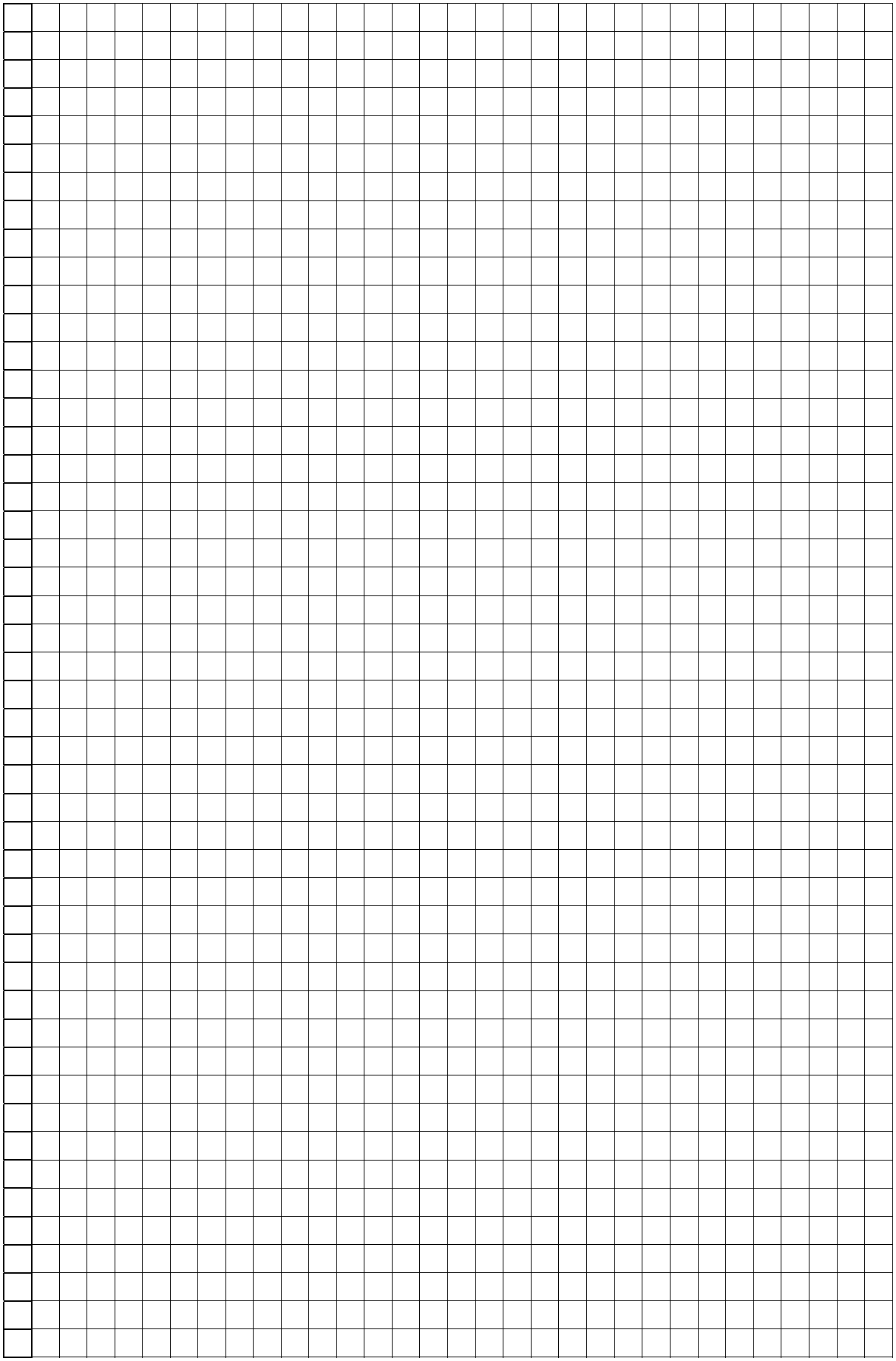
A circle of radius length 5 contains the point $(7,8)$. Its centre lies on the line $-2x+y=-4$.
Find the equations of the two circles that satisfy these conditions.



