



**PRE-LEAVING CERTIFICATE EXAMINATION, 2015**

**MARKING SCHEME**

**MATHEMATICS**

**HIGHER LEVEL**

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## OVERVIEW OF MARKING SCHEME

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scale	0, 5	0, 2, 5	0, 2, 3, 5		
10 mark scale	0, 10	0, 5, 10	0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scale	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scale	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scale	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

### Marking scales – level descriptors

#### A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

#### B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

#### C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

#### D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

#### E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

Marking categories for all questions are shown throughout the solutions. In certain cases, typically involving rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, Scale 10C\* indicates that 9 marks may be awarded.

# PAPER 1

## QUESTION 1

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Part (a)	Scale 5B
Part (b)	Scale 5B
Part (c)	Scale 5B
Part (d)	Scale 10C

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(a) What is a proof by contradiction?

A **proof by contradiction** is a proof that establishes the validity of a proposition by showing that the proposition being false would imply a contradiction **Or** similar.

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

(b) What is an irrational number? Give an example of an irrational number.

Cannot be written in the form  $\frac{a}{b}$  where  $a, b \in \mathbb{Z}$  and  $b \neq 0$ . i.e. cannot be written as a fraction

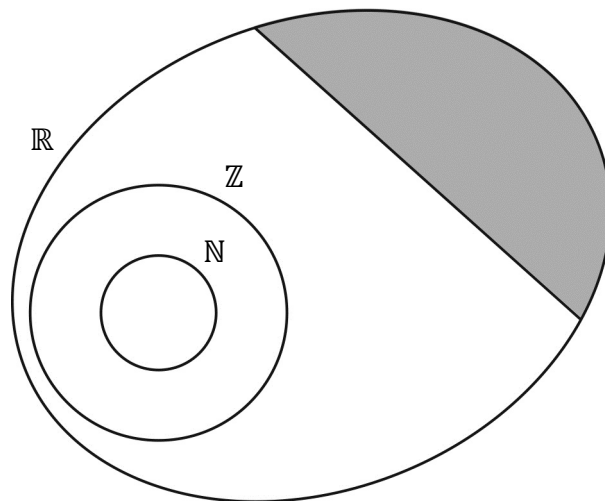
**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

- (c) Shade in the region of the following Venn diagram that represents the set of irrational numbers.



**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

- (d) Use a proof by contradiction to prove that  $\sqrt{2}$  is an irrational number.

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error in proof

**Low Partial Credit (3 Marks)**

- Any correct step in proof

## QUESTION 2

**Part (a)**  
**Part (b)**  
**Part (c)**

**Scale 5C**  
**Scale 10C**  
**Scale 10C**

(a) Simplify:

$$\frac{x^2 - 4}{3y^2 + y} \times \frac{3y^3 - 2y^2 - y}{x^2 + x - 6}$$

$$\begin{aligned} & \frac{x^2 - 4}{3y^2 + y} \times \frac{3y^3 - 2y^2 - y}{x^2 + x - 6} \\ & \frac{(x-2)(x+2)}{y(3y+1)} \times \frac{y(3y+1)(y-1)}{(x+3)(x-2)} \\ & \frac{(x+2)(y-1)}{(x+3)} \end{aligned}$$

**Full Credit (5 Marks)**

- Fully correct

**High Partial Credit (3 Marks)**

- No more than 1 error

**Low Partial Credit (2 Marks)**

- Any correct step

(b) Solve the inequality,  $\frac{3x-2}{x+4} < 2$ ,  $x \neq 4$  and  $x \in \mathbb{R}$ .

$$\begin{aligned} & \frac{(3x-2)(x+4)^2}{x+4} < 2(x+4)^2 \\ & 3x^2 + 10x - 8 < 2x^2 + 16x + 32 \\ & x^2 - 6x - 40 < 0 \\ & x = -4 \quad x = 10 \\ & -4 < x < 10 \quad \text{or on a graph correctly indicated} \end{aligned}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error and continues to end correctly

**Low Partial Credit (3 Marks)**

- Any correct step
- Does not multiply by  $(x+4)^2$  but continues

(c) Solve the simultaneous equations:

$$2x + 3y - 4z = -11$$

$$-x + 2y + z = -1$$

$$x - y + 3z = 12$$

$x = 2, \quad y = -1, \quad z = 3$
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**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- No more than 2 errors and continues to end

**Low Partial Credit (3 Marks)**

- Any correct step

### QUESTION 3

**Part (a)**

**Scale 10C**

**Part (b)**

**Scale 5B**

**Part (c)**

**Scale 10C**

- (a) Examine the graph of the polynomial shown. Write down the polynomial for the graph in the form  $ax^4 + bx^3 + cx^2 + dx + e$ , where  $a, b, c, d, e \in \mathbb{R}$ .

$$\begin{aligned}x &= -2, \quad x = 1, \quad x = 1, \quad x = 4 \\(x+2)(x-1)(x-1)(x-4) &= 0 \\x^4 - 4x^3 - 3x^2 + 14x - 8 &= 0\end{aligned}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- No more than 2 errors and continues to end

**Low Partial Credit (3 Marks)**

- Any correct step

- (b) Investigate if  $3x - 1$  is a factor of  $6x^3 - 5x^2 + 4x + 5$ .

$$\begin{aligned}x &= \frac{1}{3} \quad \therefore 6\left(\frac{1}{3}\right)^3 - 5\left(\frac{1}{3}\right)^2 + 4\left(\frac{1}{3}\right) + 5 = 6 \\ \therefore 3x - 1 & \text{ is not a factor}\end{aligned}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct
- No conclusion

(c) Given that  $x^2 + ax - 1$  is a factor of  $x^3 + px^2 + qx + r = 0$ , show that  $q = -(ar + 1)$ .

$$(x^2 + ax - 1)(x + t) = x^3 + px^2 + qx + r$$
$$x^3 + tx^2 + ax^2 + atx - x - t = x^3 + px^2 + qx + r$$

Equating Coefficients

$$t + a = p \quad \therefore t = p - a$$

$$at - 1 = q$$

$$t = -r$$

$$\therefore q = a(-r) - 1 = -(ar + 1)$$

\*\*\*Accept other valid methods for full credit

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Correct long division but fails to find correct form
- Sets up correct equation, multiplies but final answer not in correct form

**Low Partial Credit (3 Marks)**

- Any correct step



## QUESTION 4

Part (a)

