P 6

Pre-Leaving Certificate Examination, 2014

Triailscrúdú na hArdteistiméireachta, 2014

Mathematics
(Project Maths – Phase 3)

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

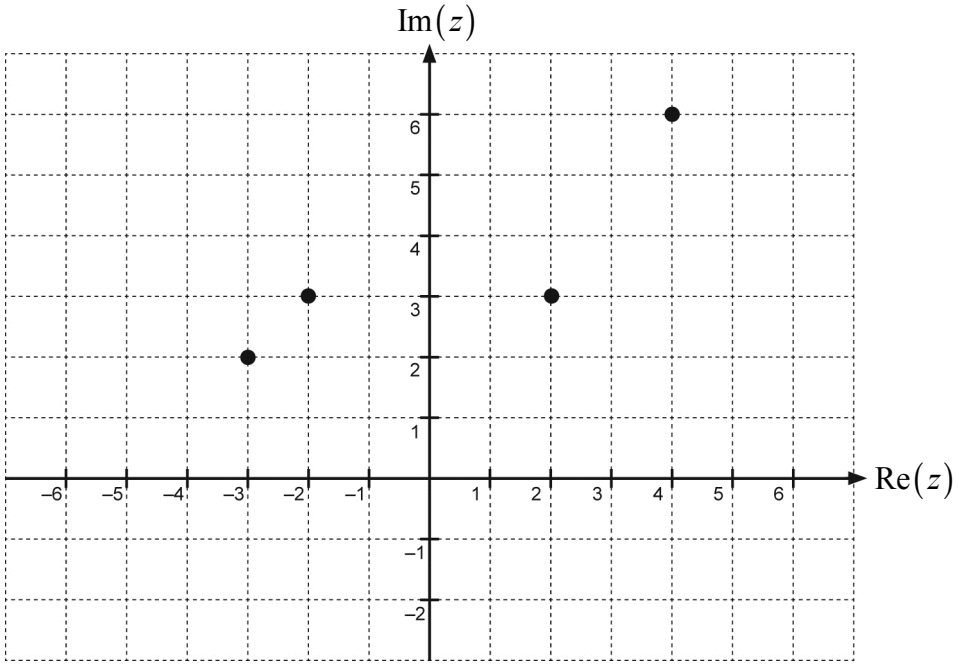
Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) If $z = a + bi$, where a and $b \in \mathbb{R}$, show that $z.\bar{z}$ simplifies to a constant.

(b) Examine the complex plane below. Identify on the diagram each complex number.



$z_2 = 2z_1$ $z_3 = iz_1$ $z_4 =$ image of z_1 under S_y

(c) Use De Moivre's theorem to prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

