PRE-LEAVING CERTIFICATE EXAMINATION, 2014

MARKING SCHEME

PROJECT MATHS (Phase 3)

HIGHER LEVEL

Table of Contents

OVERVIEW OF MARKING SCHEME (QUESTIONS 1 TO 6)

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 3, 5	0, 3, 4, 5		
10 mark scale	0, 10	0, 6, 10	0, 5, 8, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 8, 15	0, 8, 13, 15	0, 7, 10, 13, 15	
20 mark scale	0, 20	0, 10, 20	0, 10, 18, 20	0, 5, 10, 15, 20	
25 mark scale		0, 12, 25	0, 8, 20, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)
- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)
- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

Marking categories for all questions are shown throughout the solutions.
In certain cases, typically involving rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, Scale 10C* indicates that 9 marks may be awarded.

SOLUTIONS TO PAPER 1

QUESTION 1

Part (a)	**Scale 5C**
Part (b)	**Scale 10C**
Part (c)	**Scale 10C**

(a) If $z = a + bi$, where a and $b \in \mathbb{R}$, show that $z.\bar{z}$ simplifies to a constant.

$$(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2$$
$$a^2 + b^2 \quad \therefore a \text{ constant}$$

Full Credit (5 Marks)
• Fully correct

High Partial Credit (3 Marks)
• One error in multiplication
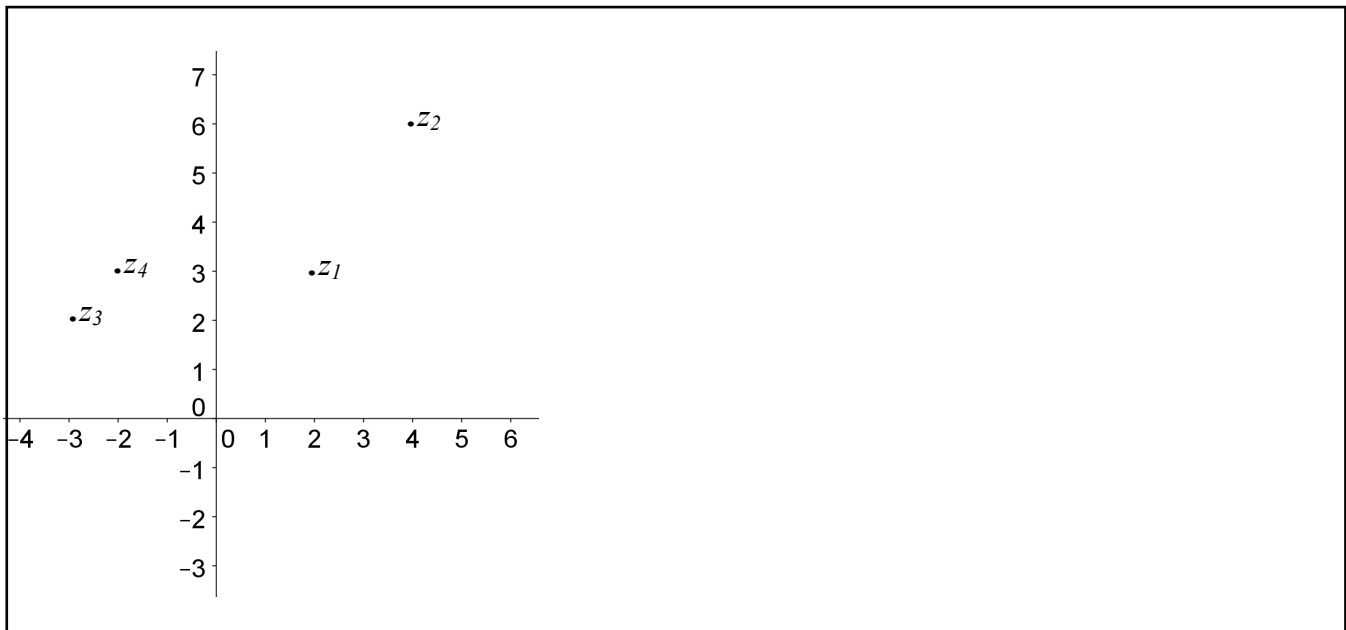
Low Partial Credit (2 Marks)
• Any correct step

(b) Examine the complex plane below. Identify on the diagram each complex number.

$$z_2 = 2z_1 \qquad z_3 = iz_1 \qquad z_4 = \text{image of } z_1 \text{ under } S_y$$

Full Credit (10 Marks)
• Fully correct

High Partial Credit (7 Marks)
• Two correctly identified

Low Partial Credit (3 Marks)
• One correctly identified

(c)   Use De Moivre's theorem to prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

$$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$$
$$\cos 3\theta + i\sin 3\theta = \cos^2\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$$
$$i\sin 3\theta = 3i\cos^2\theta\sin\theta - i\sin^3\theta$$
$$\sin 3\theta = 3(1 - \sin^2\theta)\sin\theta - \sin^3\theta$$
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

Full Credit (10 Marks)
•   Fully correct

High Partial Credit (7 Marks)
•   Error in expansion but continues to end

Low Partial Credit (3 Marks)
•   Any correct step

QUESTION 2

Part (a)	**Scale 10C**
Part (b)	**Scale 15C**

(a) Show that $a^2 + 16 + b^2 \geq 2ab$ for all values of $a, b \in \mathbb{R}$.

