



PRE-LEAVING CERTIFICATE EXAMINATION, 2013

MARKING SCHEME

PROJECT MATHS (Phase 3)

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OVERVIEW OF MARKING SCHEME

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale	0, 10	0, 6, 10	0, 5, 8, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 8, 15	0, 8, 13, 15	0, 7, 10, 13, 15	
20 mark scale	0, 20	0, 10, 20	0, 10, 18, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 20, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

All questions marking category are shown throughout the solutions.

Summary of mark allocations and scales to be applied on Paper 1.

Section A

Question 1

- (a) 5C
- (b) 10C
- (c) 10C

Question 2

- (a) 5C
- (b) 10C
- (c) 10C

Question 3

- (a) 10C
- (b) 15C

Question 4

- (a) 10C
- (b) 15C

Question 5

- (a) (i) 5B
- (ii) 5B
- (b) 15C

Question 6

- (a) 10B
- (b) 15C

Section B

Question 7

- (a) 10C
- (b) (i) 10C*
- (ii) 10C*

Question 8

- (a) 10B
- (b) (i) 10C
- (ii) 10C
- (iii) 20C

Question 9

- (a) 10B
- (b) 15C

SOLUTIONS TO PAPER 1

QUESTION 1

Part (a)

Scale 5C

Part (b)

Scale 10C

Part (c)

Scale 10C

(a)

$$2x + 3y + z = 2$$

$$2x - 2y + \cancel{2z} = 8$$

$$4x + 6y + \cancel{2z} = 4$$

$$x - y + z = 4$$

$$\frac{x - 2y - \cancel{2z} = 2}{3x - 4y = 10}$$

$$\frac{x - 2y - \cancel{2z} = 2}{5x + \cancel{4y} = 6}$$

$$x - 2y - 2z = 2$$

$$3x - 4y = 10$$

$$5x + \cancel{4y} = 6$$

$$\frac{3x - \cancel{4y} = 10}{8x = 16}$$

$$8x = 16$$

$$x = 2$$

$$\therefore y = -1$$

$$\therefore z = 1$$

Full Credit (5 Marks)

- Fully correct solution.

High Partial Credit (4 Marks)

- Solves one variable incorrectly and continues to end with no further errors.
- Solves two variables correctly.

Low Partial Credit (3 Marks)

- Solves one variable incorrectly and continues to end with further errors.

(b) Let $f(x) = 2x^3 + ax^2 - 17x + b$ where a and b are constants.

Given that $(x-1)$ and $(x+4)$ are factors of $f(x)$, find the value of a and the value of b .

$$\begin{aligned}x=1 & \quad \therefore 2(1)^3 + a(1)^2 - 17(1) + b = 0 \\ & \quad \quad \quad a + b = 15 \\ \\x=-4 & \quad \therefore 2(-4)^3 + a(-4)^2 - 17(-4) + b = 0 \\ & \quad \quad \quad 16a + b = 60 \\ & \quad \quad \quad \underline{-a - b = -15} \\ & \quad \quad \quad 15a = 45 \\ & \quad \quad \quad a = 3 \quad \therefore b = 12\end{aligned}$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- Solves one variable correctly.
- Solves both variables incorrectly with no more than two minor errors.

Low Partial Credit (5 Marks)

- Solves both variables incorrectly with more than two minor errors.
- Some correct step.

(c) Simplify: $\frac{x^2 - x - 2}{x^2 - 3x} \div \frac{x^2 + 5x + 4}{x^2 - x - 6}$

$$\begin{aligned}\frac{(x+1)(x-2)}{x(x-3)} \times \frac{(x+2)(x-3)}{(x+1)(x+4)} \\ \frac{(x-2)(x+2)}{x(x+4)}\end{aligned}$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- Incorrect factors once.
- Fails to simplify fully.

Low Partial Credit (5 Marks)

- Incorrect factors more than once.
- Some correct step.

QUESTION 2

Part (a)
Part (b)
Part (c)

Scale 5C
Scale 10C
Scale 10C

- (a) $4x-4$, $x+4$ and $2x-7$ are three consecutive terms in a sequence where $x \in \mathbb{Z}$. Investigate if the sequence is arithmetic or geometric. Explain your answer fully.

Solve: Arithmetic

$$\begin{aligned} T_3 - T_2 &= T_2 - T_1 \\ (2x-7) - (x+4) &= (x+4) - (4x-4) \\ x-11 &= -3x+8 \\ 4x &= 19 \\ x &= \frac{19}{4} \notin \mathbb{Z} \end{aligned}$$

Geometric

$$\begin{aligned} \frac{2x-7}{x+4} &= \frac{x+4}{4x-4} \\ (2x-7)(4x-4) &= (x+4)^2 \\ 8x^2 - 8x - 28x + 28 &= x^2 + 8x + 16 \\ 7x^2 - 44x + 12 &= 0 \\ (7x-2)(x-6) & \\ x = \frac{2}{7} \quad x = 6 & \\ \notin \mathbb{Z} & \\ \text{if } x = 6 \quad 20, 10, 5 & \\ a = 20 \quad r = \frac{1}{2} \quad \therefore \text{Geometric} & \end{aligned}$$

$$x = 6$$

20, 10, 5 so sequence is geometric as $r = \frac{1}{2}$

Full Credit (5 Marks)

- Fully correct solution.

High Partial Credit (4 Marks)

- Finds terms but incorrect conclusion.

Low Partial Credit (3 Marks)

- Some correct step.