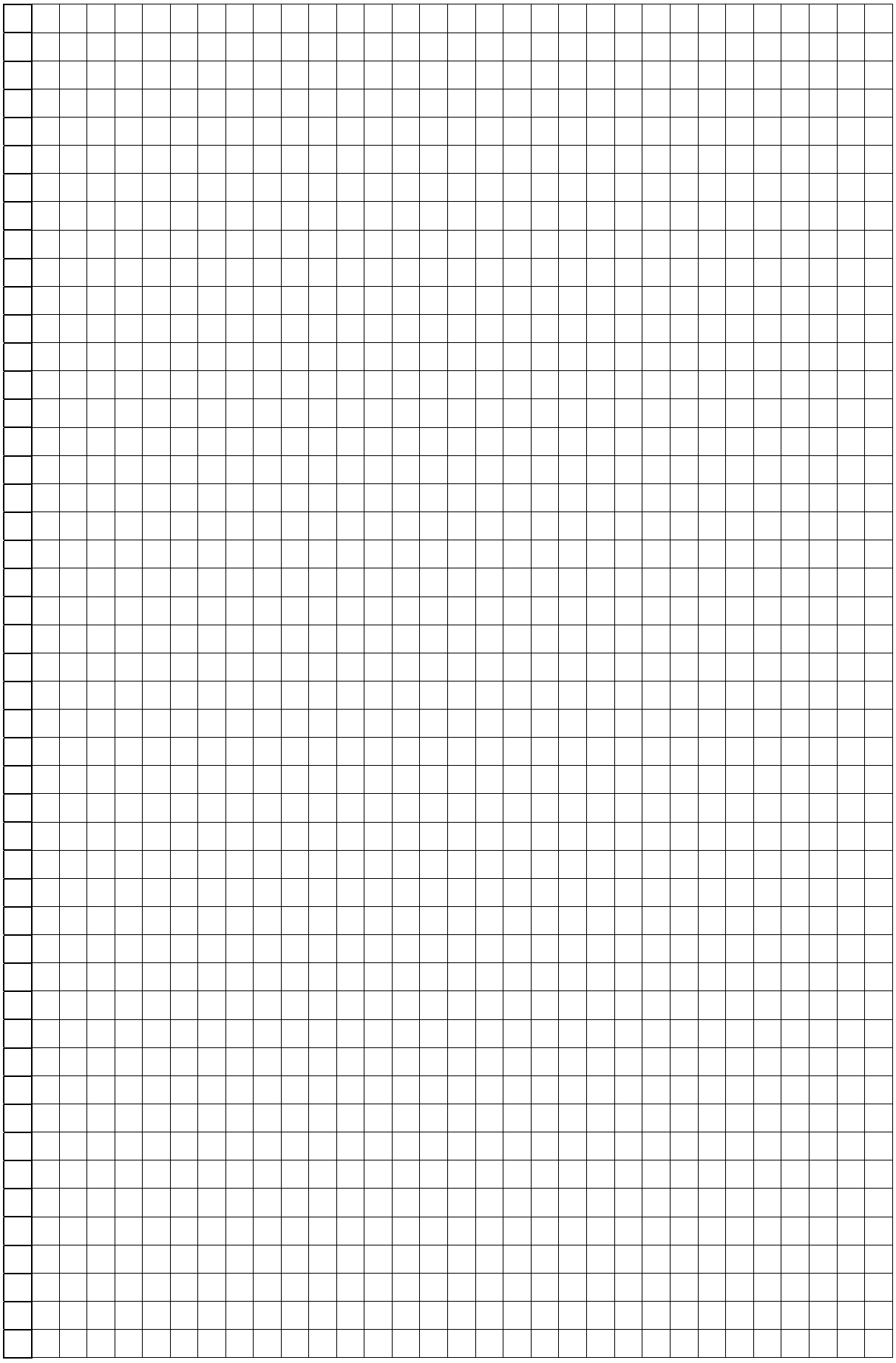
Question 5 **(25 marks)**

(a) Tap A can fill a cylindrical container in 3 minutes. Tap B can fill the same container in 15 minutes. How long would it take the two taps together to fill the container?