$a^2 + 16 + b^2 \geq 2ab$

$a^2 - 2ab + b^2 + 16 \geq 0$

$(a-b)^2 + (4)^2 \geq 0$

$\therefore$ true as all squared numbers are ≥ 0

Full Credit (10 Marks)
• Fully correct

High Partial Credit (8 Marks)
• One error in multiplication

Low Partial Credit (4 Marks)
• Any correct step

(b) Solve the simultaneous equations: $x + 3y = -5$

$2x^2 + y^2 = 41$

where $x, y \in \mathbb{Z}$

$x + 3y = -5$

$x = -3y - 5$

$2(-3y-5)^2 + y^2 = 41$

$19y^2 + 60y + 9 = 0$

$y = -3$ or $y = -\dfrac{3}{19} \notin \mathbb{Z}$

$x = 4$

Full Credit (15 Marks)
• Fully correct

High Partial Credit (13 Marks)
• Error solving the quadratic but continues to end with no further errors
• Fully correct but uses the $y = -\dfrac{3}{19}$ and continues to end

Low Partial Credit (8 Marks)
• Any correct step

QUESTION 3

Part (a)	Scale 5C
Part (b)	Scale 10C
Part (c)	Scale 10C

(a) Given that $x^2 + x - 6$ is a factor of $2x^3 - px^2 + qx - 6$, find the value of p and q.

> By inspection: $\quad (x^2 + x - 6)(2x + 1)$
>
> $\qquad\qquad\qquad 2x^3 + 3x^2 - 11x - 6$
>
> $\qquad\qquad\qquad \therefore p = -3$ and $q = -11$

*Other acceptable methods warrant full marks

Full Credit (5 Marks)
- Fully correct

High Partial Credit (3 Marks)
- One error only and solves to end

Low Partial Credit (2 Marks)
- Any correct step

(b) Prove that the function $f(x): x^3 + 7x^2 + 17x + 15$ has only one real root.

> $f(x): x^3 + 7x^2 + 17x + 15$
>
> $f(-3): \quad (-3)^3 + 4(-3)^2 + 17(-3) + 15 = 0 \quad \therefore (x + 3)$ is a factor
>
> $\dfrac{x^3 + 7x^2 + 17x + 15}{x + 3} = x^2 + 4x + 5$
>
> $x^2 + 4x + 5 = (x + 2 - i)(x + 2 + i) \quad \therefore$ Complex roots
>
> $\therefore f(x): \quad x^3 + 7x^2 + 17x + 15$ has only one real root

*Note: Students may also carry out by calculus.
*Note: Candidates may also use $b^2 - 4ac < 0$ from resulting quadratic

Full Credit (10 Marks)
- Fully correct (algebra)
- By calculus candidate must have a fully correct solution using the quadratic formula.

High Partial Credit (7 Marks)
- One error in solving but continues to end
- Omits conclusion with a fully correct solution
- Differentiates and attempts to solve the resulting quadratic

Low Partial Credit (3 Marks)
- Any correct step

(c) Solve for x.

$$3^{2x+1} + \left(26(3)^x\right) - 9 = 0$$

$$3^{2x+1} + 26\left(3^x\right) - 9 = 0$$
$$3^1 \cdot \left(3^x\right)^2 + 26\left(3^x\right) - 9 = 0$$
$$3y^2 + 26y - 9 = 0$$
$$(3y - 1)(y + 9)$$
$$y = \frac{1}{3} \text{ or } y = -9$$
$$3^x = 3^{-1} \text{ or } 3^x \neq -9$$
$$\therefore x = -1$$

Full Credit (10 Marks)
• Fully correct

High Partial Credit (7 Marks)
• One error in solving
• Fails to omit the invalid solution

Low Partial Credit (3 Marks)
• Any correct step

QUESTION 4

Part (a)	**Scale 15D**
Part (b)	**Scale 10C**

(a) Prove by induction that 3 is a factor of $5^n - 2^n$ for all $n \in \mathbb{N}$ and $n \geq 1$.

Step 1: Prove for $n = 1$
$5^1 - 2^1 = 3$ ... is divisible by 3

Step 2: Assume true for $n = k$
$5^k - 2^k = 3A$
$\therefore 5^k = 3A + 2^k$

Step 3: Prove true for $n = k + 1$
$5^{k+1} - 2^{k+1}$
$5.5^k - 2.2^k$
$5\left(3A + 2^k\right) - 2.2^k$
$15A + 3.2^k$
$3\left(5A + 2^k\right)$

3 is a factor for $n = k + 1$

Step 4: This is true for $n = 1$. If it is true for $n = k$ it will also be true for $n = k + 1$
$\therefore$ true for all $n \in \mathbb{N}$ by induction.

Full Credit (15 Marks)
• Fully correct solution

High Partial Credit (11 Marks)
• Three correct steps

Mid Partial Credit (7 Marks)
• Two correct steps.

Low Partial Credit (4 Marks)
• Any correct step

(b) Séan wants to save €50,000 over a ten-year period. If the expected interest rate is 2.5% how much does Séan need to invest at the beginning of each year to save €50,000?

$$€50,000 = P + P(1.025)^1 + P(1.025)^2 + P(1.025)^3 + \ldots + P(1.025)^9$$

$$a = P \qquad r = 1.025 \qquad n = 10$$

$$S_{10} = \frac{P(1 - 1.025^{10})}{1 - 1.025}$$

$$50,000 = \frac{P(1 - 1.025^{10})}{1 - 1.025}$$

$$50,000 = 11.20338177P$$

$$P \approx €4,462.94$$

Full Credit (10 Marks)
*   Fully correct

High Partial Credit (7 Marks)
*   Sets up geometric series but error in solving

Low Partial Credit (3 Marks)
*   Any correct step

QUESTION 5

Part (a)	**Scale 10C**
Part (b)	**Scale 15C***

(a) Evaluate $\int_0^1 6x^2 e^{x^3} dx$.

$$\int_0^1 6x^2 e^{x^3} dx$$
$$u = x^3$$
$$\frac{du}{dx} = 3x^2$$
$$2du = 6x^2 dx$$
$$2\int e^u du = 2e^u = 2e^u \Big|_0^1$$
$$2e - 2$$

Full Credit (10 Marks)
• Fully correct

High Partial Credit (7 Marks)
• One error and continues to end without over simplification of the problem

Low Partial Credit (3 Marks)
• Any correct step

(b) Find the area bounded by the curve $f(x)$, the line $g(x)$ and the x-axis.