(b) If $x^2 + 7x + 12$ is a factor of $x^3 + px^2 - 2x + r$, find the value of p and r .

$$(x^2 + 7x + 12)(x + t) = x^3 + px^2 - 2x + r$$

$$x^3 + tx^2 + 7tx + 7x^2 + 12x + 12t = x^3 + px^2 - 2x + r$$

$\frac{x^2}{t+7} = p$	$\frac{x}{7t+12} = -2$	$12t = r$
$-2+7 = p$	$7t = -14$	$12(-2) = r$
$\therefore p = 5$	$t = -2$	$\therefore r = -24$

$p = 5, \quad r = -24$

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- Finds two incorrect values by one correct method.
- Substitutes $x = -4$ and $x = -3$ into the equation..

Low Partial Credit (5 Marks)

- One correct step.

(c) Solve the following inequality: $2x^2 + 7x - 4 \leq 0$

$$(2x - 1)(x + 4)$$

$$x = \frac{1}{2} \quad x = -4$$

$$-4 \leq x \leq \frac{1}{2}$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- Finds both roots but fails to solve the inequality.

Low Partial Credit (5 Marks)

- Some correct step.

QUESTION 3

Part (a)

Scale 10C

Part (b)

Scale 15C

(a) Given that $z = a + bi$ where $a, b \in \mathbb{R}$, find the values of z if $z\bar{z} - 4i\bar{z} = 4 - 8i$.

$$\begin{aligned}(a + bi)(a - bi) - 4i(a - bi) &= 4 - 8i \\ a^2 + b^2 - 4ai - 4b &= 4 - 8i \\ a^2 - 4b + b^2 &= 4 \quad -4a = -8 \\ \therefore b^2 - 4b + 4 &= 4 \quad a = 2 \\ b^2 - 4b &= 0 \\ b(b - 4) &= 0 \\ b = 0 \quad b = 4 \quad \therefore z = 2 \quad \text{or} \quad z = 2 + 4i\end{aligned}$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- Solves with no more than two errors.

Low Partial Credit (4 Marks)

- Solves with more than two errors.
- Some correct step.

(b) Use De Moivre's Theorem to solve the equation $z^3 = 1$.

$$\text{Roots} = 1, \quad -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \quad -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Full Credit (15 Marks)

- Fully correct solution.

High Partial Credit (13 Marks)

- Writes in general polar form but mishandles indices.
- Error writing in general polar form but continues correctly to end.

Low Partial Credit (8 Marks)

- Error writing in general polar form and continues incorrectly to end.
- Some correct step.

QUESTION 4

Part (a)

Scale 10C

Part (b)

Scale 15C

(a) If $T_{n+1} = (T_n)^2 - 3n$ find T_5 , given that $T_1 = 1$.

$$T_2 = (1)^2 - 3(1) = -2$$

$$T_3 = (-2)^2 - 3(2) = -2$$

$$T_4 = (-2)^2 - 3(3) = -5$$

$$T_5 = (-5)^2 - 3(4) = 13$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- Solves with no more than two errors.

Low Partial Credit (5 Marks)

- Solves with more than two errors.
- Some correct step.

(b) Prove by induction that for any natural number, $n^3 + 2n$ is divisible by 3.

Step 1: Proves for $n = 1$

Step 2: Assume true for $n = k$

$$\text{Step 3: } (k+1)^3 + 2(k+1) = (k^3 + 3k^2 + 3k + 1) + (2k + 2)$$

$$= (k^3 + 2k) + (3k^2 + 3k + 3)$$

$$= (k^3 + 2k) + 3(k^2 + k + 1)$$

which is divisible by 3, because $(k^3 + 2k)$ is divisible by 3.

Full Credit (15 Marks)

- Fully correct solution.

High Partial Credit (13 Marks)

- Proves steps 1 and 2.

Low Partial Credit (8 Marks)

- Only states step 1.

QUESTION 5

Part (a) (i)
(ii)

Scale 5B
Scale 5B
Scale 15C

Part (b)

(a) Find:

(i) $\int 3x^2 dx$

$$x^3 + C$$

Full Credit (5 Marks)

- Fully correct solution.

Partial Credit (3 Marks)

- Some correct step.

(ii) $\int \frac{x^4 - x^7}{x^3} dx$

$$\frac{x^2}{2} - \frac{x^5}{5} + C$$

Full Credit (5 Marks)

- Fully correct solution.

Partial Credit (3 Marks)

- Some correct step.

(b) Show that $\cos^3 x = \cos x(1 - \sin^2 x)$. Hence, otherwise evaluate $\int_0^{\frac{\pi}{3}} \cos^3 x dx$.

$$\cos^3 x = \cos x \cos^2 x = \cos x(1 - \sin^2 x)$$

$$\int_0^{\frac{\pi}{3}} \cos^3 x dx = -\sin x - \frac{\sin^3 x}{3} = -\frac{5\sqrt{3}}{8}$$

Full Credit (15 Marks)

- Fully correct solution.

High Partial Credit (13 Marks)

- First step correct.

Low Partial Credit (8 Marks)

- Any correct step.

QUESTION 6

Part (a)

Scale 10B

Part (b)

Scale 15C

(a) Differentiate $\sqrt{3x^2 + 5x - 2}$ with respect to x .

$$\frac{1}{2}(3 \times 2 + 5x - 2)^{-\frac{1}{2}}(6x + 5)$$

Full Credit (5 Marks)

- Fully correct solution.

Partial Credit (3 Marks)

- Some correct step.

(b) Differentiate, from first principles, $\sin x$ with respect to x .

$$f(x) = \sin x$$

$$f(x+h) = \sin(x+h)$$

$$f(x+h) - f(x) = \sin(x+h) - \sin x$$

$$= 2 \cos\left(\frac{x+h+x}{2}\right) \sin\left(\frac{x+h-x}{2}\right)$$

$$= 2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{h} \times \frac{1}{\frac{1}{2}}$$

Full Credit (15 Marks)

- Fully correct solution.