(b) The height of a cylinder is four times its radius. If the volume of the cylinder is 108π cm^3, calculate the radius and height of the cylinder.



(c) 32 identical cylinders are packed into a square-based box. Calculate the surface area of the box.



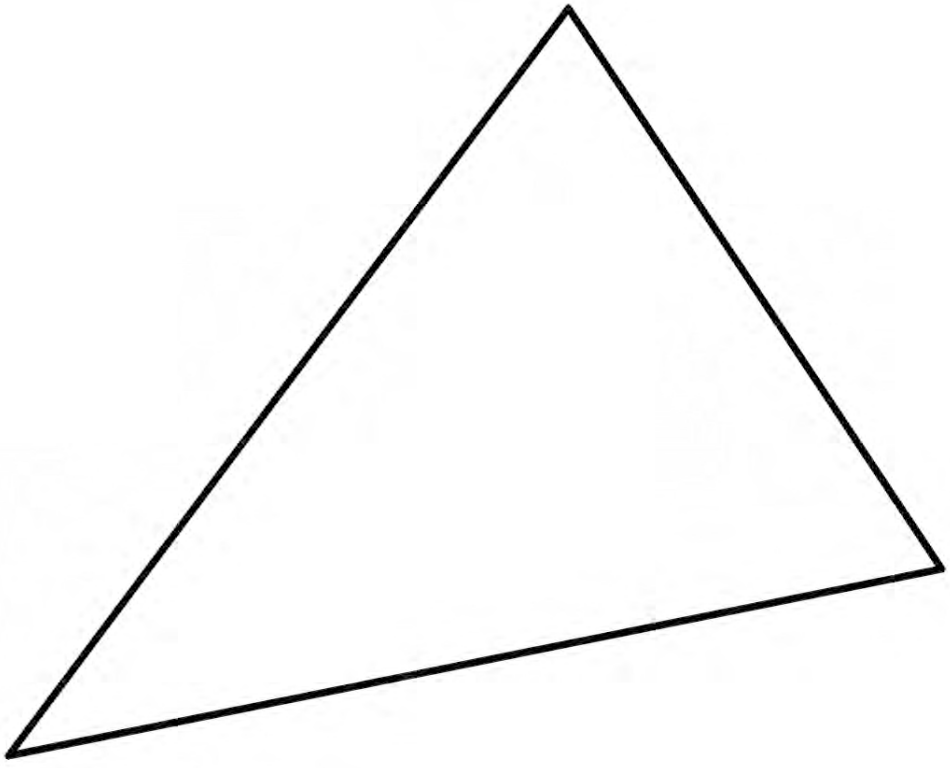
Question 6 **(25 marks)**

Answer **either** 6A **or** 6B.

Question 6A

(a) Solve $\cos\theta = 0.5$ for θ, where $0° \leq \theta \leq 360°$.

(b) A student is trying to find the centre of gravity of the following triangle. Use a suitable geometrical construction to find the point the student wishes to find.



OR

Question 6B

Prove that if two triangles *ABC* and *DEF* are similar then their sides are proportional.

Section B	Contexts and Applications	150 marks

Answer Question 7, Question 8 and Question 9.

Question 7 **(75 marks)**

The following table shows the numbers employed in certain sectors of industry in Ireland between 2005 and 2012.