Question 2 **(25 marks)**

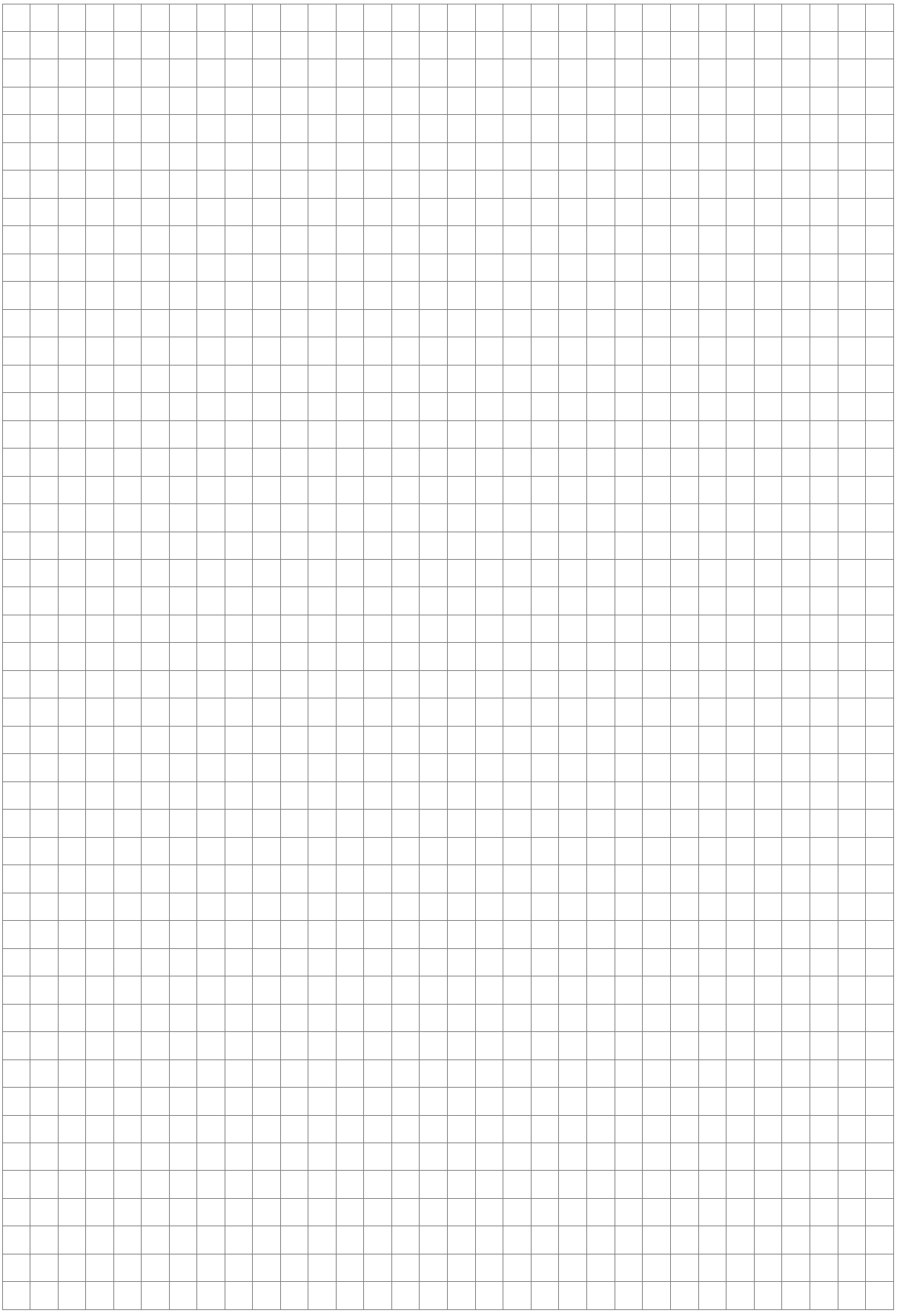
(a) Show that $a^2 + 16 + b^2 \geq 2ab$ for all values of $a, b \in \mathbb{R}$.

(b) Solve the simultaneous equations:

$$x + 3y = -5$$
$$2x^2 + y^2 = 41$$
where $x, y \in \mathbb{Z}$

Question 3 **(25 marks)**

(a) Given that $x^2 + x - 6$ is a factor of $2x^3 - px^2 + qx - 6$, find the value of p and q.



(b) Prove that the function $f(x): x^3 + 7x^2 + 17x + 15$ has only one real root.