High Partial Credit (13 Marks)

- One error in RHS.
- Omits LHS but RHS fully correct.
- Omits Limit at end.

Low Partial Credit (8 Marks)

- Any correct step.

QUESTION 7

Part (a)

Scale 10C

Part (b) (i)

Scale 10C*

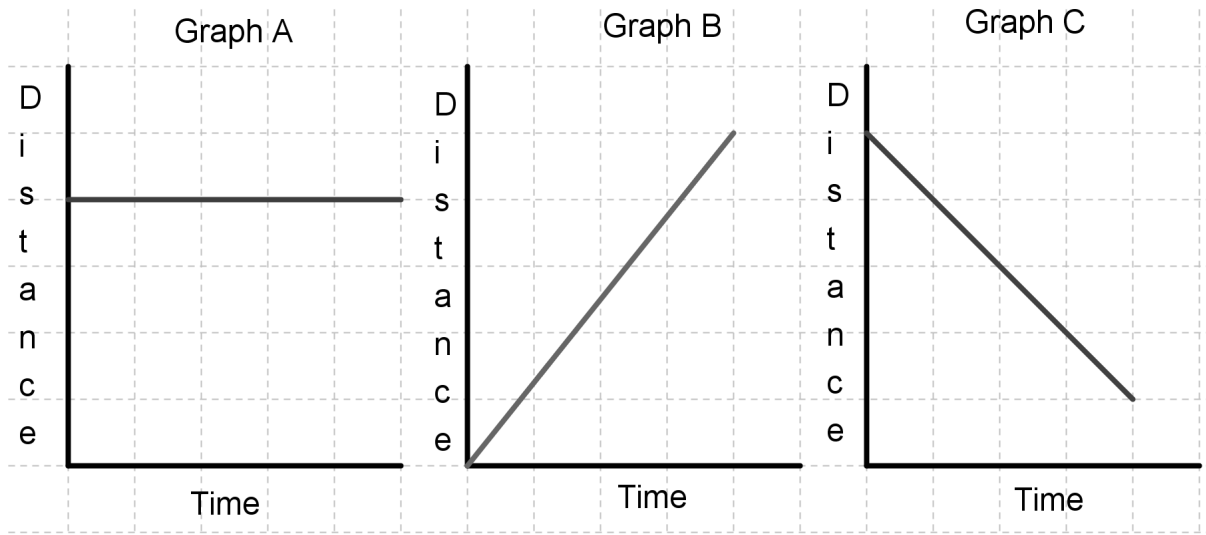
(ii)

Scale 10C*

(iii)

Scale 20C*

- (a) The following graphs show the distance a cyclist is from a marker in a race. Examine the graphs and describe the motion of the cyclist relative to the marker for each.



Graph A: Stopped a distance away from the marker.

Graph B: Moving away from the marker.

Graph C: Moving towards the marker.

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- Two correct observations.

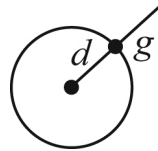
Low Partial Credit (5 Marks)

- one correct observation.

(b) The formula $g = \frac{GM}{d^2}$ can be used to determine the acceleration (g) due to gravity on an object a distance d from the centre of a planet where $G = 6.7 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

(i) Given that on the surface of the Earth, the acceleration (g) due to gravity is 9.81 ms^{-2} and the mass of the Earth $M = 6 \times 10^{24} \text{ kg}$, calculate the radius of the Earth, correct to two significant figures.

$$g = \frac{GM}{d^2}$$



$d = \text{radius} = r$

$$d^2 = \frac{GM}{g} = \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(9.81)}$$

$$d = \sqrt{\frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(9.81)}}$$

$$d = r = 6.4 \times 10^6 \text{ m}$$

***Deduct 1 mark from a fully correct solution for incorrect or no units.**

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- Two correct observations.

Low Partial Credit (5 Marks)

- One correct observation.

(ii) Find the acceleration (g) due to gravity 500 km above the earth's surface, correct to two decimal places.

$$g = \frac{(6.7 \times 10^{-11})(6 \times 10^{24})}{(6.4 \times 10^6 + 500,000)^2}$$

$$= 8.44 \text{ ms}^{-2}$$

***Deduct 1 mark from a fully correct solution for incorrect or no units.**

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- Two correct observations.

Low Partial Credit (5 Marks)

- One correct observation.

- (iii) The distance between the centre of the Earth and the centre of the Moon is 3.8×10^8 m and the mass of the Moon is 7.41×10^{22} kg. At what point between the Earth and the Moon is the force of gravity on an object zero?

$$\frac{GM_E}{x^2} = \frac{GM_M}{y^2}$$

$$x + y = 3.8 \times 10^8$$

$$M_E y^2 = M_M x^2$$

$$(6 \times 10^{24}) y^2 = (7.41 \times 10^{22}) (3.8 \times 10^8 - y)^2$$

$$y = 38,000,608 \text{ km}$$

Zero gravity when 38,000,608 km from the Moon or 341,999,392 km from the Earth.

***Deduct 1 mark from a fully correct solution for incorrect or no units.**

Full Credit (20 Marks)

- Fully correct.

High Partial Credit (18 Marks)

- Answer found but no more than two errors and method almost correct.

Low Partial Credit (10 Marks)

- Any correct step.

QUESTION 8

Part (a)	Scale 10B
Part (b) (i)	Scale 10C
(ii)	Scale 10C
(iii)	Scale 20C

- (a) If you borrow €1,000 for one year at an interest rate of 12% per year compounded quarterly, how much do you owe at the end of the year?

$$1000(1+0.12)^4 = \text{€}1,573.52$$

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Any correct step.