$f(x) = (x+2)(x-3) = -x^2 + x + 6$

$g(x) = -x + 3$

$f(x) = g(x)$

$-x^2 + x + 6 = -x + 3$
$x^2 - 2x - 3 = 0$
$x = -1 \ \text{or} \ x = 3$

$\text{Area} = \int_{-2}^{-1} f(x) + \text{Area} \ \Delta \ \text{from} -1 \ \text{to} \ 3$

$\text{Area} = \int_{-2}^{-1} (-x^2 + x + 6) + \frac{1}{2}(4)(4)$

$\text{Area} = \left(-\frac{x^3}{3} + \frac{x^2}{2} + 6x \right)\Big|_{-2}^{-1} + 8$

$\text{Area} = \frac{61}{6} \ \text{square units}$

*If candidate takes points from graph High Partial at most can be achieved.

Full Credit (15 Marks)
- Fully correct solution

High Partial Credit (11 Marks)
- Reads points from graph and finishes correctly
- Finds points algebraically and finishes to end with no more than two errors

Mid Partial Credit (7 Marks)
- Reads points form graph and finishes with no more than two errors

Low Partial Credit (4 Marks)
- Any correct step

QUESTION 6

Part (a)	**Scale 10C**
Part (b)	**Scale 5C**
Part (c) (i)	**Scale 5C**
(ii)	**Scale 5C**

(a) Differentiate $-2x^2 + 3x - 5$ from first principles.

$$f(x) = -2x^2 + 3x - 5$$

$$f(x+h) = -2(x+h)^2 + 3(x+h) - 5 = -2x^2 - 4xh - 2h^2 + 3h$$

$$f(x+h) - f(x) = -4hx - 2h^2 + 3h$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-4xh}{h} - \frac{2h^2}{h} + \frac{3h}{h}$$

$$\frac{f(x+h) - f(x)}{h} = -4x - 2h + 3$$

$$\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h} = -4x - 0 + 3$$

$$\lim_{h \to \infty} \frac{f(x+h) - f(x)}{h} = -4x + 3$$

Full Credit (10 Marks)
- Fully correct

High Partial Credit (7 Marks)
- Error expanding brackets but continues to end correctly
- Error handling limits

Low Partial Credit (3 Marks)
- Any correct step

(b)   Find the equation of the tangent to the curve $y = x^2 + 3x - 5$ at the point $(1, 2)$.

$$\frac{dy}{dx} = 2x + 3$$
$$2(1) + 3 = 5$$
$$m = 5 \quad (1, -6)$$
$$y - y_1 = m(x - x_1)$$
$$y - 2 = 5(x - 1)$$
$$5x - y = 3$$

Full Credit (5 Marks)
•   Fully correct

High Partial Credit (3 Marks)
•   Error finding slope but continues to end correctly

Low Partial Credit (2 Marks)
•   Any correct step

(c) The function $g(x) = \dfrac{2}{2-3x}$ is defined for all $x \in \mathbb{R} \setminus \left\{ \dfrac{2}{3} \right\}$.

(i) Prove that the function has no turning points.

$$g'(x) = \frac{6}{(2-3x)^2} \neq 0$$

$\therefore g(x)$ has no turning points

Full Credit (5 Marks)
• Fully correct

High Partial Credit (3 Marks)
• Error differentiation but continues to end correctly
• Correct with no conclusion

Low Partial Credit (2 Marks)
• Any correct step

(ii) Jane states that the graph of $g(x)$ would be increasing for all values of x.
Would you agree with Jane? Explain your answer fully.

$$g'(x) = \frac{6}{(2-3x)^2}$$

$\therefore g'(x)$ is always positive

$\therefore g(x)$ always increasing for all values of x.   Yes.

Full Credit (5 Marks)
• Fully correct

High Partial Credit (3 Marks)
• Error differentiation but continues to end correctly
• Correct with no conclusion

Low Partial Credit (2 Marks)
• Any correct step

QUESTION 7

Part (a)	**Scale 5A***
Part (b)	**Scale 15C**
Part (c)	**Scale 15C**
Part (d)	**Scale 20D***

(a) Using the information above, write an equation to represent the volume of water in the cone at any time t.

$V = 0.02t$ litres/second

Full Credit (5 Marks)
• Fully correct

(b) Write the radius of the cone r, in terms of the height of the water h.

$$\tan 30 = \frac{r}{h}$$

$$\therefore r = h \tan 30 \text{ or } r = \frac{h}{\sqrt{3}}$$

Full Credit (15 Marks)
• Fully correct

High Partial Credit (13 Marks)
• Incorrect trigonometric ratio and continues

Low Partial Credit (8 Marks)
• Any correct step

(c) Show that the height of the water h at any time t can be expressed as $h = \sqrt[3]{\dfrac{0.18t}{\pi}}$.

$$V = \frac{1}{3}\pi r^2 h = 0.02$$

$$\therefore \frac{1}{3}\pi \left(\frac{h}{\sqrt{3}}\right)^2 h = 0.02t$$

$$h = \sqrt[3]{\frac{0.18t}{\pi}}$$

Full Credit (15 Marks)
• Fully correct

High Partial Credit (13 Marks)
• No more than two errors transposing the formula

Low Partial Credit (8 Marks)
• Any correct step

(d) Find the rate of change in the height of the water after 3 seconds.