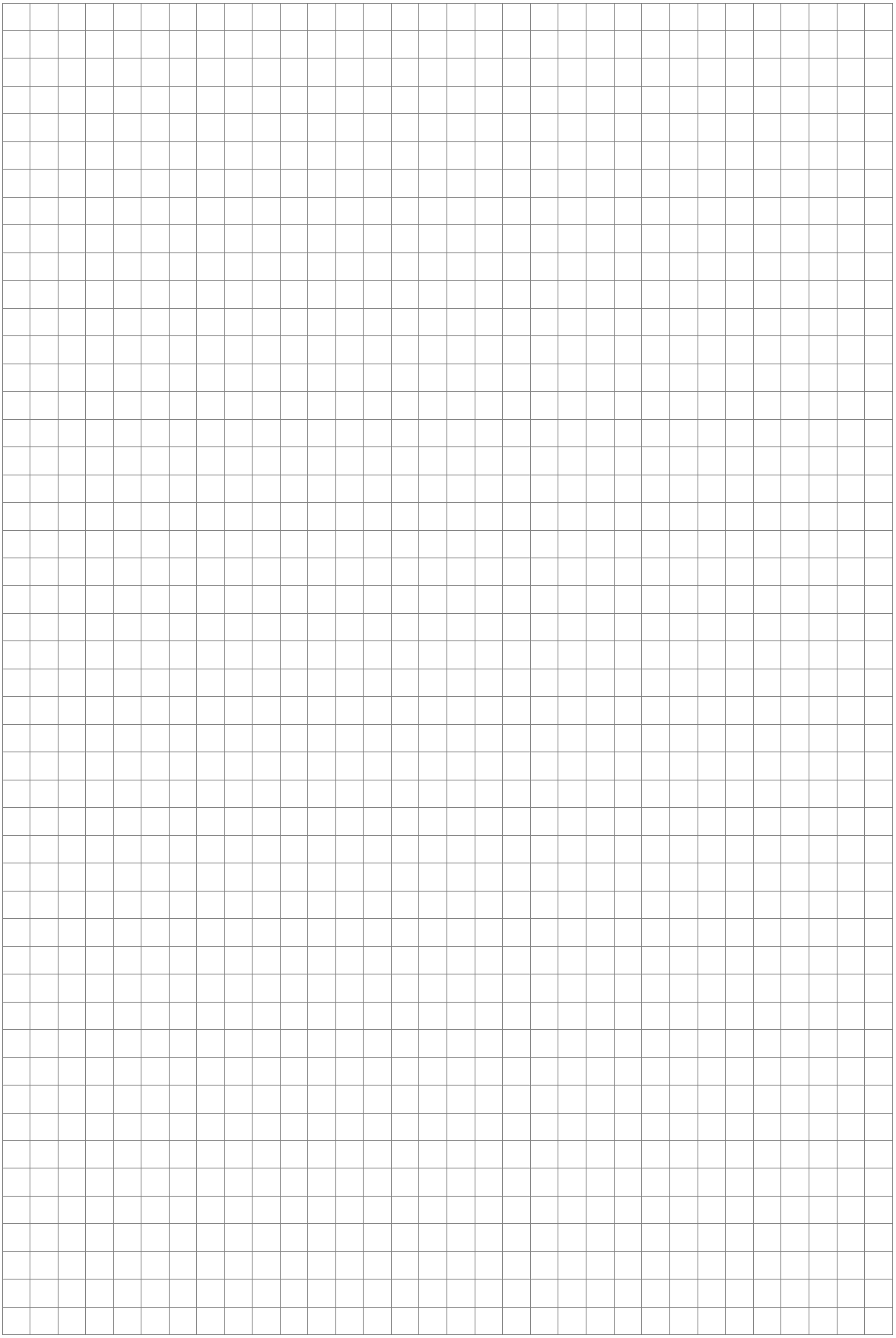
(c) Solve for x.

$$3^{2x+1} + \left(26(3)^{x}\right) - 9 = 0$$

Question 4 **(25 marks)**

(a) Prove by induction that 3 is a factor of $5^n - 2^n$ for all $n \in \mathbb{N}$ and $n \geq 1$.