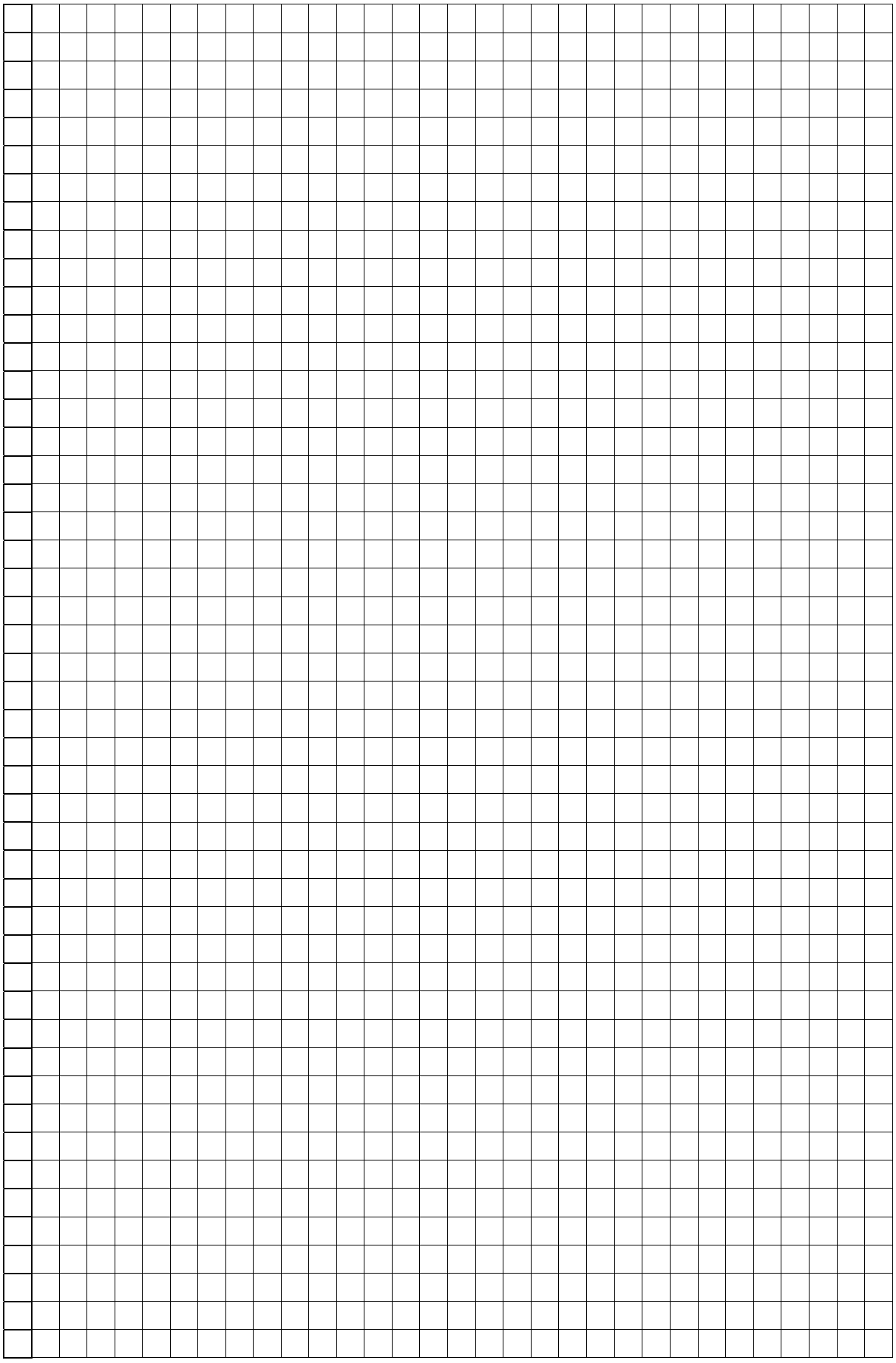
Employment and Unemployment (ILO) '000s

Economic Sector	Apr - Jun 05	Apr - Jun 06	Apr - Jun 07	Apr - Jun 08	Apr - Jun 09	Apr - Jun 10	Apr - Jun 11	Apr - Jun 12
Agriculture, Forestry & Fishing	110	110	109	115	97	85	86	87
Construction	228	252	270	241	155	125	106	99
Accommodation and food service activities	117	125	131	15	120	120	107	114
Information and communication	66	70	71	71	74	74	75	78
Education	126	136	141	146	150	150	147	144
Human health and social work activities	186	201	210	221	228	235	238	237
Total in Employment	**833**	**894**	**932**	**809**	**824**	**789**	**759**	**759**
Total Unemployed	96	98	103	127	265	294	305	309
Total Labour Force								

(Adapted from: CSO http://www.cso.ie/en/statistics/labourmarket/principalstatistics)

(a) Calculate the total labour force for each Apr-Jun period.