- (b) An individual who plans to retire in 20 years has decided to put an amount A, in the bank at the beginning of each month for the next 20 years.

- (i) Show that the present value of the deposits can be expressed in the form

$$A \left(\frac{1 \left(1 - \left(\frac{1}{1+r} \right)^{240} \right)}{1 - \left(\frac{1}{1+r} \right)} \right)$$

The present value of all her deposit is:

$$P = A + \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + \dots + \frac{A}{(1+r)^{239}} = A \left(\frac{1 \left(1 - \left(\frac{1}{1+r} \right)^{240} \right)}{1 - \left(\frac{1}{1+r} \right)} \right)$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- At least three terms of the progression but does not express as the sum of a geometric progression.
- Error in the handling of indices.

Low Partial Credit (5 Marks)

- Any correct term.

- (ii) After this time the individual plans to withdraw an amount W at the beginning of each month for the following 30 years. Express the present value of the deposits in terms of W and r .

$$P = \frac{W}{(1+r)^{240}} + \frac{W}{(1+r)^{241}} + \dots + \frac{W}{(1+r)^{599}} = W \left(\frac{\frac{1}{(1+r)^{240}} \left(1 - \left(\frac{1}{1+r} \right)^{360} \right)}{1 - \left(\frac{1}{1+r} \right)} \right)$$

Full Credit (10 Marks)

- Fully correct solution.

High Partial Credit (8 Marks)

- At least three terms of the progression but does not express as the sum of a geometric progression.
- Error in the handling of indices.

Low Partial Credit (5 Marks)

- Any correct term.

- (iii) Assuming a nominal yearly interest rate of 6% compounded monthly, how large does A need to be to sustain a monthly withdrawal of €1,000 for the 30 years?

$$A \left(\frac{1 \left(1 - \left(\frac{1}{1+r} \right)^{240} \right)}{1 - \left(\frac{1}{1+r} \right)} \right) = W \left(\frac{\frac{1}{(1+r)^{240}} \left(1 - \left(\frac{1}{1+r} \right)^{360} \right)}{1 - \left(\frac{1}{1+r} \right)} \right)$$

Let $r = 0.06 / 12 = 0.005$ be the monthly interest rate and $W = 1000$.

$$\therefore A = \text{€} 360.99$$

A saving of € 361 per month for 20 years will enable the individual to withdraw € 1000 A month for the following 30 years.

Full Credit (20 Marks)

- Fully correct solution.

High Partial Credit (18 Marks)

- Error handling interest rate.
- Incorrect substitution but continues to end.

Low Partial Credit (10 Marks)

- Any correct step.

QUESTION 9

Part (a)

Scale 10B

Part (b)

Scale 15C

- (a) A particle is fired vertically upwards from a point P . It's distance, s , in metres, from the point P is given by the formula $s = 5t^2 - 90t + 1000$, where t is measured in seconds. Find the height the particle reaches before falling back to earth.

$$\frac{ds}{dt} = 10t - 90 \quad \therefore t = 9$$
$$s = 595m$$

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Some correct step.

- (b) The equation $x^3 - 4x + 1 = 0$ has three real roots that lie in the domain $-4 \leq x \leq 4$.

(i) Find two consecutive integers in the domain, between which a root exists.

Between -3 and -2 or 0 and 1 or 1 and 2.

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Some correct step.

(ii) Taking one of the integers from part (i) as the first approximation of the real root of the equation $x^3 - 4x + 1 = 0$ use the Newton-Raphson method to find x_2 , the second approximation.

Between -3 and -2 or 0 and 1 or 1 and 2.

$$x_2 = (-3) - \frac{(-3)^3 - 4(-3) + 1}{3(-3)^2 - 4} = -2.39$$

Note: Candidate can use any integer from part (i).

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Some correct step.

(c) The equation of a curve is $\frac{2}{x-4}$, where $x \neq 4$.

(i) Show that the curve has no tangent parallel to the x -axis.

$$f'(x) = \frac{-2}{(x-4)^2} \neq 0 \quad \therefore \text{no tangent parallel to } x\text{-axis}$$

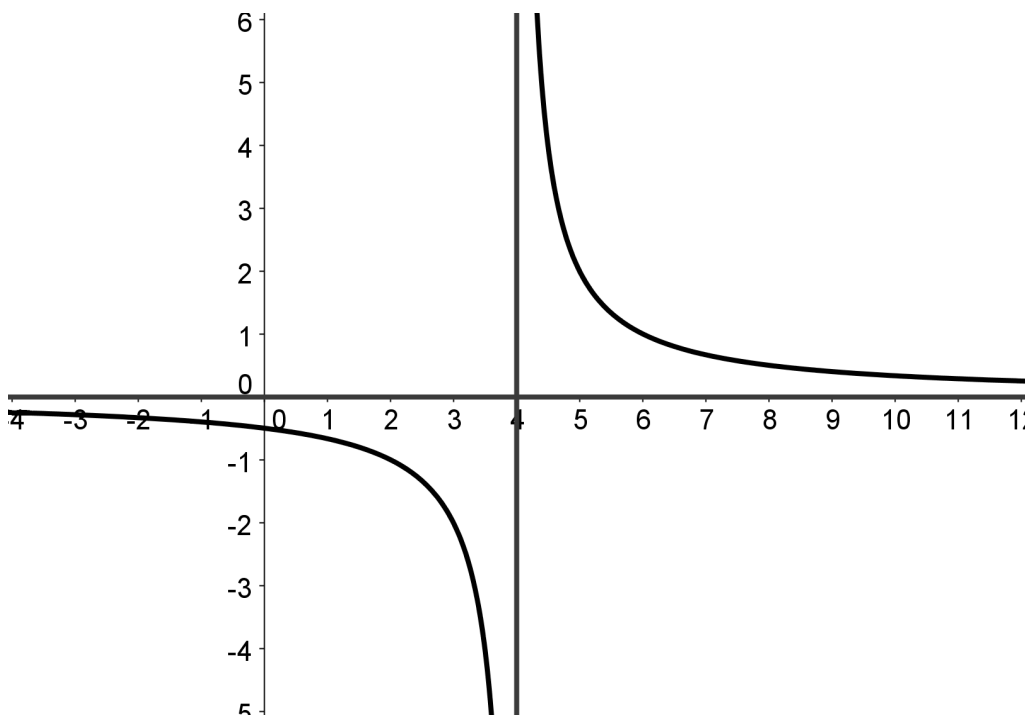
Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Some correct step.