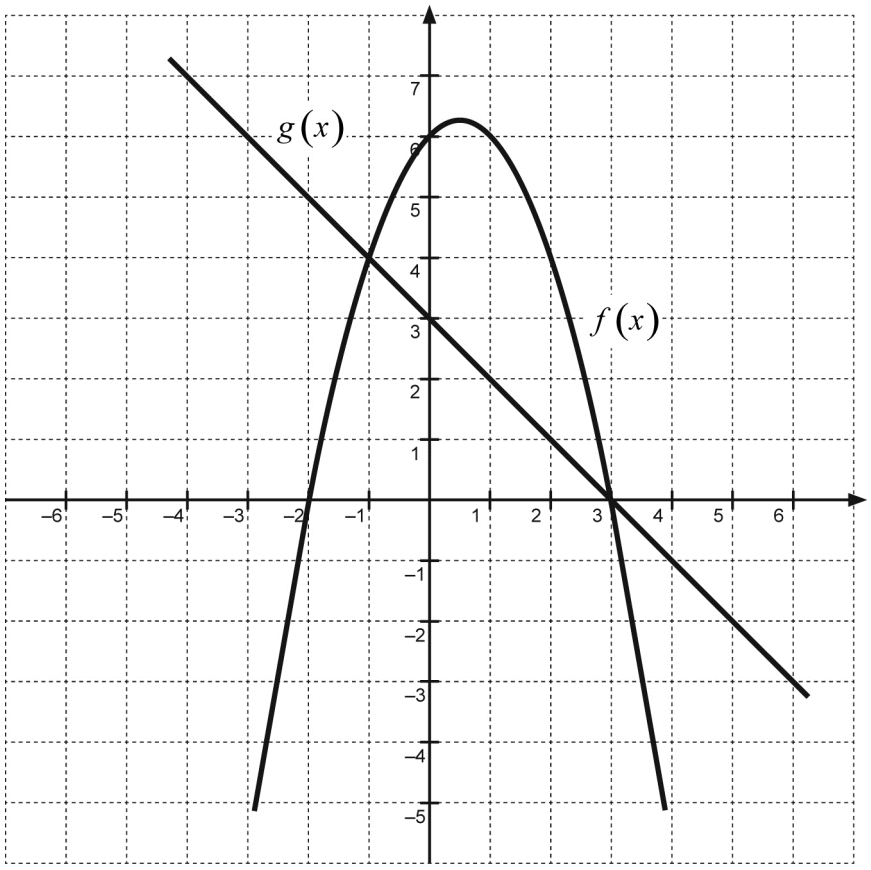
(b) Séan wants to save €50,000 over a ten-year period. If the expected interest rate is 2.5% how much does Séan need to invest at the beginning of each year to save €50,000?



Question 5 **(25 marks)**

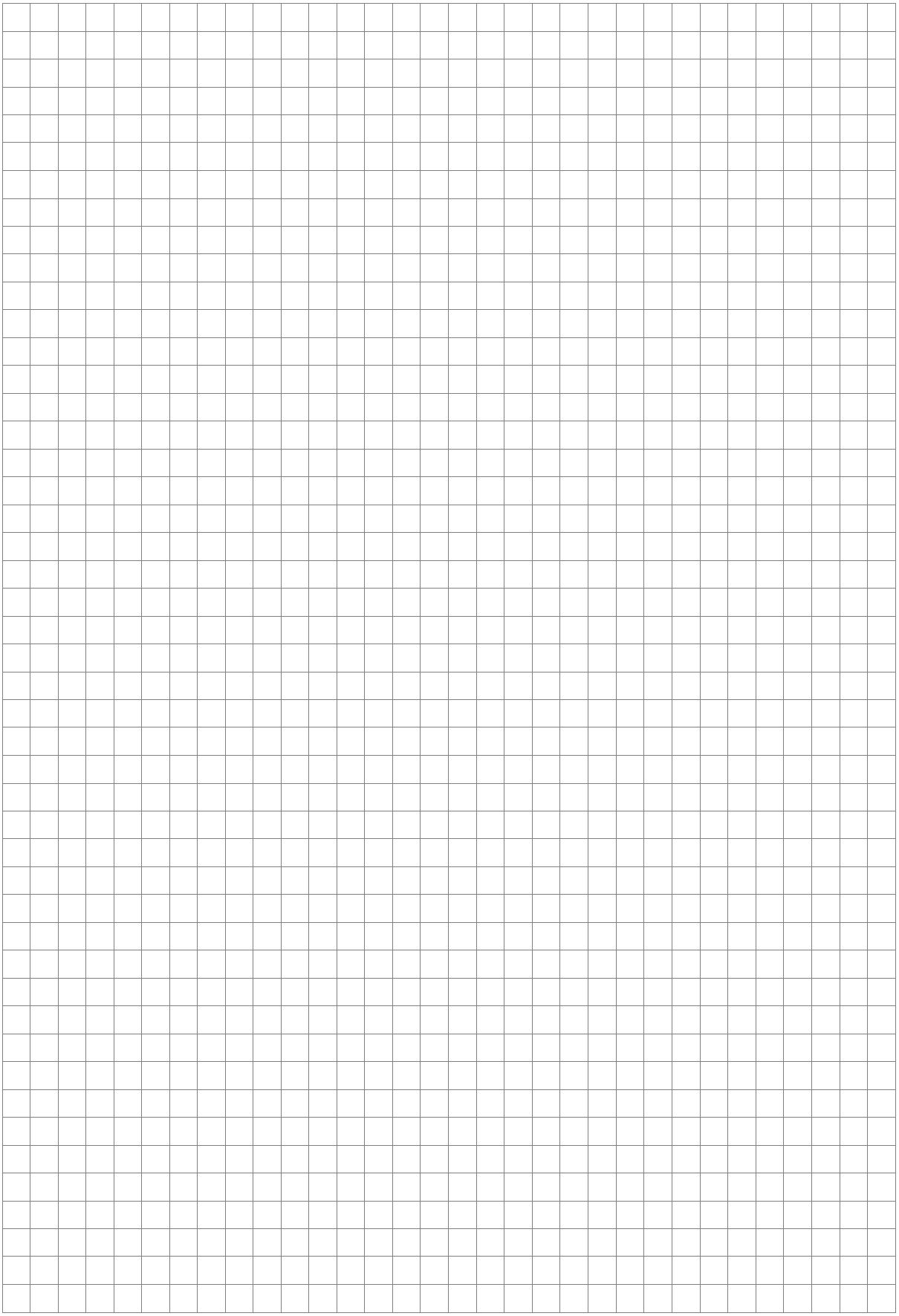
(a) Evaluate $\int_0^1 6x^2 e^{x^3}\,dx$.