(b) Choose a suitable graphical display to compare the employment figures in three of the sectors in the table above from 2005 to 2012.



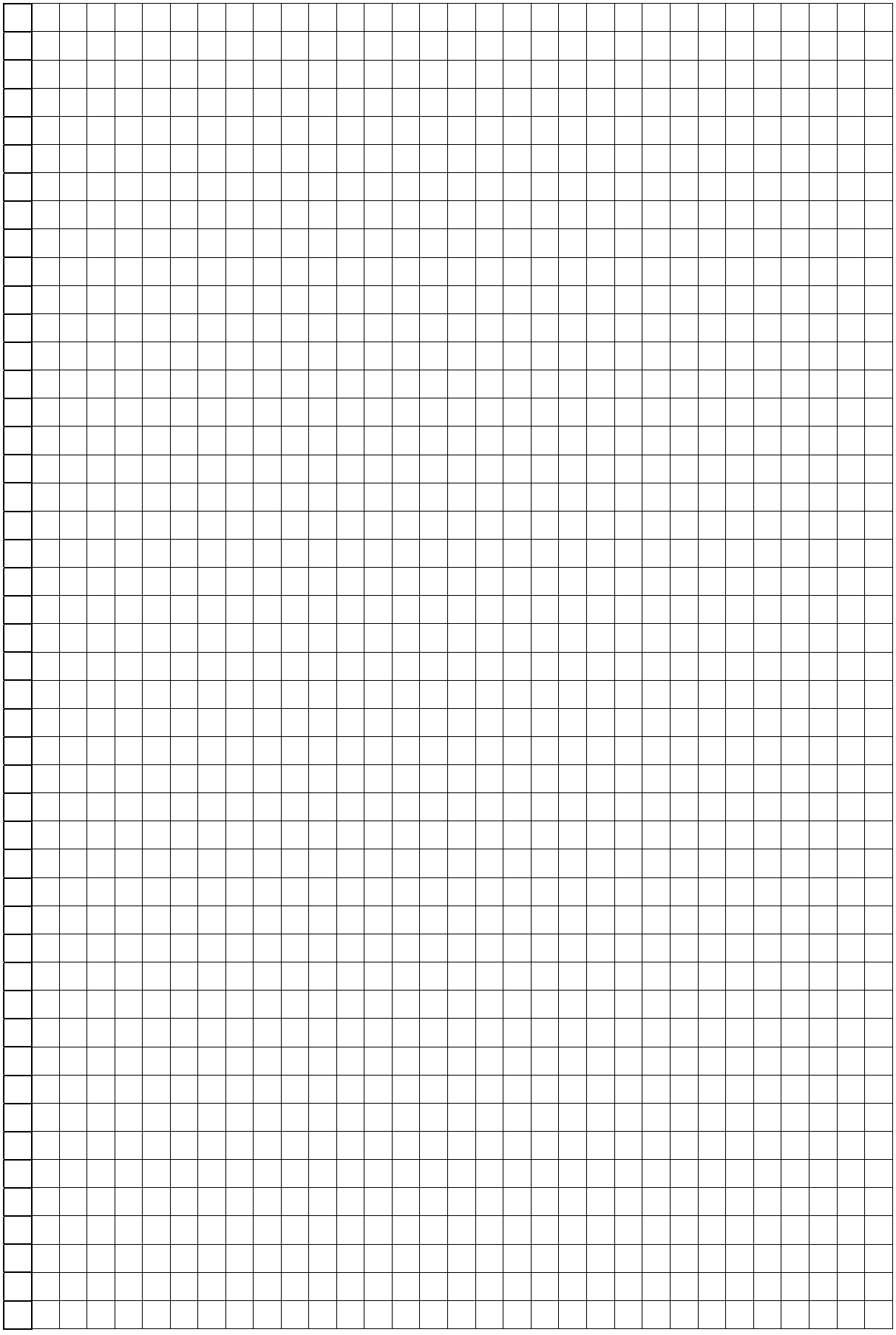
(c) Discuss the shape and distribution of each sector you have chosen. In your opinion, why do you think these sectors have the distribution in