(ii) Finding the equations of the vertical and horizontal asymptotes, and hence sketch the curve.



Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Some correct step.

Summary of mark allocations and scales to be applied on Paper 2.

Section A

Question 1

- (a) (i) 5B
- (ii) 5B
- (b) 15C

Question 2

- (a) 5B
- (b) 10B
- (c) 10C

Question 3

- (a) 15C
- (b) 10C

Question 4

25D

Question 5

- (a) 10C
- (b) 10C
- (c) 5B

Question 6A

- (a) 10B
- (b) 15C

Question 6B

25D

Section B

Question 7

- (a) 10B
- (b) 15C
- (c) 15C
- (d) 15C
- (e) (i) 5A
- (ii) 10B
- (iii) 5B

Question 8

- (a) 20C*
- (b) 10C*
- (c) 10B
- (d) 10B

Question 9

- (a) 15C*
- (b) 10B*

SOLUTIONS TO PAPER 2

QUESTION 1

Part (a) (i)

Scale 5B

(ii)

Scale 5B

Part (b)

Scale 15C

(a) 20% of consignment of oranges are known to be bad. If five oranges are selected at random, find the probability that:

(i) all five are bad.

$$P(\text{all bad}) = \left(\frac{20}{100}\right)^5 = \frac{1}{3125}$$

Full Credit (5 Marks)

- Fully correct.

Partial Credit (3 Marks)

- Partially correct.

(ii) at least one is bad.

$$P(\text{at least one bad}) = 1 - \left(\frac{80}{100}\right)^5 = \frac{2101}{3125}$$

Full Credit (5 Marks)

- Fully correct.

Partial Credit (3 Marks)

- Partially correct.

- (b) A bag contains 8 blue marbles, 4 red marble and x white marbles. A marble is drawn at random and not replaced. A second marble is drawn at random. If the probability that both are white is $\frac{5}{51}$, how many white marbles are in the bag?

$$P(\text{both white}) = \frac{x}{x+12} \times \frac{x-1}{x+11} = \frac{5}{51}$$

$$23x^2 - 83x - 330 = 0$$

$x = 6$ as second answer is invalid

Full Credit (15 Marks)

- Fully correct.

High Partial Credit (13 Marks)

- Correct fractions for each probability but error solving.
- One error in fractions and continues to solve.
- Fails to cancel invalid answer or does not clearly identify valid answer.

Low Partial Credit (8 Marks)

- Incorrect fractions for each probability but continues to solve.
- Any correct step indicated.

QUESTION 2

Part (a)
Part (b)
Part (c)

Scale 5B
Scale 10B
Scale 10C

- (a) Explain what is meant by stratified sampling and give an example of this type of sampling.

Stratified sampling is when the population is divided into sub-populations based on factors such as gender or age, etc.

Full Credit (5 Marks)

- Fully correct.

Partial Credit (3 Marks)

- Partially correct.

- (b) Name one measure of central tendency and give one advantage and one disadvantage of the named measure.

Average	Disadvantages	Advantages
mean	Can be distorted by outliers	Every value is used.
median	Requires ordering of values	Easy to identify. Ignores outliers.
mode	A unique mode may not exist.	Easy to identify. Ignores outliers. Shows what is most typical.

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Partially correct.

- (c) A market research company is carrying out a national poll to find out peoples opinion on the involvement of the E.U. in Irelands banking crisis. The company picks 15 towns at random from a map of Ireland. They then choose 50 phone numbers from each town at random. These people will form the sample. Discuss the validity of choosing the sample in this fashion.

Sample too small.
Excludes people who don't have land line.
15 towns not representative of the country.
People might not understand the topic in question.

Full Credit (10 Marks)

- Fully correct explanation with two reasons why this is a not a valid sample.

High Partial Credit (8 Marks)

- One valid reason given.

Low Partial Credit (5 Marks)

- Partially correct.

QUESTION 3

Part (a)

Scale 15C

Part (b)

Scale 10C

- (a) The lines d , e and f intersect as shown. By using the formula $\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$ investigate the precise nature of the polygon formed by the lines d , e and f .

$$m_p = -\frac{13}{7}, \quad m_q = \frac{13}{9}, \quad m_r = 1$$

$$\text{Angle between } p \text{ and } r = 73.3^\circ$$

$$\text{Angle between } q \text{ and } r = 44.4^\circ$$

$$\text{Angle between } p \text{ and } q = 62.3^\circ$$

\therefore Scalene Triangle

Full Credit (15 Marks)

- Fully correct solution with statement.

High Partial Credit (13 Marks)

- No statement or incorrect statement.
- One error in formula.

Low Partial Credit (8 Marks)

- Partially correct.