Part (b) (i)  
(ii)

Scale 5C

Scale 10C

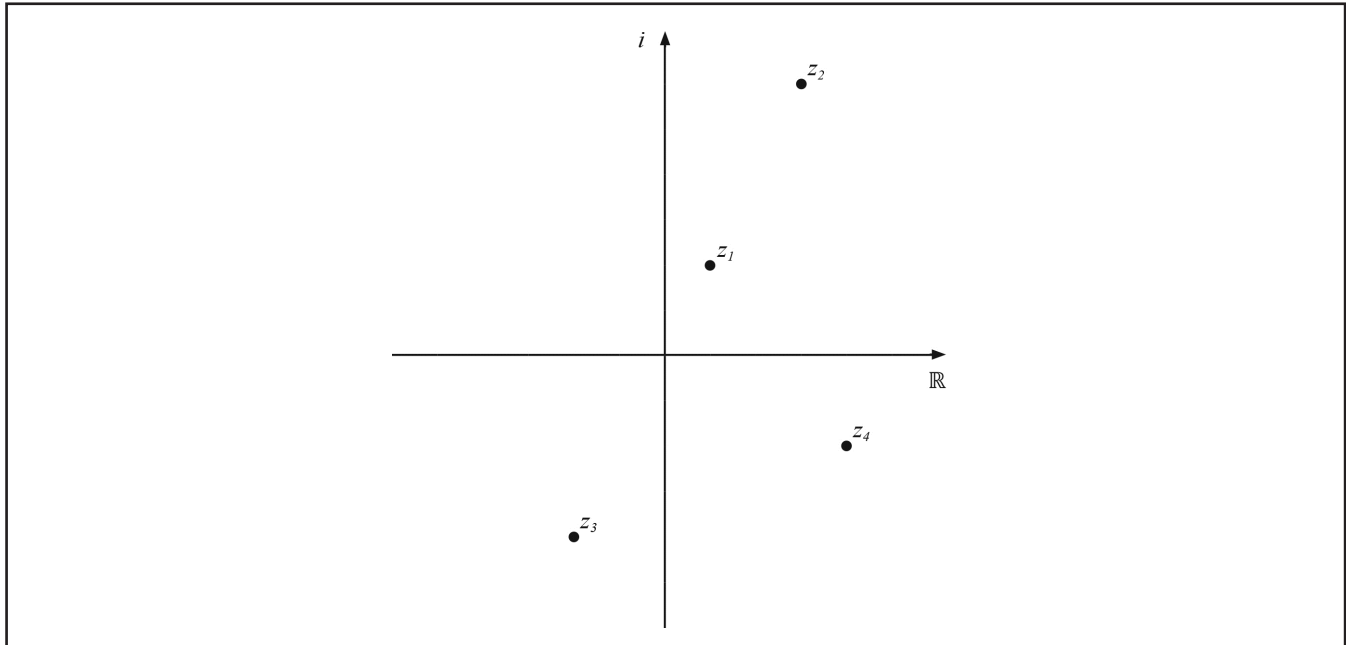
Scale 10C

(a) Label each of the complex numbers given the following information.

$$z_2 = 3z_1$$

$$z_4 = iz_3$$

$$z_3 = -2z_1$$



**Full Credit (5 Marks)**

- Fully correct

**High Partial Credit (3 Marks)**

- 2 correct

**Low Partial Credit (2 Marks)**

- 1 correct

(b) (i) Given  $\omega = 2 + 2\sqrt{3}i$ , write  $\omega$  in Polar Form.

$$2 + 2\sqrt{3}i = 4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Correct Modulus
- Correct Angle

**Low Partial Credit (3 Marks)**

- Any correct step
- Sketch of  $\omega$

(ii) Hence solve the equation  $z^4 = \omega$ .

$$z^4 = 4 \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

$$z = \left[ 4 \left( \cos \left( \frac{\pi}{3} + 2n\pi \right) + i \sin \left( \frac{\pi}{3} + 2n\pi \right) \right) \right]^{\frac{1}{4}}$$

$$z = \sqrt{2} \left[ \cos \left( \frac{\pi}{12} + \frac{n\pi}{2} \right) + i \sin \left( \frac{\pi}{12} + \frac{n\pi}{2} \right) \right]$$

$$\text{If } n = 0 \quad z = \frac{1 + \sqrt{3}}{2} + i \left( \frac{-1 + \sqrt{3}}{2} \right)$$

$$\text{If } n = 1 \quad z = \frac{1 - \sqrt{3}}{2} + i \left( \frac{1 + \sqrt{3}}{2} \right)$$

$$\text{If } n = 2 \quad z = -\frac{1 + \sqrt{3}}{2} + i \left( \frac{1 - \sqrt{3}}{2} \right)$$

$$\text{If } n = 3 \quad z = \frac{-1 + \sqrt{3}}{2} - i \left( \frac{1 + \sqrt{3}}{2} \right)$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- No more than 2 errors and continues to end