(b) Find the area bounded by the curve $f(x)$, the line $g(x)$ and the x-axis.



Question 6 **(25 marks)**

(a) Differentiate $-2x^2 + 3x - 5$ from first principles.



(b) Find the equation of the tangent to the curve $y = x^2 + 3x - 5$ at the point $(1, 2)$.

(c) The function $g(x) = \dfrac{2}{2-3x}$ is defined for all $x \in \mathbb{R} \setminus \left\{\dfrac{2}{3}\right\}$.

(i) Prove that the function has no turning points.

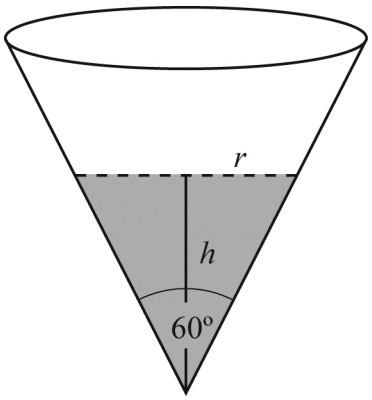
(ii) Jane states that the graph of $g(x)$ would be increasing for all values of x.
Would you agree with Jane? Explain your answer fully.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(55 marks)**

Water is poured into an empty conical container as shown at a rate of 0.02 litres per second.



(a) Using the information above, write an equation to represent the volume of water in the cone at any time t.

(b) Write the radius of the cone r, in terms of the height of the water h.

(c) Show that the height of the water h at any time t can be expressed as $h = \sqrt[3]{\dfrac{0.18t}{\pi}}$.

(d) Find the rate of change in the height of the water after 3 seconds.