$$h = \sqrt[3]{\frac{0.18t}{\pi}}$$

$$h = \left(\sqrt[3]{\frac{0.18}{\pi}} \right)\left(t^{\frac{1}{3}} \right)$$

$$\frac{dh}{dt} = \frac{1}{3}\left(\sqrt[3]{\frac{0.18}{\pi}} \right)\left(t^{-\frac{2}{3}} \right)$$

At $t = 3$

$$\frac{dh}{dt} = \frac{1}{3}\left(\sqrt[3]{\frac{0.18}{\pi}} \right)\left((3)^{-\frac{2}{3}} \right)$$

$$\frac{dh}{dt} = 0.062 \text{ litres/second}$$

*Accept candidate's "h" from previous section

Full Credit (20 Marks)
•   Fully correct solution

High Partial Credit (15 Marks)
•   Correct differential and stops
•   Incorrect differential and continues to end correctly

Mid Partial Credit (10 Marks)
•   Incorrect differential and stops

Low Partial Credit (5 Marks)
•   Any correct step

QUESTION 8

Part (a)	**Scale 10C**
Part (b)	**Scale 10C***
Part (c)	**Scale 15C**

In 1935, while working at the California Institute of Technology, Charles Francis Richter invented the Richter scale. The Richter scale is used to measure earthquake intensity on a machine called a seismograph.

The formula for the Richter is given by $M = \log_{10}\left[\dfrac{I}{I_0}\right]$

where M is the magnitude on the Richter scale in millimetres, I_0 is the 'threshold quake intensity', i.e. a movement that can barely be detected, and I is the intensity of the earthquake in millimetres. $I_0 = 10^{-3}$ mm.

(a) Calculate the magnitude of an earthquake with an intensity of 500 mm.

$$M = \log_{10}\left[\frac{500}{10^{-3}}\right]$$
$$M = 5.69897 \approx 5.7$$

Full Credit (10 Marks)
• Fully correct

High Partial Credit (8 Marks)
• Error in the calculation

Low Partial Credit (4 Marks)
• Any correct step

(b) In 2005 an earthquake in Kashmir in Pakistan measured 7.5 on the Richter scale. Calculate the intensity of the earthquake.

$$7.5 = \log_{10}\left[\frac{I}{10^{-3}}\right]$$
$$\left(10^{7.5}\right)\left(10^{-3}\right) = I$$
$$I = 31622.7766 \ mm$$

Full Credit (10 Marks)
• Fully correct

High Partial Credit (8 Marks)
• One error in the calculation

Low Partial Credit (4 Marks)
• Any correct step

(c) A heavy goods lorry rumbling on a road can cause a micro quake with a rating of up to 3.3 on the Richter scale. An at-home seismograph records an intensity of $40,000\,I_0$ while the owner is away on holidays. What event was likely to have occurred while the owner was away, an earthquake or a heavy goods lorry passing his house?

$$M = \log_{10}\left[\frac{40,000I_0}{I_0}\right]$$
$$M = 4.6$$

Likely to have been a small earthquake and not a heavy goods lorry as magnitude is too big for a heavy goods lorry.

Full Credit (15 Marks)
• Fully correct with explanation

High Partial Credit (13 Marks)
• Correct with no explanation
• Incorrect but with a correct explanation

Low Partial Credit (8 Marks)
• Any correct step

QUESTION 9

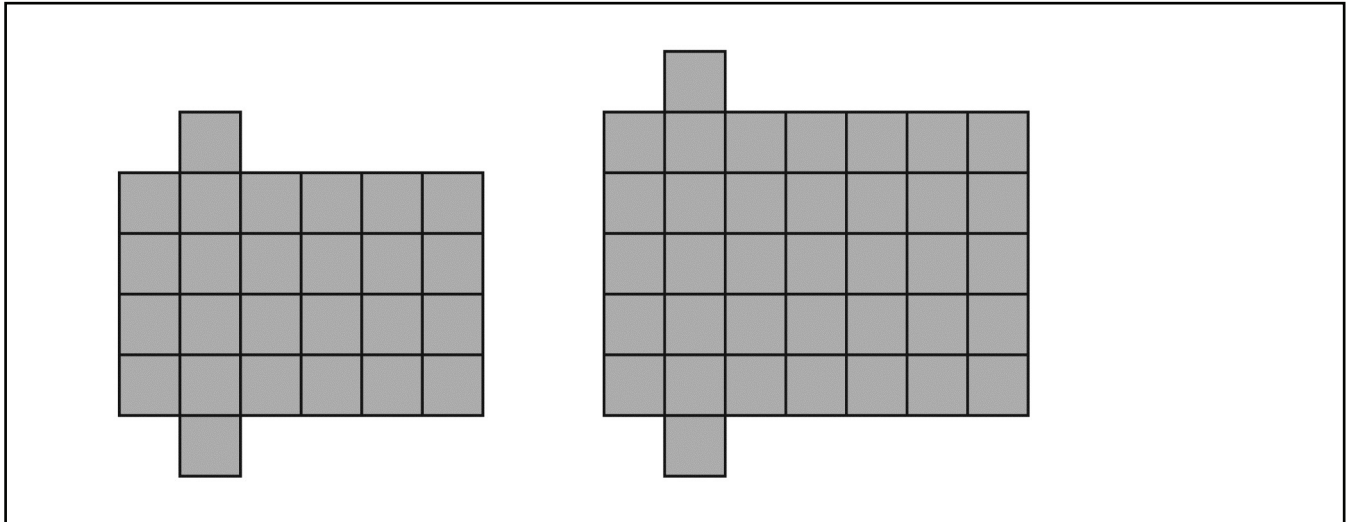|	
Part (a)	**Scale 10C**
Part (b)	**Scale 15C**
Part (c)	**Scale 10B**
Part (d)	**Scale 20D**
Part (e)	**Scale 5B**

Paul is creating designs using green blocks.