**Low Partial Credit (3 Marks)**

- Any correct step

## QUESTION 5

**Part (a)**

**Scale 5B**

**Part (b) (i)**

**Scale 5B**

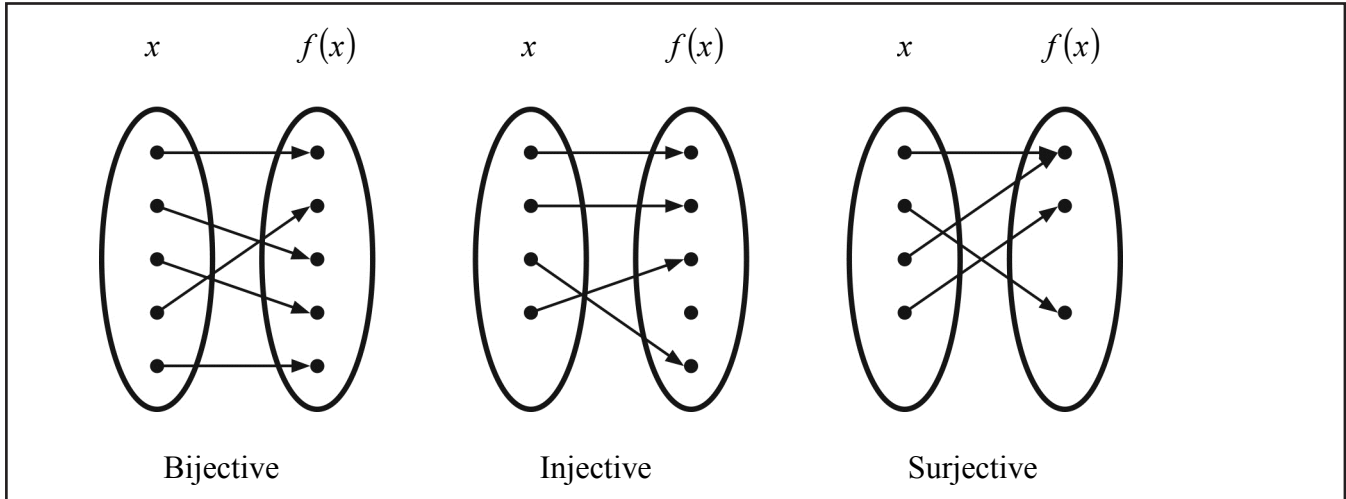
**(ii)**

**Scale 5B**

**Part (c)**

**Scale 10C**

**(a)** Identify the following functions as injective, surjective or bijective.



**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (3 Marks)**

- One correct

**(b)** The growth of a certain species of bacteria can be modelled with the equation  $N = Re^{kt}$ . A science teacher puts 100 bacteria into nutrient agar plates (agar will act as a food source for the bacteria). Five hours later, there are 600 bacteria.

**(i)** Calculate the value of  $k$  correct to four decimal places.

$$600 = 100e^{k(5)}$$

$$\frac{\ln 6}{5} = k$$

$$k = 0.3584$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(ii) After how many hours will the number of bacteria in the plate be 12,000?

$$12,000 = 100e^{(0.3584)t}$$
$$\frac{\ln 120}{0.3584} = k$$
$$k = 13.36 \text{ hours}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(c) Solve for x correct to two decimal places:

$$3^{2x+1} - 10(3^x) - 8 = 0$$

$$3^{2x+1} - 10(3^x) - 8 = 0$$
$$3(3^x)^2 - 10(3^x) - 8 = 0$$
$$3y^2 - 10y - 8 = 0$$
$$(3y + 2)(y - 4)$$
$$y = -\frac{2}{3} \quad \text{or} \quad y = 4$$
$$3^x \neq -\frac{2}{3} \quad \text{or} \quad 3^x = 4$$
$$\therefore x = 1.26$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Correct but fails to cancel incorrect solution
- No more than 1 error in solving and cancels incorrect solution

**Low Partial Credit (3 Marks)**

- Any correct step

## QUESTION 6

Part (a) (i)  
(ii)

Scale 5B

Scale 5B

Part (b)

Scale 5B

Part (c)

Scale 10C

(a) Find:

(i)  $\int (2x^2 + 4x - 2) dx$

$$\frac{2x^3}{3} + 2x^2 - 2x + c$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(ii)  $\int (\sin 3x - \cos 2x) dx$

$$-\frac{1}{3} \cos 3x - \frac{1}{2} \sin 2x + c$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(b) Evaluate  $\int_1^3 \left( \frac{x^3 - 8}{x - 2} \right) dx$ .

$$\begin{aligned} \int_1^3 \left( \frac{x^3 - 8}{x - 2} \right) &= \int_1^3 \left( \frac{(x-2)(x^2 + 2x + 4)}{(x-2)} \right) \\ &= \int_1^3 (x^2 + 2x + 4) \\ &= \left( \frac{x^3}{3} + x^2 + 4x \right) \Big|_1^3 \\ &= (9 + 9 + 12) - \left( \frac{1}{3} + 1 + 4 \right) = 24 \frac{2}{3} \end{aligned}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(c) Find the area enclosed by the  $x$ -axis and the curve  $y = -x^2 + 2x + 3$ .

$$-x^2 + 2x + 3 = 0 \quad \therefore x = -1 \quad \text{or} \quad x = 3$$

$$\int_{-1}^3 -x^2 + 2x + 3$$

$$\left( \frac{-x^3}{3} + x^2 + 3 \right)_{-1}^3$$

$$(-9 + 9 + 9) - \left( \frac{1}{3} + 1 - 3 \right) = 10 \frac{2}{3}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Error in solving but continues to end with no further error
- 1 error in integration

**Low Partial Credit (3 Marks)**

- Any correct step

## QUESTION 7

<b>Part (a)</b>	<b>Scale 10B</b>
<b>Part (b)</b>	<b>Scale 15C</b>
<b>Part (c) (i)</b>	<b>Scale 10C</b>
<b>(ii)</b>	<b>Scale 10C</b>
<b>(iii)</b>	<b>Scale 15C</b>

- (a) John wishes to borrow € $X$  from his bank over  $n$  years. Interest of  $r\%$  APR will be applied to the loan. He will make an annual repayment of € $A$ . Show the amount owing at the end of the first year,  $P_1$ , can be written as:

$$P_1 = X(1+r)A$$

$$F = P(1+r)^t$$

$$F = X(1+r)^t$$

$$\therefore P_1 = X(1+r) - A$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

- (b) If  $P_n = 0$  after the final year of the mortgage, show that the annual repayment  $A$  can be

written as  $A = \frac{Xr(1+r)^n}{(1+r)^n - 1}$ .

$$P_n = X(1+r)^n - A(1+r)^{n-1} - A(1+r)^{n-2} - \dots - A(1+r) - A$$

$$P_n = 0$$

$$\therefore 0 = X(1+r)^n - A(1+r)^{n-1} - A(1+r)^{n-2} - \dots - A(1+r) - A$$

$$A + A(1+r) + A(1+r)^2 + \dots + A(1+r)^{n-2} + A(1+r)^{n-1} = A(1+r)^n$$

$$\frac{A(1 - (1+r)^n)}{-r} = X(1+r)^n$$

$$A = \frac{Xr(1+r)^n}{(1+r)^n - 1}$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- Error in geometric progression and continues correctly to end

**Low Partial Credit (5 Marks)**

- Any correct step

(c) John borrows €150,000 over 20 years at 4% compound interest per annum.

(i) Calculate the monthly interest rate correct to five significant figures.

$$(1.04)^{\frac{1}{12}} - 1 = 0.00327$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Correct calculation but incorrect significant figures

**Low Partial Credit (3 Marks)**

- Any correct step

(ii) Calculate John's monthly repayment correct to the nearest euro.

$$A = \frac{(150,000)(0.00327)(1 + 0.00327)^{240}}{(1 + 0.00327)^{240} - 1} = 902.97$$
$$A = \text{€}903$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error in calculation
- Treats as power of 20 with no other error

**Low Partial Credit (3 Marks)**

- Any correct step



- (iii) After 12 years, John decides to pay off the remainder of the loan in one lump sum. Calculate the remaining balance on the loan correct to the nearest euro.