Question 8 **(35 marks)**

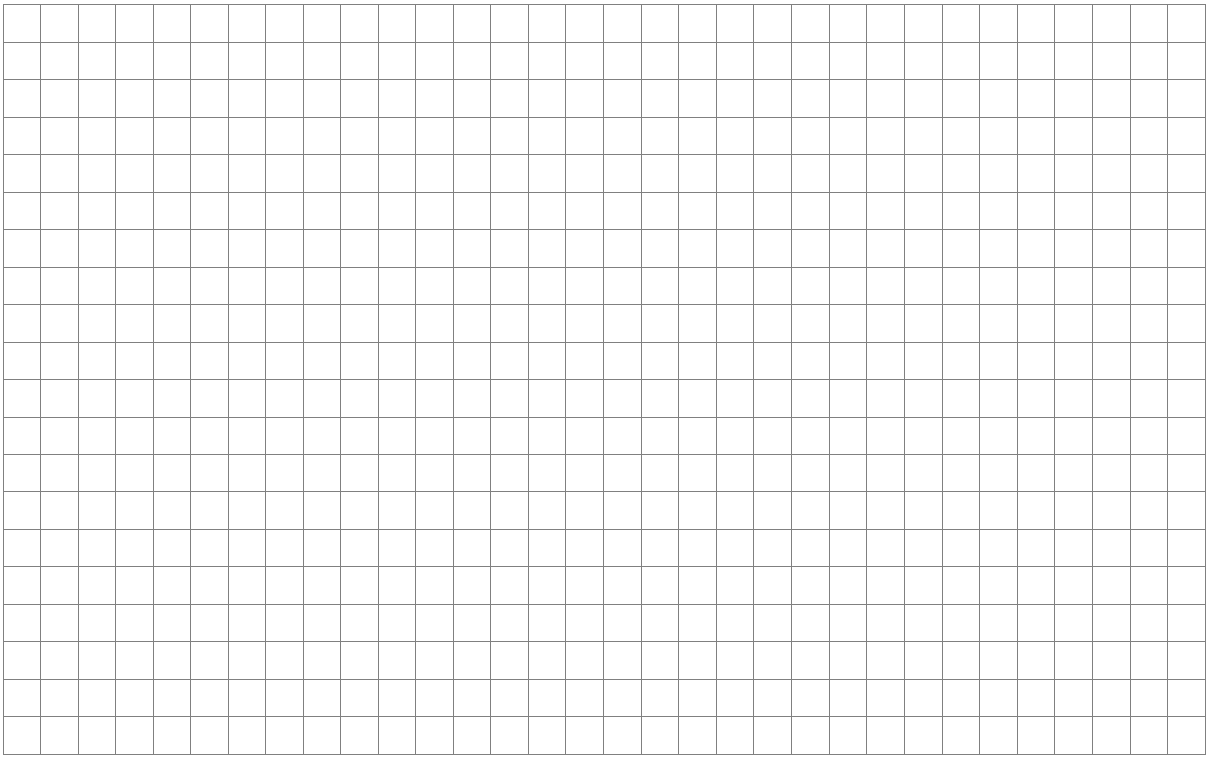
In 1935, while working at the California Institute of Technology, Charles Francis Richter invented the Richter scale. The Richter scale is used to measure earthquake intensity on a machine called a seismograph.

The formula for the Richter is given by $M = \log_{10}\left[\frac{I}{I_0}\right]$

where M is the magnitude on the Richter scale in millimetres, I_0 is the 'threshold quake intensity', i.e. a movement that can barely be detected, and I is the intensity of the earthquake in millimetres. $I_0 = 10^{-3}$ mm.

(a) Calculate the magnitude of an earthquake with an intensity of 500 mm.

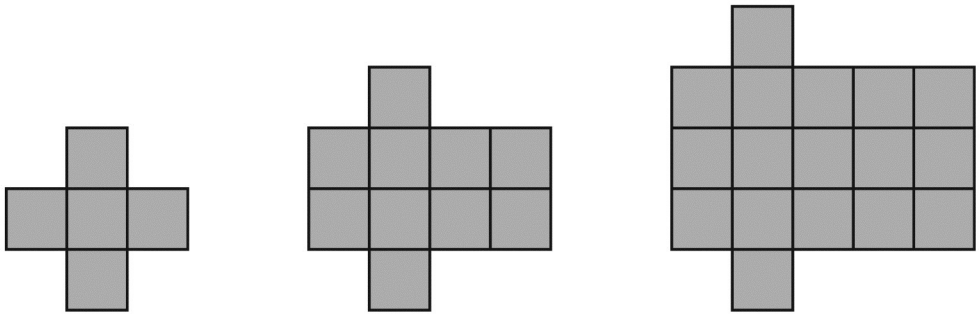
(b) In 2005 an earthquake in Kashmir in Pakistan measured 7.5 on the Richter scale. Calculate the intensity of the earthquake.