(a)   Draw the next two patterns in the sequence.

Full Credit (10 Marks)
• 　　Both correct

High Partial Credit (7 Marks)
• 　　One correct

Low Partial Credit (3 Marks)
• 　　One partially correct

(b) John states the pattern is linear. Mary disagrees. Would you agree with John or Mary? Explain your answer fully.

Pattern	5	10	17	26
1st Difference		5	7	9
2nd Difference			2	2

Pattern is quadratic as the second differences are the same.

Full Credit (15 Marks)
- Fully correct with explanation

High Partial Credit (13 Marks)
- Correct with no explanation
- Incorrect but with a correct explanation

Low Partial Credit (8 Marks)
- Any correct step

(c) The general term for the sequence is given by $T_n = an^2 + bn + c$. Find the values of a, b and c.

$T_1 : a + b + c = 5, \quad T_2 : 4a + 2b + c = 10, \quad T_3 : 9a + 3b + c$
Gives $a = 1, \quad b = 2, \quad c = 2$
$T_n = 1n^2 + 2n + 2$

Full Credit (20 Marks)
- Fully correct

High Partial Credit (13 Marks)
- Forms three correct equations but has no more than two errors in solving
- Forms three incorrect equations and has no errors in solving

Low Partial Credit (7 Marks)
- Any correct step

(d) What pattern in the sequence will contain 122 green blocks? Verify your solution.

$$T_n = n^2 + 2n + 2 = 122$$
$$n^2 + 2n - 120 = 0$$
$$n = 10 \quad \therefore \text{Tenth pattern}$$

Full Credit (15 Marks)
- Fully correct and verified

High Partial Credit (13 Marks)
- Error setting up the quadratic but continues to end and tries to verify
- Correct but fails to verify

Low Partial Credit (8 Marks)
- Any correct step

(e) How many blocks would Paul need to create 21 patterns?

3815 blocks

Full Credit (5 Marks)
- Fully correct

Partial Credit (2 Marks)
- Any correct attempt to find total sum

SOLUTIONS TO PAPER 2

QUESTION 1

Part (a)	**Scale 5B**
Part (b)	**Scale 5B**
Part (c)	**Scale 5B**
Part (d)	**Scale 10B**

(a) Explain the term Stratified Sampling.

> Where the population is split based on certain factors, e.g. Age, Sex

Full Credit (5 Marks)
- Fully correct
- Explains with a valid example

Partial Credit (3 Marks)
- Partially Correct

(b) Explain the term Cluster Sampling.

> Sample or population splits naturally into clusters, e.g. geographical areas.

Full Credit (5 Marks)
- Fully correct
- Explains with a valid example

Partial Credit (3 Marks)
- Partially correct

(c) Are Stratified and Cluster Sampling random or non-random methods of sampling? Explain your answer fully.

> Both are random as everyone in the each population has an equal change of being included in the sample.

Full Credit (5 Marks)
- Fully correct
- Explains with a valid example

Partial Credit (3 Marks)
- Partially correct

(d) An estate agent wishes to find out the prices of the houses in a newly developing area of a city. Which type of sampling above would be most suitable to use? Explain your answer fully.

> Cluster sampling. As he wants to find the prices in a certain area, i.e. a geographical cluster.

Full Credit (10 Marks)
- Fully correct
- Explains with a valid example

Partial Credit (6 Marks)
- Partially correct

QUESTION 2

Part (a) (i)	Scale 5B
(ii)	Scala 5B
Part (b) (i)	Scala 5C
(ii)	Scale 10C

(a) **(i)** What is a Bernoulli Trial?

> A Bernoulli trial is a random experiment with exactly two possible outcomes, "success" and "failure", in which the probability of success is the same every time the experiment is conducted.

Full Credit (5 Marks)
* Fully correct
* Explains with a valid example

Partial Credit (3 Marks)
* Partially correct

(ii) Give two characteristics of a binomial distribution.

> Finite number of trials
> Two outcomes. Success or failure
> Trials are independent of each other
> The probability of a success is the same for each trial

Full Credit (5 Marks)
* Two correct

Partial Credit (3 Marks)
* One correct
* One Partially correct

(b) At a certain Garda checkpoint one in every 10 cars has a faulty headlamp.

(i) Find the probability that the first three cars will have a faulty headlamp.

$$\binom{5}{3}\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^2 = \frac{81}{10,000}$$

Full Credit (5 Marks)
• Fully correct with work shown

High Partial Credit (3 Marks)
• Correct with no work shown
• Incorrect with workings shown

Low Partial Credit (2 Marks)
• Any correct step

(ii) Find the probability that at least two of the first four cars stopped will have a faulty headlamp.

$$1-(\text{none have a faulty lamp or 1 has a faulty lamp})$$

$$1-\left(\binom{5}{5}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^5+\binom{5}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^4\right)$$

$$\frac{4,073}{50,000}$$

Full Credit (10 Marks)
• Fully correct with work shown

High Partial Credit (8 Marks)
• Correct with no work shown
• Incorrect with workings shown

Low Partial Credit (4 Marks)
• Any correct step

QUESTION 3

Part (a)	**Scale 10C**
Part (b)	**Scale 15C**

(a) Calculate the shortest distance between the line $6x+7y-10=0$ and the point $(3,6)$.

$$\frac{\left|6(3)+7(6)-10\right|}{\sqrt{(6)^2+(7)^2}}=\frac{10\sqrt{85}}{17}$$

Full Credit (10 Marks)
*   Fully correct with work shown

High Partial Credit (8 Marks)
*   Correct with no work shown
*   Incorrect with workings shown
*   Uses distance between two points correctly to end with a second point on the line