8 years left = 96 months

$$€ 903 + \frac{903}{1.00327} + \frac{903}{(1.00327)^2} + \frac{903}{(1.00327)^3} + \dots + \frac{903}{(1.00327)^{95}}$$

$$\therefore a = 903 \text{ and } r = \frac{1}{1.00327}$$

$$S_{96} = \frac{903 \left( 1 - \left( \frac{1}{1.00327} \right)^{96} \right)}{1 - \frac{1}{1.00327}}$$

$$S_{96} = € 74.540$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- Error in geometric progression and continues correctly to end

**Low Partial Credit (5 Marks)**

- Any correct step

## QUESTION 8

<b>Part (a)</b>	<b>Scale 10B*</b>
<b>Part (b)</b>	<b>Scale 10B*</b>
<b>Part (c)</b>	<b>Scale 10B*</b>
<b>Part (d)</b>	<b>Scale 10B*</b>
<b>Part (e)</b>	<b>Scale 10C*</b>

A water trough is in the shape of a prism as shown.

- (a) Calculate the internal volume of the water trough.

$$\text{Vol} = \frac{1}{2}(2)(1.5)(6) = 9 \text{ m}^3$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

- (b) Write an equation to represent the volume ( $V$ ) of water in the tank at any time  $t$  in terms of  $h$  and  $w$ .

$$V = \frac{1}{2}(w)(h)(6)$$
$$V = 3wh \text{ m}^3$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

- (c) Show that  $w = \frac{4}{3}h$  metres.

$$\frac{w}{2} = \frac{h}{1.5}$$
$$w = \frac{4}{3}h \text{ metres}$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

(d) Show that the volume of the water tank can be expressed as:  $V = 4h^2$ .

$$V = 3wh$$

$$V = 3h\left(\frac{4}{3}h\right)$$

$$V = 4h^2 \text{ m}^3$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (Marks)**

- Any correct step

(e) If water is being pumped in at a constant rate of  $5 \text{ m}^3/\text{s}$ , at what rate is the height of the water changing when the water has a height of 120 cm?

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

$$\frac{dV}{dh} = 8h$$

$$\frac{dh}{dt} = (5) \left( \frac{1}{8(1.2)} \right)$$

$$\frac{dh}{dt} = 0.52 \text{ m/s}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error in calculation

**Low Partial Credit (3 Marks)**

- Any correct step

## QUESTION 9

<b>Part (a)</b>	<b>Scale 5C</b>
<b>Part (b)</b>	<b>Scale 10B</b>
<b>Part (c)</b>	<b>Scale 5B</b>
<b>Part (d)</b>	<b>Scale 10C</b>
<b>Part (e)</b>	<b>Scale 10B</b>

(a) Complete the following table

<b>Shape</b>	1	2	3	4
<b>Number of blocks</b>	1	7	19	37

**Full Credit (5 Marks)**

- Fully correct

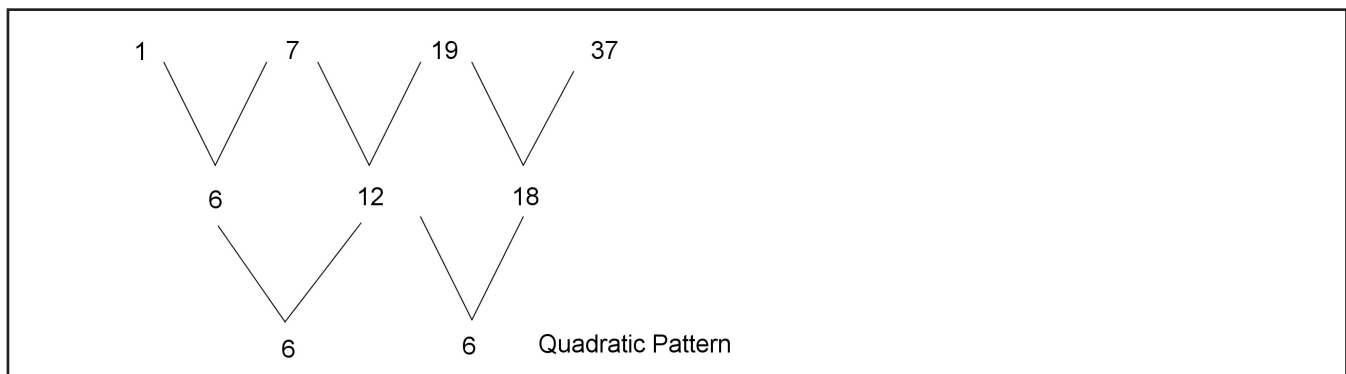
**High Partial Credit (3 Marks)**

- 1 incorrect

**Low Partial Credit (2 Marks)**

- Any 1 correct

(b) Describe how the number of blocks is changing.



**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

(c) How many blocks would be in the 5<sup>th</sup> shape?

61

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(d) Write an expression in  $n$  for the number of blocks in the  $n^{\text{th}}$  pattern in the sequence.

$$\begin{aligned} 2^{\text{nd}} \text{ Diff} &= 6 \quad \therefore a = 3 \\ T_n &= an^2 + bn + c \\ T_n &= 3n^2 + bn + c \\ n = 1 \quad T_1 &= 3(1)^2 + b(1) + c = 1 \\ &b + c = -2 \\ n = 2 \quad T_1 &= 3(2)^2 + b(2) + c = 7 \\ &2b + c = -5 \\ &\therefore b = -3, c = 1 \\ &\therefore T_n = 3n^2 - 3n + 1 \end{aligned}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error solving to end

**Low Partial Credit (3 Marks)**

- Any correct step

(e) What shape will require 397 blocks to build it?

$$\begin{aligned} 3n^2 - 3n + 1 &= 397 \\ 3n^2 - 3n - 396 &= 0 \\ n^2 - n - 132 &= 0 \\ (n - 12)(n + 11) & \\ \therefore n &= 12^{\text{th}} \text{ shape} \end{aligned}$$

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

# PAPER 2

## QUESTION 1

Part (a)

Scale 5B

Part (b)

Scale 10C

Part (c)

Scale 10C

Two lines make an angle of  $\tan^{-1} \frac{1}{4}$  with the line  $t$ , where  $t : 2x + y = 3$ .