Sector 1:

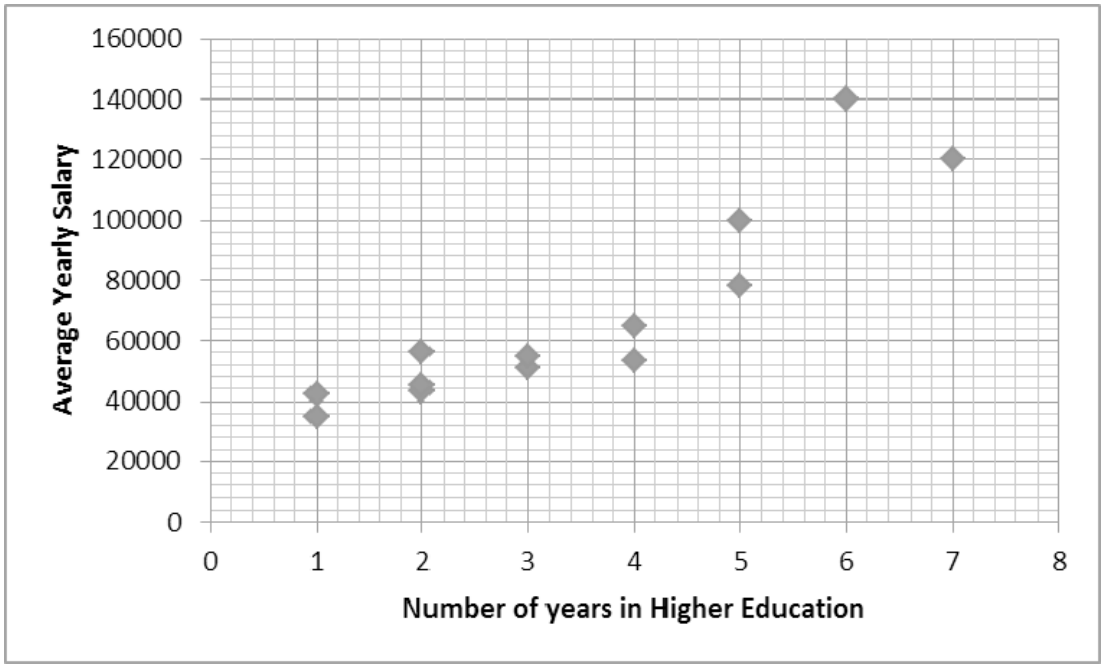
Sector 2:

Sector 3:

(d) Compare and contrast the figures in the construction and the education sector using a measure of central tendency and a measure of spread. Explain why you think the employment figures in both industries have followed the patterns in the table.



(e) Examine the following scatter plot.



(i) Describe the correlation coefficient of the plot.

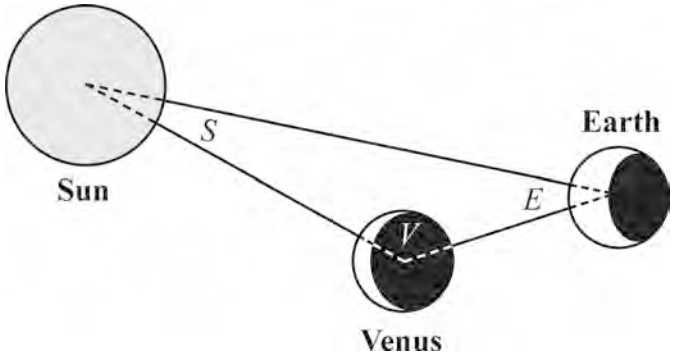
(ii) Calculate the correlation coefficient of the plot.

(iii) In your opinion, what is the plot attempting to show?