Low Partial Credit (4 Marks)
*   Any correct step
*   Uses distance between two points correctly to end with a second point not on the line

(b) Find the equations of the lines through the point $(-4,-2)$ which make an angle of $45°$ with the line $x+2y=7$.

$$\tan 45 = \left|\frac{-\dfrac{1}{2}-m}{1+\dfrac{1}{2}m}\right|$$

$$1 = \frac{\dfrac{1}{4}+m+m^2}{1-m+\dfrac{1}{4}m^2}$$

$$3m^2+8m-3=0 \qquad m=\frac{1}{3} \text{ or } m=-3 \qquad x-3y-2=0 \text{ and } 3x+y+14=0$$

Full Credit (15 Marks)
*   Fully correct solution

High Partial Credit (11 Marks)
*   Finds the slopes correctly but omits the equations of one or both lines
*   Finds the slopes with no more than two errors and finds the equations of both lines
*   Finds the slopes correctly and has errors in one or both equations of the lines

Mid Partial Credit (7 Marks)
*   Finds the slopes with more than two errors and finds the equations of both lines

Low Partial Credit (4 Marks)
*   Any correct step

QUESTION 4

Part (a)	**Scale 15D**
Part (b)	**Scale 10C**

(a) Find the equation of the circle c, which passes through the points $(-1,-3)$ and $(8,-2)$ and whose centre lies on the line $4x + 5y = 22$.

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Equation 1:
$$(-1)^2 + (-3)^2 + 2g(-1) + 2f(-3) + c = 0$$
$$-2g - 6f + c = -10$$

Equation 2:
$$(8)^2 + (-2) + 2g(8) + 2f(-2) + c = 0$$
$$16g - 4f + c = -68$$

Equation 3:
$$4(-g) + 5(-f) = 22$$
$$-4g - 5f = 22$$

Solving: $\quad g = -3 \quad f = -2 \quad c = -28$
$$\therefore x^2 + y^2 - 6x - 4y - 28 = 0$$

Full Credit (15 Marks)
• Fully correct with work shown

High Partial Credit (11 Marks)
• Finds three equations and attempts and stops
• No more than two errors solving correct equations

Mid Partial Credit (7 Marks)
• More than two errors solving correct equations
• Finds two correct equations

Low Partial Credit (4 Marks)
• Any correct step

(b) The line $5x - 4y + k = 0$ is a tangent to the circle c. Find two possible values for k.

$$\sqrt{(-3)^2 + (-2)^2 - (-28)} = \sqrt{41}$$

$$\left| \frac{5(3) - 4(2) + k}{\sqrt{(5)^2 + (4)^2}} \right| = \sqrt{41}$$

$$(7 + k)^2 = 41$$

$$k^2 + 14k - 1632 = 0$$

$$\frac{-(14) \pm \sqrt{(14)^2 - 4(1)(-1632)}}{2(1)}$$

$$k = 35 \quad \text{or} \quad k = -49$$

Full Credit (10 Marks)
• Fully correct with work shown

High Partial Credit (7 Marks)
• No more than two errors solving for k

Low Partial Credit (3 Marks)
• Any correct step

QUESTION 5

(a)   Find two values for θ, where $\sin\theta = -\dfrac{\sqrt{3}}{2}$ and $0° \leq \theta \leq 360°$.

$$\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60°$$

$$\theta_1 = 240° \quad \text{and} \quad \theta_2 = 300°$$

Full Credit (10 Marks)
•   Fully correct with work shown

High Partial Credit (8 Marks)
•   Only finds one value for θ
•   Finds two incorrect values for θ with work shown

Low Partial Credit (4 Marks)
•   Any correct step

(b)   If the point D is the midpoint of $[AC]$, find $[\angle BCD]$, correct to the nearest degree.

$$|AD|^2 = (7.6)^2 + (8)^2 - (7.6)(8)\cos(50)$$

$$|AD| = 6.6$$

$$(8)^2 = (6.6)^2 + (12)^2 - 2(6.6)(12)\cos|\angle BCD|$$

$$|\angle BCD| = 39°$$

Full Credit (15 Marks)
•   Fully correct with work shown

High Partial Credit (13 Marks)
•   Solves with nn more than two errors

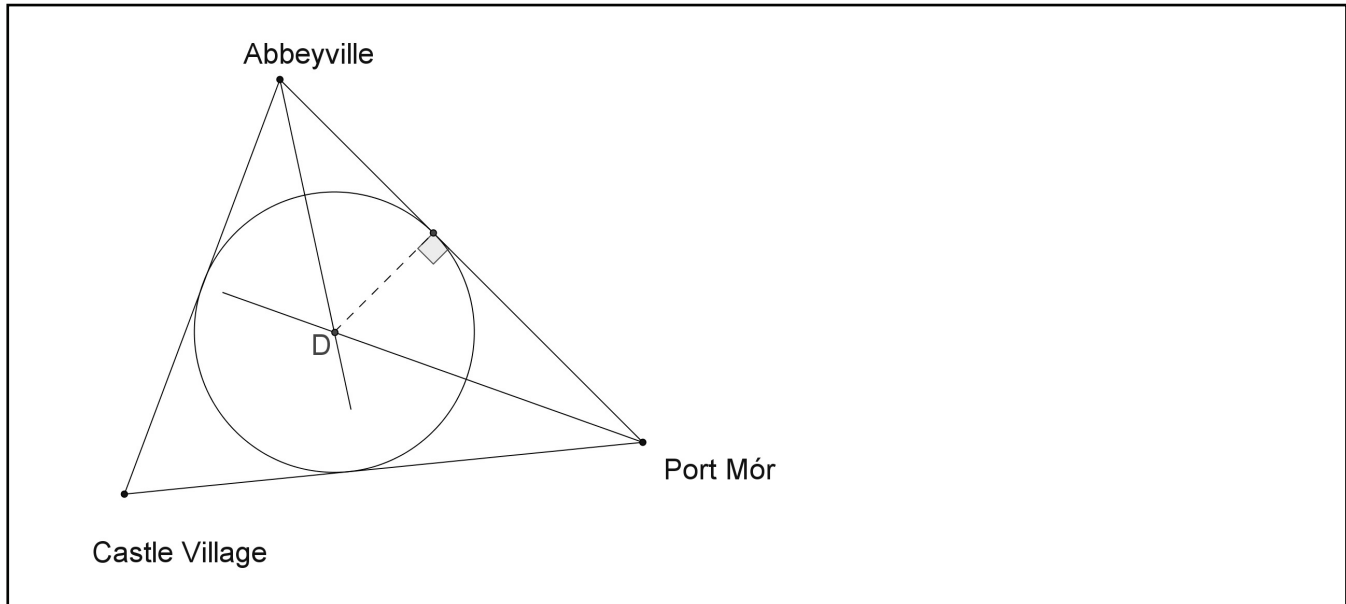**Low Partial Credit (8 Marks)**
•   Any correct step