(a) Write down the slope of the line  $t$ .

$$m = -2$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Some correct step towards finding the slope

(b) Find the values of  $m$ , where  $m$  is the slope of the lines that intersect the line  $t$ .

$$\frac{1}{4} = \pm \frac{-2 - m}{1 + (-2)(m)}$$
$$\frac{1}{4} = \frac{-2 - m}{1 + (-2)(m)} \quad \text{or} \quad \frac{1}{4} = -\frac{-2 - m}{1 + (-2)(m)}$$
$$1(1 - 2m) = 4(-2 - m) \quad \text{or} \quad 1(1 - 2m) = -4(-2 - m)$$
$$1 - 2m = -8 - 4m \quad \text{or} \quad 1 - 2m = 8 + 4m$$
$$m = -\frac{9}{2} \quad \text{or} \quad m = -\frac{7}{6}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error and continues to end correctly

**Low Partial Credit (3 Marks)**

- Any correct step

(c) Find the equations of both lines given that they both contain the point (1, 5).

$$\begin{array}{l} y - 5 = -\frac{9}{2}(x - 1) \quad \text{or} \quad y - 5 = -\frac{7}{6}(x - 1) \\ 2y - 10 = -9x + 9 \quad \text{or} \quad 6y - 30 = -7x + 7 \\ 9x + 2y - 19 = 0 \quad \text{or} \quad 7x + 6y - 37 = 0 \end{array}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- 1 error and continues to end correctly

**Low Partial Credit (3 Marks)**

- Any correct step

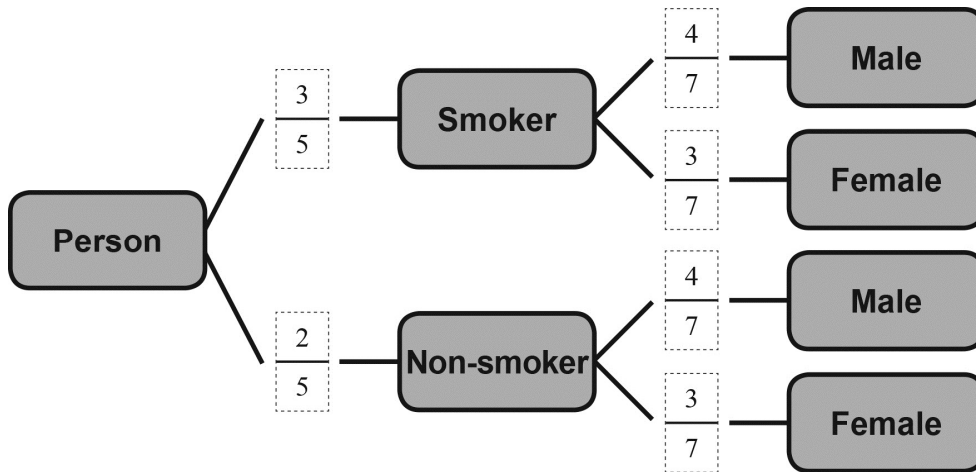
## QUESTION 2

Part (a)  
Part (b)  
Part (c)  
Part (d)

Scale 10C  
Scale 5B  
Scale 5B  
Scale 5B

In a survey of 80 men and 60 women, 60% of each gender said that they smoked cigarettes.

(a) Complete the following tree diagram.



**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Correct tree diagram with no probabilities shown
- Correct tree diagram with incorrect consistent probabilities shown

**Low Partial Credit (3 Marks)**

- Any correct step

A person is chosen at random and sent a follow-up questionnaire.

(b) Find the probability that the person chosen is a male who does not smoke.

$$\frac{8}{35}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Some correct step towards finding the slope
- 75% of the non-smokers having never smoked. 50% of those who have smoked are males.



(c) How many females were smokers but have quit?

7

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Some correct step towards finding the slope

The smoking ban was introduced in Ireland in 2004. A government official claims that the number of smokers has fallen steadily since 2004 and is significantly lower now than in 2004.

(d) Discuss the validity of his claim.

- Regular and heavy smokers numbers on the decline
- Light and occasional on the increase.
- Need further figures to determine if numbers have fallen significantly.

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partial explanation

## QUESTION 3

Scale 25D

Find the equation of a circle which contains the points  $(-3, 5)$  and  $(7, 11)$ , and whose centre lies on the line  $3x - 5y = -34$ .

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$(-3)^2 + (5)^2 + 2g(-3) + 2f(5) + c = 0$$

$$-6g + 10f + c = -34 \quad \text{eg 1}$$

$$(7)^2 + (11)^2 + 2g(7) + 2f(11) + c = 0$$

$$14g + 22f + c = -170 \quad \text{eg 2}$$

Centre  $(-g, -f)$

$$3(-g) - 5(-f) = -34$$

$$-3g + 5f = -34 \quad \text{eg 3}$$

$$20g + 12f = 136 \quad (\times 3) \Rightarrow 60g + 36f = 408$$

$$-3g + 5f = -34 \quad (\times 20) \Rightarrow \underline{-60g + 100f = -680}$$

$$136f = -272$$

$$f = -2$$

### Full Credit (25 Marks)

- Fully correct

### High Partial Credit (19 Marks)

- Error in equations but continues to end
- Equation of circle not written at end

### Mid Partial Credit (12 Marks)

- Sets up two correct equations

### Low Partial Credit (6 Marks)

- Any correct step

## QUESTION 4

**Part (a)**

**Scale 5B**

**Part (b) (i)**

**Scale 5B**

**(ii)**

**Scale 10C**

**(iii)**

**Scale 5B**

The probability of a certain make of car breaking down within two years after it is purchased is 0.18.