Question 8 **(50 marks)**

European astronomers carried out an experiment that started to measure the distance of Venus from the Sun.

When the centre of Venus is at right angles to both the centres of the Earth and the Sun it was found that Venus was 1.05×10^8 km from the Sun and that $|\angle SEV| = 44.4°$.

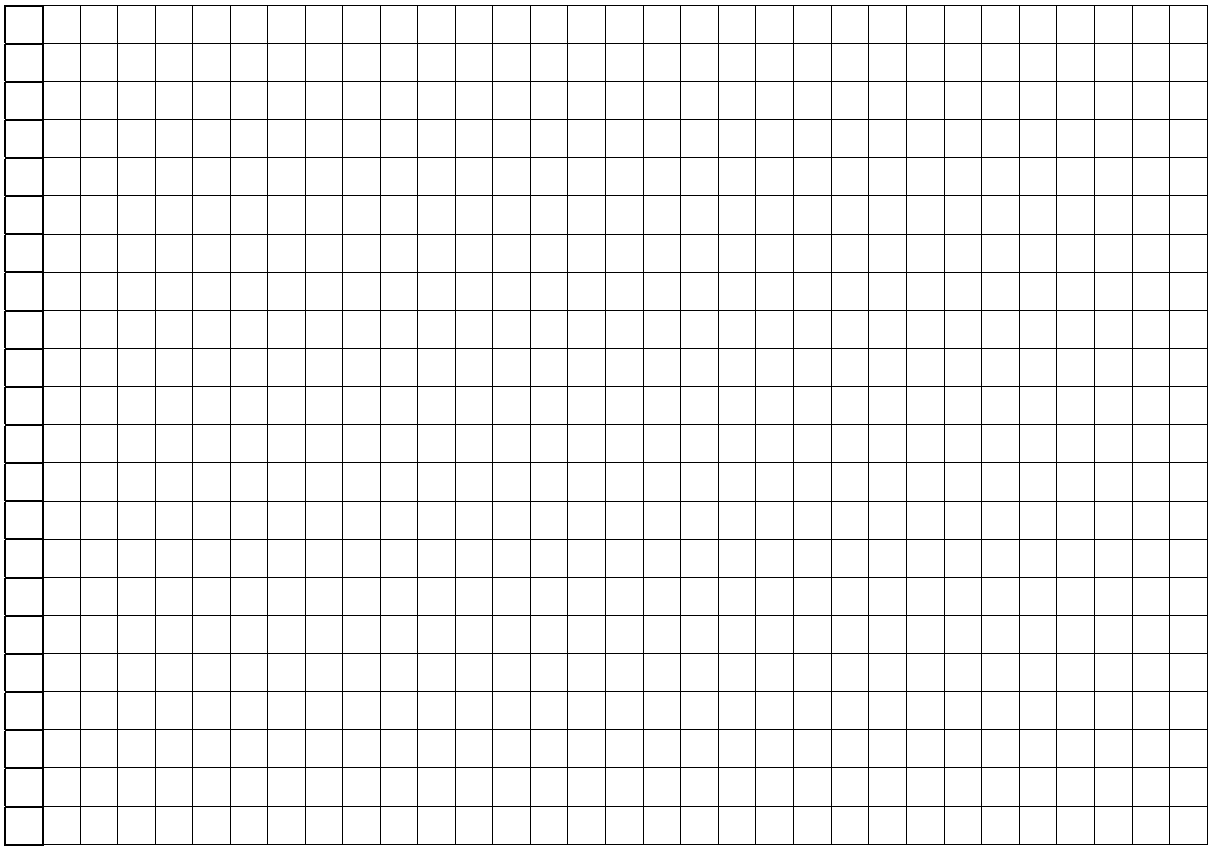


(a) Calculate the distance from the centre of the Earth to the centre of the Sun correct to two significant figures.