QUESTION 6A

A fire station is being built to service the three towns shown so that it is equidistant from the roads adjoining each town.

(a)   Show, using a suitable construction, where the fire station should be built.

Full Credit (20 Marks)
*   Fully correct with all constructions shown

High Partial Credit (15 Marks)
*   Fully correct with no/some constructions shown
*   Constructs an incorrect centre fully correct with constructions shown

Mid Partial Credit (10 Marks)
*   Constructs an incorrect centre not fully correct with some constructions shown

Low Partial Credit (5 Marks)
*   Any correct step

(b)   What is the name given to the point where the fire station will be built?

Incentre

Full Credit (5 Marks)
*   Fully correct

Partial Credit (3 Marks)
*   Mentions any other centre

QUESTION 6B

(a) $[BC]$ and $[DE]$ are two parallel chords of a circle with centre A and radius r.

       $[AF] \perp [BC]$ and $[\angle BAC] = 2[\angle DAE]$. If the area ΔABC = area ΔADE, show that $a = \dfrac{\pi}{6}$ rads.

$$\frac{1}{2}(r)(r)\sin 4a = \frac{1}{2}(r)(r)\sin 2a$$
$$2\sin 2a \cos 2a = \sin 2a$$
$$\cos 2a = \frac{1}{2}$$
$$a = \frac{\pi}{6}$$

Full Credit (15 Marks)
•   Fully correct with work shown

High Partial Credit (13 Marks)
•   Solves with no more than two errors

Low Partial Credit (8 Marks)
•   Any correct step (check diagram for valid work).

(b) Find the value of r, where $r \in \mathbb{N}$, if $|BC|^2 + |DE|^2 = 36$.

$$\sin 2a = \frac{\frac{1}{2}|BC|}{r} \qquad\qquad \sin a = \frac{\frac{1}{2}|DE|}{r}$$

$$\therefore \quad |BC| = 2r\sin 2a \qquad\qquad |DE| = 2r\sin a$$

$$(2r\sin 2a)^2 + (2r\sin a)^2 = 36$$

$$r^2\left(\sin^2 2a + \sin^2 a\right) = 9$$

$$r^2(1) = 9$$

$$r = \sqrt{9}$$

$$r = 3$$

Full Credit (10 Marks)
• Fully correct with work shown

High Partial Credit (8 Marks)
• Finds $|BC|$ and $|DE|$ correctly and continues to end with errors
• Finds $|BC|$ and $|DE|$ incorrectly and continues to end with no further errors

Low Partial Credit (4 Marks)
• Any correct step

QUESTION 7

Part (a) (i)		**Scale 10C**
(ii)		**Scale 5A**
(iii)		**Scale 10 C**
(iv)		**Scale 5B**
(v)		**Scale 15B**
(vi)		**Scale 5B**
Part (b) (i)		**Scale 5A**
(ii)		**Scale 10B**
(iii)		**Scale 10B**

(a) Each year the Central Statistics Office, or CSO, compile vital statistics about life in Ireland. The following table from the CSO shows the number of people employed and unemployed in the Irish workforce, in thousands, from 1987 to 2011.

> **(i)** Examining the figures in the table discuss the trends in the unemployment figures between 1987 and 2011.

> Three factors to correct:
> 1. High unemployment in 1987
> 2. Unemployment decreased to 2001 (might mention small rise from in early 90's)
> 3. On the rise since 2001

Full Credit (10 Marks)
• Three facts from table

High Partial Credit (7 Marks)
• Two facts from table

Low Partial Credit (3 Marks)
• One fact from table

> **(ii)** Ireland has seen a dramatic change in the employment figures since the turn of the century. Why, in your opinion, have the figures changed?

> Recession.
> Building trade burst.
> Bank crisis.

Full Credit (5 Marks)
• Any correct reason

The following table shows the changes in Ireland's population from 1987 to 2011. All the figures are in thousands.

(iii) Discuss the trends in the Immigration/Emigration figures for the years shown.

Three facts correct

Full Credit (10 Marks)
• Three facts from table

High Partial Credit (7 Marks)
• Two facts from table

Low Partial Credit (3 Marks)
• One fact from table

Paul states "Recent emigration figures are affected by the lack of jobs in Ireland".

(iv) Would you agree with Paul? Explain your answer fully.

Yes with a valid reason.