(a) Write the probability that the car will not break down within the two years.

$$0.82 = 82\% = \frac{41}{50}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(b) A local car dealer sold 20 of these cars in January 2014. Calculate the probability that:

(i) None of the cars will break down within the two year period.

$$\binom{20}{0}(0.82)^{20}(0.18)^0 = 0.01889$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

(ii) No more than two of the cars will break down within the two year period.

$$\binom{20}{0}(0.82)^{20}(0.18)^0 + \binom{20}{1}(0.82)^{19}(0.18)^1 + \binom{20}{2}(0.82)^{18}(0.18)^2 = 0.2747$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- One error

**Low Partial Credit (3 Marks)**

- Any correct step

(iii) At least three of the cars will break down within the two year period.

$$1 - 0.2747 = 0.7253$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Any correct step

## QUESTION 5

**Part (a)**

**Scale 5C**

**Part (b)**

**Scale 5B**

**Part (c)**

**Scale 15C**

(a) If  $\sin A = -0.6293$ , find two values of  $A$ , correct to the nearest degree, where  $0^\circ \leq A \leq 360$ .

$$A = 219^\circ, 321^\circ$$

**Full Credit (5 Marks)**

- Fully correct

**High Partial Credit (3 Marks)**

- Correct reference angle found with work towards finding two values of  $A$

**Low Partial Credit (2 Marks)**

- Any correct step

(b) The line  $q$  makes an acute angle with the  $x$ -axis as shown.

(i) Write down the slope of the line  $q$ .

$$m = \frac{1}{5}$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Some correct step towards finding the slope

- (ii) The line  $p$  makes an angle  $\beta$  with the  $x$ -axis such that  $\tan(\alpha + \beta) = \frac{11}{3}$ .

Find the slope of the line  $p$ .

$$\frac{\frac{1}{5} + \tan \beta}{1 - \frac{1}{5} \tan \beta} = \frac{11}{3}$$

$$\frac{3}{5} + 3 \tan \beta = 11 - \frac{11}{5} \tan \beta$$

$$\frac{26}{5} \tan \beta = \frac{52}{5}$$

$$\tan \beta = 2$$

$$\therefore m_p = \tan \beta = 2$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- Solves with no more than 1 error

**Low Partial Credit (5 Marks)**

- Any correct step

## QUESTION 6

**Part (a)**

**Scale 20D**

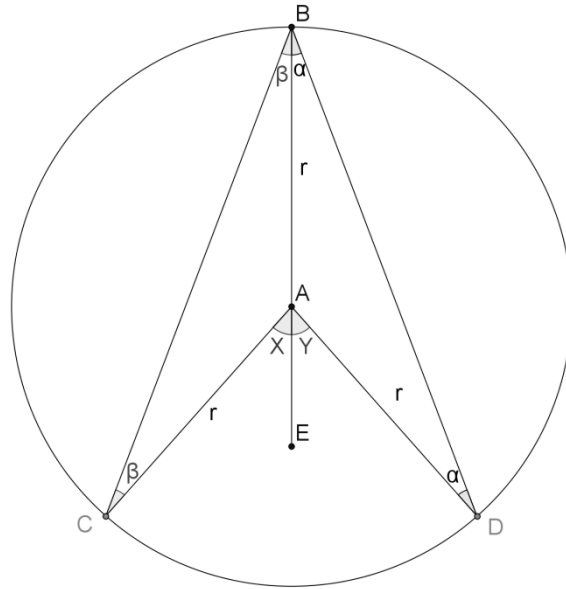
**Part (b)**

**Scale 5B**

- (a) Prove that the angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.

**Examine five sections of answer**

Diagram:



To Prove:  $|\angle CAD| = 2|\angle CBD|$

Given: A circle with centre  $A$  and Points  $B$ ,  $C$  and  $D$  on the circle as shown

Construction: Join  $B$  to  $A$  and extend to  $E$

Proof: In the triangle  $BCE$

$$|AB| = |AC| \quad \text{Radii}$$

$$|\angle ABC| = |\angle ACB| \quad \text{Theorem 2 (isosceles } \Delta)$$

$$|\angle X| = |\angle ABC| + |\angle ACB| \quad \text{Theorem 6 (exterior angle)}$$

$$|\angle X| = |\angle ABC| + |\angle ABC|$$

$$|\angle X| = 2|\angle ABC|$$

$$\text{Similarly } |\angle EAD| = 2|\angle ABD|$$

$$|\angle CAD| = 2|\angle CBD|$$

**Full Credit (20 Marks)**

- 5 sections fully correct

**High Partial Credit (15 Marks)**

- 3 or 4 sections fully correct

**Mid Partial Credit (10 Marks)**

- 2 sections fully correct

**Low Partial Credit (5 Marks)**

- Any correct step

(b) Hence find  $|\angle CGD|$ .

$$|\angle CGD| = 108^\circ$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct



## QUESTION 7

<b>Part (a)</b>	<b>Scale 10C*</b>
<b>Part (b)</b>	<b>Scale 15C</b>
<b>Part (c)</b>	<b>Scale 5A</b>
<b>Part (d)</b>	<b>Scale 15C</b>
<b>Part (e)</b>	<b>Scale 10B</b>
<b>Part (f)</b>	<b>Scale 15D</b>

(a) Calculate the distance from Ballyline to Dun Mór correct to one decimal place.

$$\frac{x}{\sin 75} = \frac{6.6}{\sin 47.2}$$
$$x = 8.7 \text{ km}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- One error

**Low Partial Credit (3 Marks)**

- Any correct step

Due to the installation of water meters, the road from Ballyline to Dun Mór will be closed for a week. Traffic will be diverted via Deepdale.

(b) Calculate the percentage increase in the distance motorists will have to travel due to the diversion if travelling from Ballyline to Dun Mór via Deepdale.

$$\frac{x}{\sin 57.8} = \frac{6.6}{\sin 47.2}$$
$$x = 7.6$$
$$\% \text{ increase} = \frac{(6.6 + 7.6) - 8.7}{8.7} \times 100 = 63.2\%$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- One error

**Low Partial Credit (5 Marks)**

- Any correct step

Each town will receive its water from a central reservoir after the water meters are installed. The reservoir will be equidistant from each town.