- (b) Calculate the area of the polygon formed.

$$(0, 0), (8, 0), (4.5, 6.5)$$

$$\text{Area} = \frac{1}{2} |(8)(6.5) - (4.5)(0)| = 24 \text{ sq. units}$$

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- One translation incorrect.
- Does not translate but continues correctly to end.

Low Partial Credit (5 Marks)

- Partially correct.

QUESTION 4

Scale 25D

A circle of radius length 5 contains the point $(7, 8)$. Its centre lies on the line $-2x + y = -4$.

Find the equations of the two circles that satisfy these conditions.

$$\sqrt{g^2 + f^2 - c} = 5 \qquad \therefore g^2 + f^2 - c = 25 \qquad (1)$$

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad \therefore 14g + 16f + c = -113 \qquad (2)$$

$$(-g, -f) \text{ belongs to } -2x + y = -4 \qquad \therefore 2g + 4 = f \qquad (3)$$

$$5g^2 + 62g + 168 = 0 \qquad (4)$$

$$g = -4, f = -4, c = 7 \qquad \text{or} \qquad g = -8.4, f = -12.9, c = 209.4$$

$$\text{Equation 1: } (x - 4)^2 + (y - 4)^2 = 25$$

$$\text{Equation 2: } (x - 8.4)^2 + (y - 12.8)^2 = 25$$

Full Credit (25 Marks)

- Fully correct.

High Partial Credit (19 Marks)

- Quadratic Correct.
- Equations not in correct form or not stated.
- Error in equations 1, 2 or 3 but continues correctly to end.

Mid Partial Credit (12 Marks)

- Two equations from 1, 2 and 3 correct.

Low Partial Credit (6 marks)

- Any correct step.

QUESTION 5

Part (a)
Part (b)
Part (c)

Scale 10C
Scale 10C
Scale 5B

- (a) Tap A can fill a cylindrical container in 3 minutes. Tap B can fill the same container in 15 minutes. How long would it take the two taps together to fill the container?

$$\frac{1}{3} + \frac{1}{15} = \frac{6}{15} \text{ of the tank per minute}$$

\therefore 132 seconds to fill the tank

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- Both fractions correct and stops.
- One fraction incorrect but continues to end correctly.

Low Partial Credit (5 Marks)

- Partially correct.

- (b) The height of a cylinder is four times its radius. If the volume of the cylinder is $108\pi\text{cm}^3$, calculate the radius and height of the cylinder.

$$\pi(r)^2(4r) = 108\pi$$

$\therefore r = 3\text{cm}, h = 12\text{cm}$

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (8 Marks)

- One error only.

Low Partial Credit (5 Marks)

- Partially correct.

- (c) 32 identical cylinders are packed into a square based box. Calculate the surface area of the box.

$$24 \times 24 \times 6 = 3456\text{cm}^2$$

Full Credit (5 Marks)

- Fully correct.

Partial Credit (3 Marks)

- Partially correct.

QUESTION 6A

Part (a)

Scale 10B

Part (b)

Scale 15C

(a) Solve $\cos \theta = 0.5$ for θ , where $0^\circ \leq \theta \leq 360^\circ$.

$$\theta = 60^\circ, 300^\circ$$

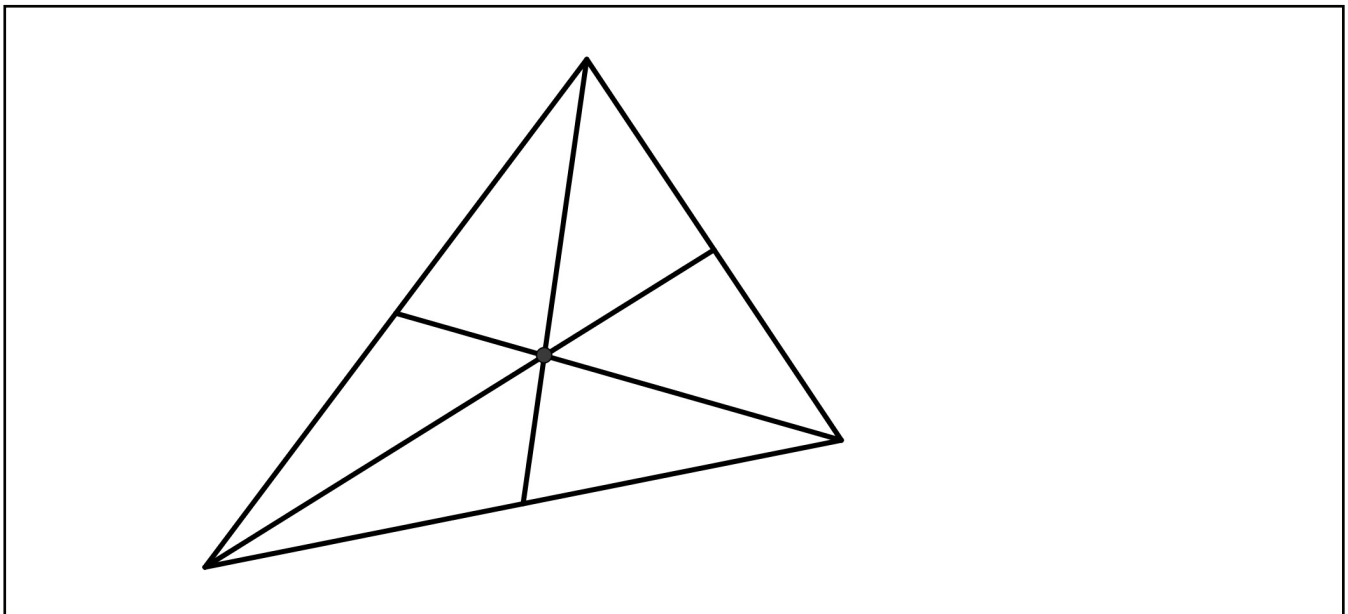
Full Credit (10 Marks)

- Fully correct solution.