(c) A heavy goods lorry rumbling on a road can cause a micro quake with a rating of up to 3.3 on the Richter scale. An at-home seismograph records an intensity of $40,000\ I_0$ while the owner is away on holidays. What event was likely to have occurred while the owner was away, an earthquake or a heavy goods lorry passing his house?

Question 9 **(60 marks)**

Paul is creating designs using green blocks.

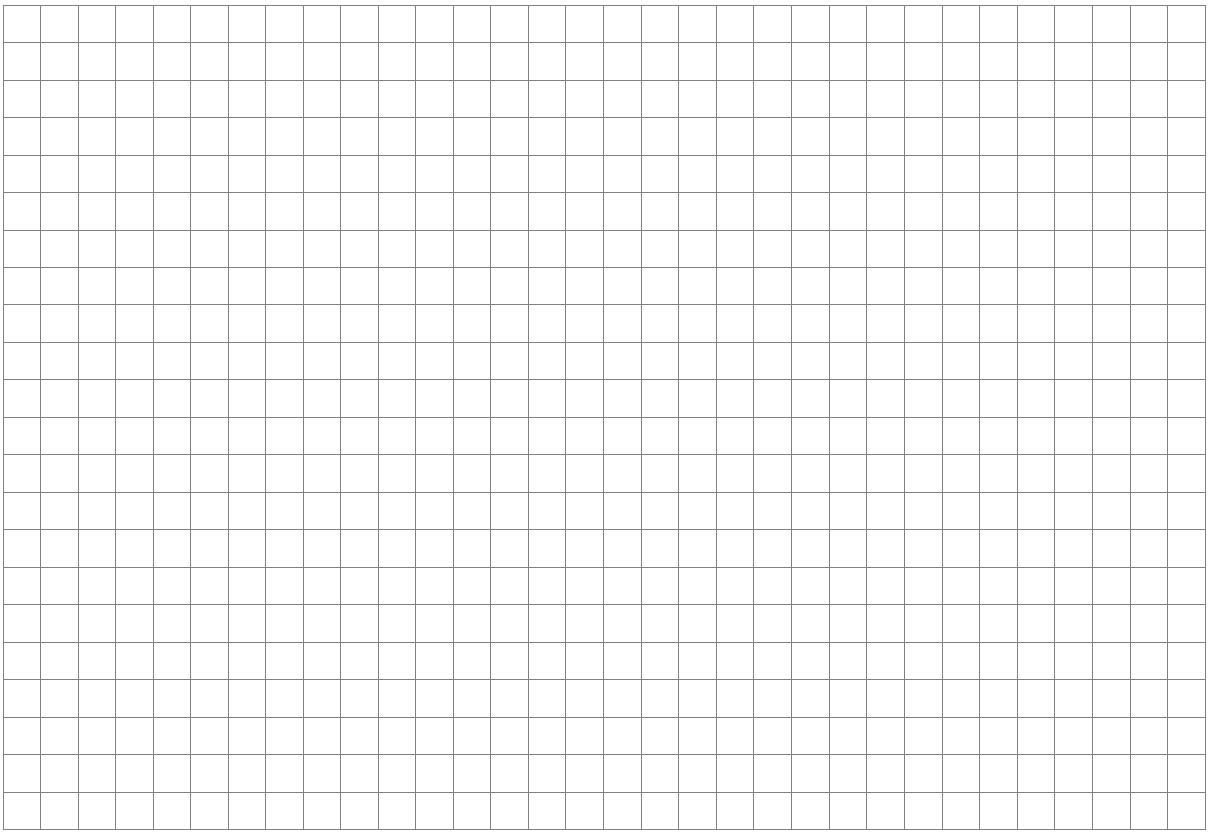