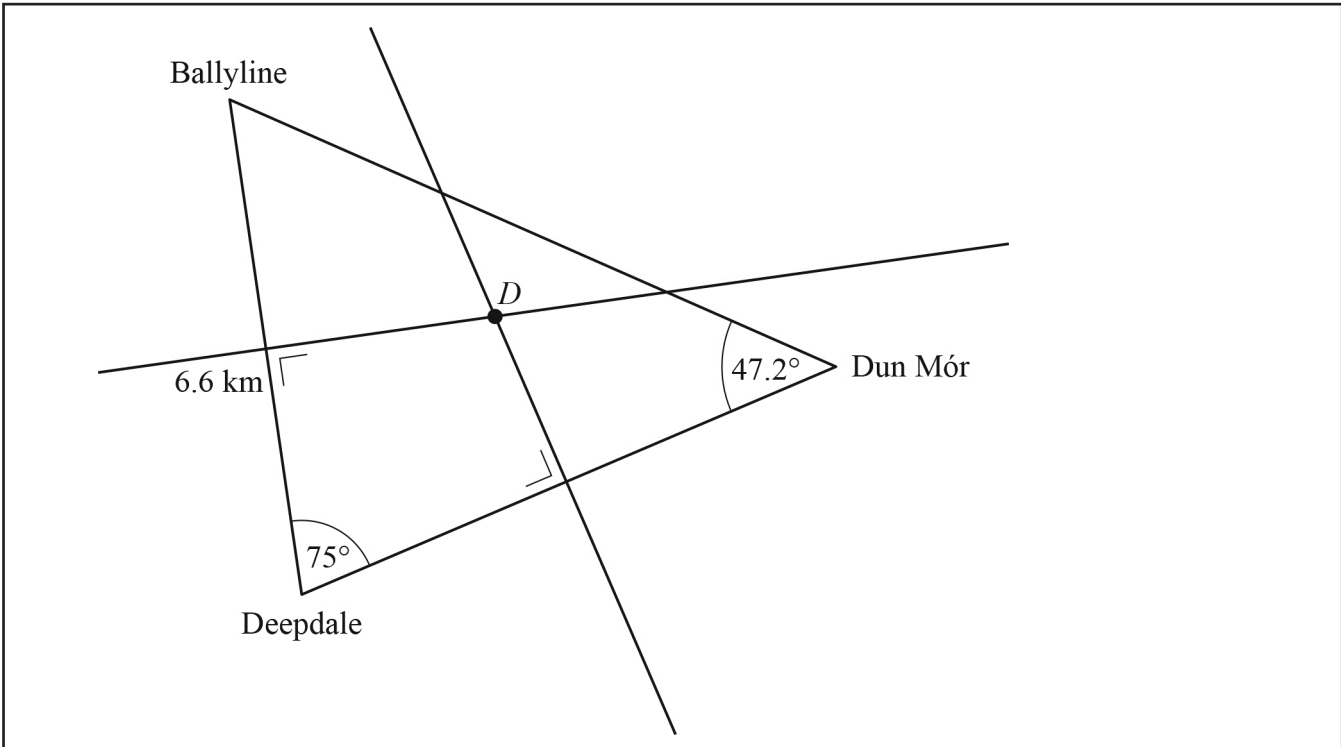
(c) What name is given to the point equidistant from the three vertices of a triangle?

Circumcentre.

**Full Credit (5 Marks)**

- Fully correct

(d) Using a suitable construction, show where the reservoir will be situated on the diagram above.



**Full Credit (15 Marks)**

- Fully correct

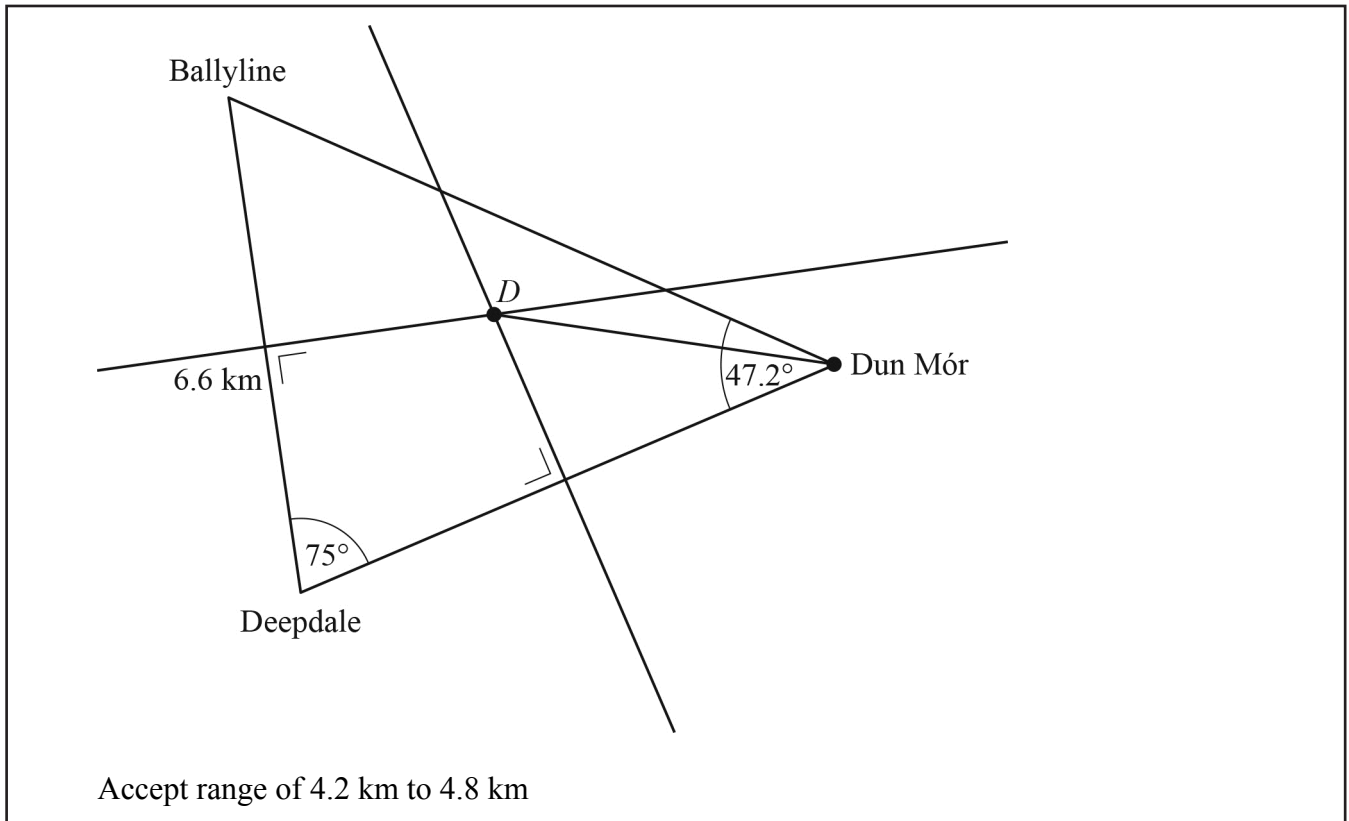
**High Partial Credit (10 Marks)**

- One bisector found correctly

**Low Partial Credit (5 Marks)**

- Any correct step

- (e) Taking a scale of 1 cm : 2 km, estimate the shortest distance from the reservoir to each town.