Partial Credit (6 Marks)

- One correct solution.
- Any correct diagram showing four quadrants and CAST.

(b) A student is trying to find the centre of gravity of the following triangle. Use a suitable geometrical construction to find this point.



Full Credit (15 Marks)

- Fully correct.

High Partial Credit (13 Marks)

- Constructs another centre correctly.

Low Partial Credit (8 Marks)

- One median correct.

QUESTION 6B

Scale 25D

Prove that if two triangles ABC and DEF are similar then their sides are in proportion.

Full Credit (25 Marks)

- Fully correct proof.

High Partial Credit (19 Marks)

- No statement or incorrect statement.
- One error in formula.

Mid Partial Credit (12 Marks)

- Partially correct.

Low Partial Credit (6 Marks)

- Any correct step.

QUESTION 7

Part (a)	Scale 10B
Part (b)	Scale 15C
Part (c)	Scale 15C
Part (d)	Scale 15C
Part (e) (i)	Scale 5A
(ii)	Scale 10B
(iii)	Scale 5B

The following table shows the numbers employed in certain sectors of industry in Ireland between 2005 and 2012.

Employment and Unemployment (ILO) '000s

Economic Sector	Apr - Jun 05	Apr - Jun 06	Apr - Jun 07	Apr - Jun 08	Apr - Jun 09	Apr - Jun 10	Apr - Jun 11	Apr - Jun 12
Agriculture, Forestry & Fishing	110	110	109	115	97	85	86	87
Construction	228	252	270	241	155	125	106	99
Accommodation and food service activities	117	125	131	15	120	120	107	114
Information and communication	66	70	71	71	74	74	75	78
Education	126	136	141	146	150	150	147	144
Human health and social work activities	186	201	210	221	228	235	238	237
Total in Employment	833	894	932	809	824	789	759	759
Total Unemployed	96	98	103	127	265	294	305	309
Total Labour Force	929	182	1035	936	1089	1083	1064	1068

(a) Calculate the total labour force for each Apr-Jun period.

Full Credit (10 Marks)

- Fully correct.

Partial Credit (6 Marks)

- Partially correct.

- (b) Choose a suitable graphical display to compare the employment figures in 3 of the sectors in the table above from 2005 to 2012.

Full Credit (15 Marks)

- Fully correct suitable display.

High Partial Credit (13 Marks)

- Axis labelled incorrectly or not labelled.
- Axis scaled incorrectly or not scaled.
- Unsuitable display chosen but correct.
- Two sets only displayed on one chart.
- Three individual displays but fully correct.

Low Partial Credit (8 Marks)

- Some correct attempt to display data.
- Only one set displayed.
- Three individual display but not fully correct.

- (c) Discuss the shape and distribution of each sector you have chosen. In your opinion, why do you think these sectors have the distribution in the table?

Full Credit (15 Marks)

- Fully correct description shape of each sector.

High Partial Credit (13 Marks)

- Two shapes correct with mention.

Low Partial Credit (8 Marks)

- One shape correct.

- (d) Compare and contrast the figures in the construction and the education sector using a measure of central tendency and a measure of spread. Explain why you think the employment figures in both industries have followed the trends in the table.

Full Credit (15 Marks)

- Fully correct explanation with mention of a measure of central tendency and a measure of spread.

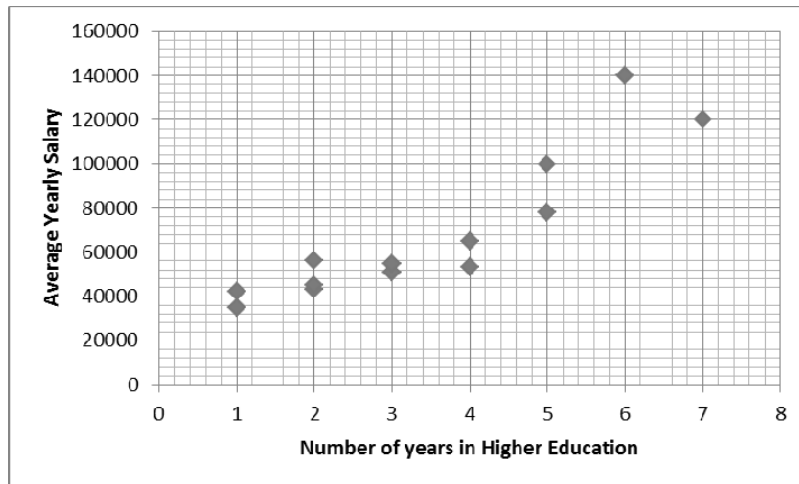
High Partial Credit (13 Marks)

- Partially correct with mention of a measure of central tendency and a measure of spread.
- Incorrect explanation with mention of a measure of central tendency and a measure of spread.

Low Partial Credit (8 Marks)

- Partially correct with mention of a measure of central tendency or a measure of spread.
- Incorrect explanation with mention of a measure of central tendency or a measure of spread.

(e) Examine the following scatter plot.



(i) Describe the correlation coefficient of the plot.

Strong Positive correlation.

Full Credit (5 Marks)

- Correct.

(ii) Calculate the correlation coefficient of the plot correct to 6 decimal places.

$$r = 0.900662$$

Full Credit (10 Marks)

- Fully correct.

High Partial Credit (6 Marks)

- Partially correct.