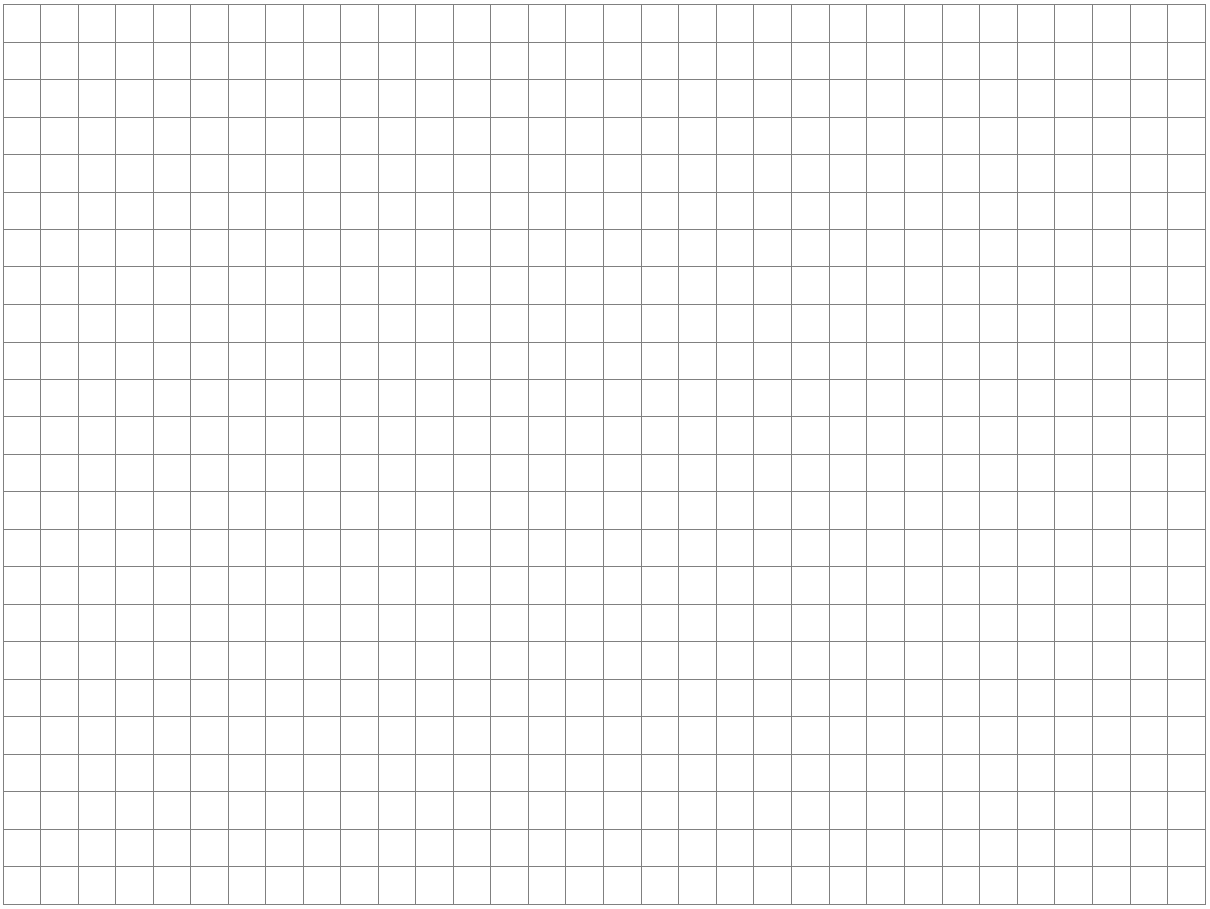


(a) Draw the next two patterns in the sequence.

(b) John states the pattern is linear. Mary disagrees. Would you agree with John or Mary? Explain your answer fully.

(c) The general term for the sequence is given by $T_n = an^2 + bn + c$. Find the values of a, b and c.

(d) What pattern in the sequence will contain 206 green blocks?



(e) How many blocks would Paul need to create 21 patterns?