Full Credit (5 Marks)
• Fully correct

Partial Credit (2 Marks)
• Partially correct reason

(v) Paul decides to represent the data on a suitable graph. He wants to investigate if there is a relationship between unemployment figures and emigration in Ireland. Display the data on a suitable graph.

Any valid suitable display.

Full Credit (15 Marks)
• Fully correct display

High Partial Credit (7 Marks)
• Partially correct display

(vi) Explain how the graph supports/does not support Paul's claim. Make reference to statistical analysis of the given data.

Valid explanation to support display.

Full Credit (5 Marks)
• Fully correct

Partial Credit (2 Marks)
• Partially correct reason

(b) **(i)** What is the relationship between the mean, mode and median of a normal distribution?

> All equal.

Full Credit (5 Marks)
- Fully correct

(ii) Which of the following would you expect to be normally distributed? Explain your answer fully.
- The lifespan of a television.
  OR
- The number of accidents on a stretch of M4 motorway in a year.

> Answer: The lifespan of a television.
> Reason: Quality control procedures ensure they all last roughly the same amount of time.
>   Small few have problems early and small number last a very long time.

Full Credit (10 Marks)
- Fully correct

Partial Credit (6 Marks)
- Partially correct reason

(iii) 400 samples of size 10 are taken from a production line producing jars of gravy. The mean weight of a jar is 200g, with a standard deviation of 1.5g. Calculate the limits between which you would expect 95% of the sample means to lie.

> $$200 \pm 2\left(\frac{1.5}{\sqrt{10}}\right)$$
> 199.05g to 200.95g
>
> or
>
> $$200 \pm 2\left(\frac{1.5}{\sqrt{10}}\right)$$
> 199.53g to 200.47g

Full Credit (10 Marks)
- Fully correct

Partial Credit (6 Marks)
- Partially correct

QUESTION 8

Part (a)	**Scale 10C**
Part (b)	**Scale 10C**
Part (c)	**Scale 10C**
Part (d)	**Scale 10C**

A rear windshield wiper has a radius of *r*. It cleans half the width of the windshield. The highest point of the arc of the semicircle is 60% of the height of the windshield.

(a) Show that the distance *x* can be written as $\frac{2}{3}r$.

$$\frac{3}{5}h = r \qquad \therefore h = \frac{5}{3}r$$

$$x = \frac{5}{3}r - r$$

$$x = \frac{2}{3}r$$

Full Credit (10 Marks)
*   Fully correct with work shown

High Partial Credit (7 Marks)
*   Correct answer with no work shown
*   Finds *x* and stops

Low Partial Credit (3 Marks)
*   Any correct step (check diagram)

(b) Show that the total area of the windshield is $\frac{20}{3}r^2$.

$$(4r)\left(\frac{5}{3}r\right) = \frac{20}{3}r^2$$

Full Credit (10 Marks)
*   Fully correct with work shown

High Partial Credit (7 Marks)
*   Correct answer with no work

Low Partial Credit (3 Marks)
*   Any correct step (check diagram)

(c)   Show that the ratio of the area cleaned to the area unclean is $\dfrac{3}{40}\pi$ square units.

$$\frac{\dfrac{1}{2}\pi r^2}{\dfrac{20}{3}r^2} = \frac{3}{40}\pi$$

Full Credit (10 Marks)
*   Fully correct with work shown

High Partial Credit (7 Marks)
*   Correct answer with no work shown

Low Partial Credit (3 Marks)
*   Any correct step (check diagram)

The design of the windshield is altered so that the width of the bottom of the windshield is reduced by 25% and the angle at the centre of the arc is reduced by 20%.

(d)   Calculate the area of the sector cleaned in terms of *r* in radians.

$$\left(\frac{1}{2}\right)\left(r^2\right)\left(\frac{4}{5}\pi\right)$$
$$\frac{2}{5}\pi r^2$$

Full Credit (10 Marks)
*   Fully correct with work shown

High Partial Credit (7 Marks)
*   Correct answer with no work shown

Low Partial Credit (3 Marks)
*   Any correct step (check diagram)

QUESTION 9

Part (a)	**Scale 10C**
Part (b)	**Scale 15C**
Part (c)	**Scale 10B**

A sign is attached to a vertical wall at the point E. $[AE] \perp$ wall. The sign is then suspended by two wires which are attached to the wall at the points B and C as shown.

(a) Calculate the length of the wires.

$$|BE|^2 = 120^2 + 50^2$$
$$|BE| = 130cm$$
$$|BA|^2 = |BE|^2 + |EA|^2$$
$$|BA|^2 = 130^2 + 120^2$$
$$|BA| = 176.918cm$$
$$|BA| = 177cm$$

Full Credit (10 Marks)
• Fully correct with work shown

High Partial Credit (7 Marks)
• Correct answer with no work shown
• Works out any correct side that may lead to correct answer

Low Partial Credit (3 Marks)
• Any correct step (check diagram)

(b) Calculate the angle that each wire makes with the wall.

$$177^2 = 100^2 + 177^2 - 2(100)(177)\cos|\angle ABC|$$
$$\angle ABC = 73.59°$$

Full Credit (15 Marks)
• Fully correct with work shown

High Partial Credit (10 Marks)
• Correct answer with no work shown
• Works out any correct side or angle correctly that may lead to correct answer (not from earlier parts)

Low Partial Credit (5 Marks)
• Any correct step (check diagram)

(c) Calculate the $|\angle AEC|$, correct to two decimal places.

$$\frac{\sin x}{120} = \frac{\sin 90}{177}$$
$$x = 42.68°$$

Full Credit (10 Marks)
• Fully correct with work shown

High Partial Credit (7 Marks)
• Correct answer with no work shown
• Works out any correct side or angle correctly that may lead to correct answer (not from earlier parts)

Low Partial Credit (3 Marks)
• Any correct step (check diagram)

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