(iii) In your opinion what is the plot attempting to show?

Time in higher education can lead to a better salary or similar.

Full Credit (5 Marks)

- Fully correct.

QUESTION 8

Part (a)	Scale 20C*
Part (b)	Scale 10C*
Part (c)	Scale 10B
Part (d)	Scale 10B

European astronomers carried out an experiment that started to measure the distance of Venus from the Sun. When the centre of Venus is at a right angles to both the centres of the Earth and the Sun it as found that Venus was 1.05×10^8 km from the Sun and that $|\angle SEV| = 44.4^\circ$.

- (a) Calculate the distance the centre of the earth to the centre of the Sun correct to two significant figures.

$$\sin 44.4 = \frac{1.05 \times 10^8 \text{ km}}{|SE|}$$
$$|SE| = 1.5 \times 10^8 \text{ km}$$

Full Credit (20 Marks)

- Fully correct solution .

High Partial Credit (18 Marks)

- Incorrect formula or ratio but continues to end correctly.

Low Partial Credit (10 Marks)

- Incorrect formula or ratio but continues to end incorrectly.
- Any correct addition to the diagram.

A satellite is put in orbit so that it is equidistance from all three centres when they are positioned as above.

- (b) Where would the satellite be located? Explain your answer fully.

The satellite would be located on the midpoint of $|SE|$.
Circumcentre is equidistance from all three centres and in a right angled triangle the circumcentre lies on the hypotenuse.

Full Credit (20 Marks)

- Correct location and statement.

High Partial Credit (18 Marks)

- Incorrect location but correct statement.
- Correct location and incorrect statement.

Low Partial Credit (10 Marks)

- Mention of orthocentre, incentre or circumcentre.
- Additions to diagram (not previously awarded).

(c) At what ratio does Venus orbit the sun compared with the earths orbit?

$$\frac{105}{150} = 0.7$$

Full Credit (10 Marks)

- Correct ratio.

Partial Credit (6 Marks)

- Some correct step.

(iv) Will this ratio hold for all positions of the three planets? Explain your answer fully.

Yes.
The planets are on fixed circular orbit and the ratio will hold as the both orbit the Sun.

Full Credit (10 Marks)

- Correct ratio.

Partial Credit (6 Marks)

- Partially correct.

QUESTION 9

Part (a)

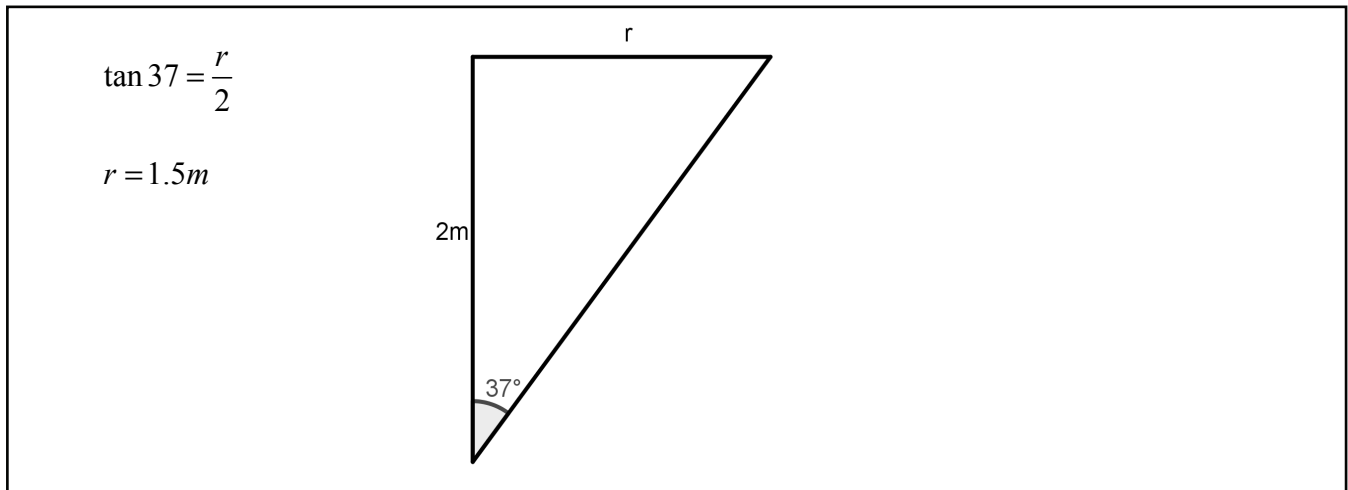
Scale 15C*

Part (b)

Scale 10B*

- (a) **Snell's window** is a phenomenon by which an underwater viewer sees everything above the surface through a cone of light. It is caused by the refraction of light. Only light which strikes the surface of the water within a circle of a certain radius, r , will reach a point P below the surface.

A light at the bottom of a pool 2 m deep emits light upwards in all directions. The maximum angle created by a ray of light and the vertical is 37° . Calculate the radius of the disc through which the light leaves the surface of the water, correct to one decimal place.



*Incorrect or omitted unit – 1 from a fully correct solution

Full Credit (15 Marks)

- Fully correct solution.

High Partial Credit (13 Marks)

- Correct right angled triangle.
- Incorrect triangle but continues to end.

Low Partial Credit (8 Marks)

- Attempts to draw triangle.
- Trigonometric ratio with 1 substitution.
- Any valid attempt to solve.

- (b) Calculate the volume of water contained in the cone, correct to one decimal place.

$$\frac{1}{3}\pi(1.5)^2(2) = 4.7m^3$$

Full Credit (10 Marks)

- Fully correct solution.

Partial Credit (6 Marks)

- Some correct Step.