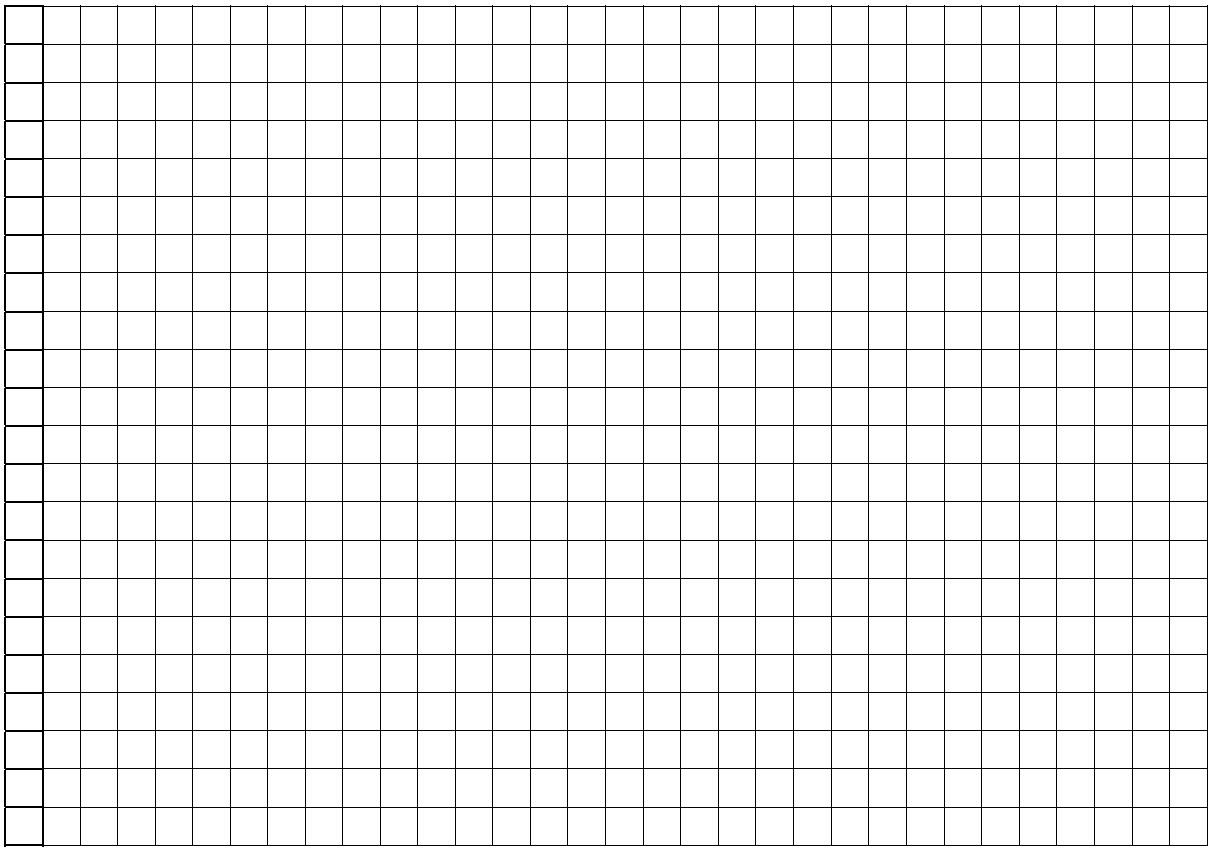
A satellite is put in orbit so that it is equidistant from the centres of all three planets when they are positioned as above.

(b) Where would the satellite be located? Explain your answer fully.

(c) At what ratio does Venus orbit the Sun compared with the Earth's orbit?



(d) Will this ratio hold for all positions of the three planets? Explain your answer fully.



Question 9 **(25 marks)**

(a) **Snell's window** is a phenomenon by which an underwater viewer sees everything above the surface through a cone of light.
It is caused by the diffraction of light. Only light which strikes the surface of the water within a circle of a certain radius, ***r***, will reach a point ***P*** below the surface.

A light at the bottom of a pool 2 m deep emits light upwards in all directions.
The maximum angle created by a ray of light is 37°. Calculate the radius of the disc through which the light leaves the surface of the water, correct to one decimal place.

(b) Calculate the volume of water contained in the cone, correct to one decimal place.