**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Any correct step

The County Planning Officer decides to double-check the above information by overlapping the information with a coordinate plane.

- (f) By choosing (0.1, 4.5) as coordinates for Ballyline, find the coordinates of the reservoir and verify your answer from part (e).

**\*\*Note:** Students' answers may vary slightly due to rounding of decimals.

**Correct in Steps:**

1. Two Midpoints
2. 2 Slopes
3. 2 equations of perpendicular bisectors
4. Simultaneous equation leading to a suitable  $x$  and  $y$  value
5. Distance within acceptable limits

**Deepdale to Dun Mór:**

$$Mid = \left( \frac{1+8}{2}, \frac{-2+1}{2} \right) = (4.5, 0.5)$$

$$m = \frac{1+2}{8-1} = \frac{3}{7} \quad \therefore \perp m = -\frac{7}{3}$$

$$y - 0.5 = -\frac{7}{3}(x - 4.5)$$

$$3y - 1.5 = -7x + 31.5$$

$$7x + 3y = 33$$

**Deepdale to Ballyline:**

$$Mid = \left( \frac{1+0.1}{2}, \frac{-2+4.5}{2} \right) = (0.55, 1.25)$$

$$m = \frac{4.5+2}{0.1-1} = -\frac{65}{9} \quad \therefore \perp m = \frac{9}{65}$$

$$y - 1.25 = \frac{9}{65}(x - 0.55)$$

$$65y - 81.25 = 9x - 4.95$$

$$-9x + 65y = 76.3$$

$$\begin{aligned} 7x + 3y &= 33 \\ -9x + 65y &= 76.3 \end{aligned}$$

$$\begin{aligned} 63x + 27y &= 297 \\ -63x + 455y &= 534.1 \end{aligned}$$

$$\begin{aligned} 482y &= 831.1 \\ y &= 1.6 \text{ to } 1.8 \\ \therefore x &= 3.5 \text{ to } x = 3.9 \end{aligned}$$

$$\sqrt{(8-3.9)^2 + (1-1.7)^2}$$

$$4.2 \text{ km}$$

Accept range 4.1 km to 4.8 km

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (11 Marks)**

- 3 or 4 correct steps

**Low Partial Credit (7 Marks)**

- 1 or 2 correct steps

**Low Partial Credit (4 Marks)**

- Any partially correct step

## QUESTION 8

<b>Part (a) (i)</b>	<b>Scale 5B</b>
<b>(ii)</b>	<b>Scale 15C</b>
<b>(iii)</b>	<b>Scale 10B</b>
<b>Part (b) (i)</b>	<b>Scale 5B</b>
<b>(ii)</b>	<b>Scale 5B</b>
<b>(iii)</b>	<b>Scale 15C</b>

(a) A light bulb manufacturer claims that the lengths of the life of its bulbs have a mean of 16 months and a standard deviation of 3 months. An independent research company tests a random sample of 60 bulbs to investigate the company's claim.

(i) Calculate an approximate value for the standard deviation of the sample means.

$$\frac{3}{\sqrt{60}} = 0.387$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

(ii) Assuming the company's claim is true, what is the probability that the research company's sample has a mean life of 15 months or less?

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$
$$z = \frac{15 - 16}{0.387} = -2.58$$
$$P(z \leq -2.58) = 1 - P(z < 2.58)$$
$$= 1 - 0.9951$$
$$= 0.0049 = 0.49\%$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- Error in  $z$  value but continues to end correctly

**Low Partial Credit (5 Marks)**

- Any correct step

- (iii) Based on this information, would you agree or disagree with the manufacturing company's claims? Explain your answer fully.

As  $|z| > 2$ , the company's claim we do not agree with the company's claims.  
The calculated  $z$  score suggests the true mean is lower.

**Full Credit (10 Marks)**

- Fully correct

**Partial Credit (5 Marks)**

- Partially correct

- (b) The manufacturing company claims that 90% of its customers find their bulbs good value for money. Out of 400 customers surveyed, 346 said that the light bulbs were good value for money.

- (i) State the null hypothesis.

$$H_0 : P = 0.9$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

- (ii) State the alternate hypothesis.

$$H_1 : P \neq 0.9$$

**Full Credit (5 Marks)**

- Fully correct

**Partial Credit (2 Marks)**

- Partially correct

(iii) Use a hypothesis test to investigate the company's claim.

Step 1:  $\hat{p} = \frac{346}{400} = 0.865$

Step 2:  $E = \frac{1}{\sqrt{400}} = 0.05$

Step 3:  $0.865 - 0.05 < p < 0.865 + 0.05$   
 $0.815 < p < 0.915$

Step 4:  $\therefore$  population proportion is within the confidence interval.  
No evidence to reject the company's claims.

\*\*\*Deduct one mark from a fully correct solution if a candidate states "We accept the company's claim"

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- 2 or 3 correct steps

**Low Partial Credit (5 Marks)**

- Any correct step

## QUESTION 9

**Part (a)**

**Scale 15C\***

**Part (b)**

**Scale 10C\***

A ranger in a forest can view two campsites from the top of the lookout tower which is 60 m high. The angle of depression to Campsite 1 is  $37.3^\circ$  while the angle of depression with Campsite 2 is  $18.4^\circ$ .

- (a) Calculate the distance from the bottom of the tower to each campsite correct to one decimal place.

$$\tan 37.3 = \frac{60}{x}$$
$$x = 78.8 \text{ m}$$

$$\tan 18.4 = \frac{60}{x}$$
$$x = 180.4 \text{ m}$$

**Full Credit (15 Marks)**

- Fully correct

**High Partial Credit (10 Marks)**

- Both calculated with one error

**Low Partial Credit (5 Marks)**

- Any correct step

- (b) It is intended to open a walking trail that extends through each campsite. Calculate the distance from Campsite 1 to Campsite 2 correct to one decimal place.

$$x^2 = (78.8)^2 + (180.4)^2 - 2(78.8)(180.4)\cos(121)$$
$$x = 231.1 \text{ m}$$

**Full Credit (10 Marks)**

- Fully correct

**High Partial Credit (7 Marks)**

- Calculated with one error to end

**Low Partial Credit (3 Marks)**

- Any correct step