P 6

Pre-Leaving Certificate Examination, 2015

Triailscrúdú na hArdteistiméireachta, 2015

Mathematics

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

Section A — Concepts and Skills — 150 marks

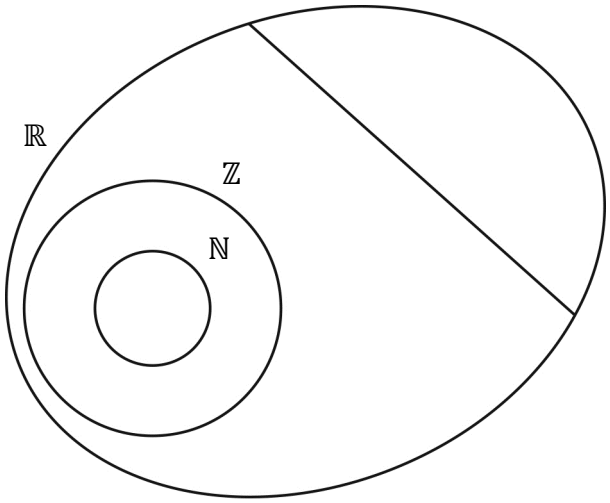
Answer **all six** questions from this section.

Question 1 (25 marks)

(a) What is a proof by contradiction?

(b) What is an irrational number? Give an example of an irrational number.

(c) Shade in the region of the following Venn diagram that represents the set of irrational numbers.



(d) Use a proof by contradiction to prove that $\sqrt{2}$ is an irrational number.

Question 2 **(25 marks)**

(a) Simplify:

$$\frac{x^2-4}{3y^2+y}\times\frac{3y^3-2y^2-y}{x^2+x-6}$$

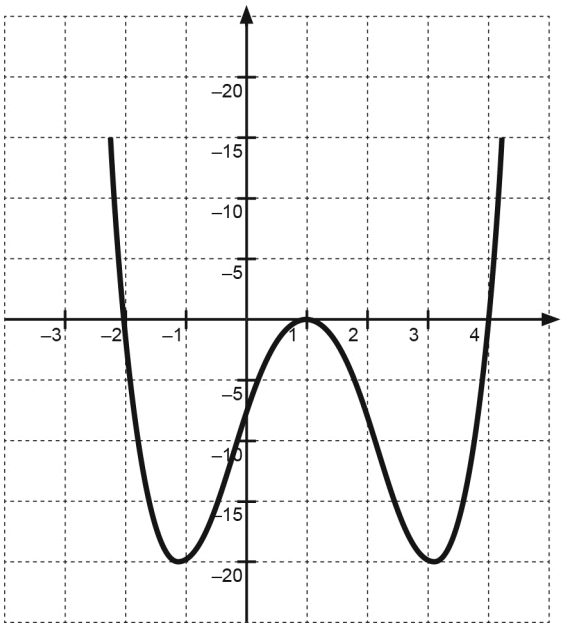
(b) Solve the inequality, $\frac{3x-2}{x+4}<2, x\neq 4$ and $x\in\mathbb{R}$.

(c) Solve the simultaneous equations:

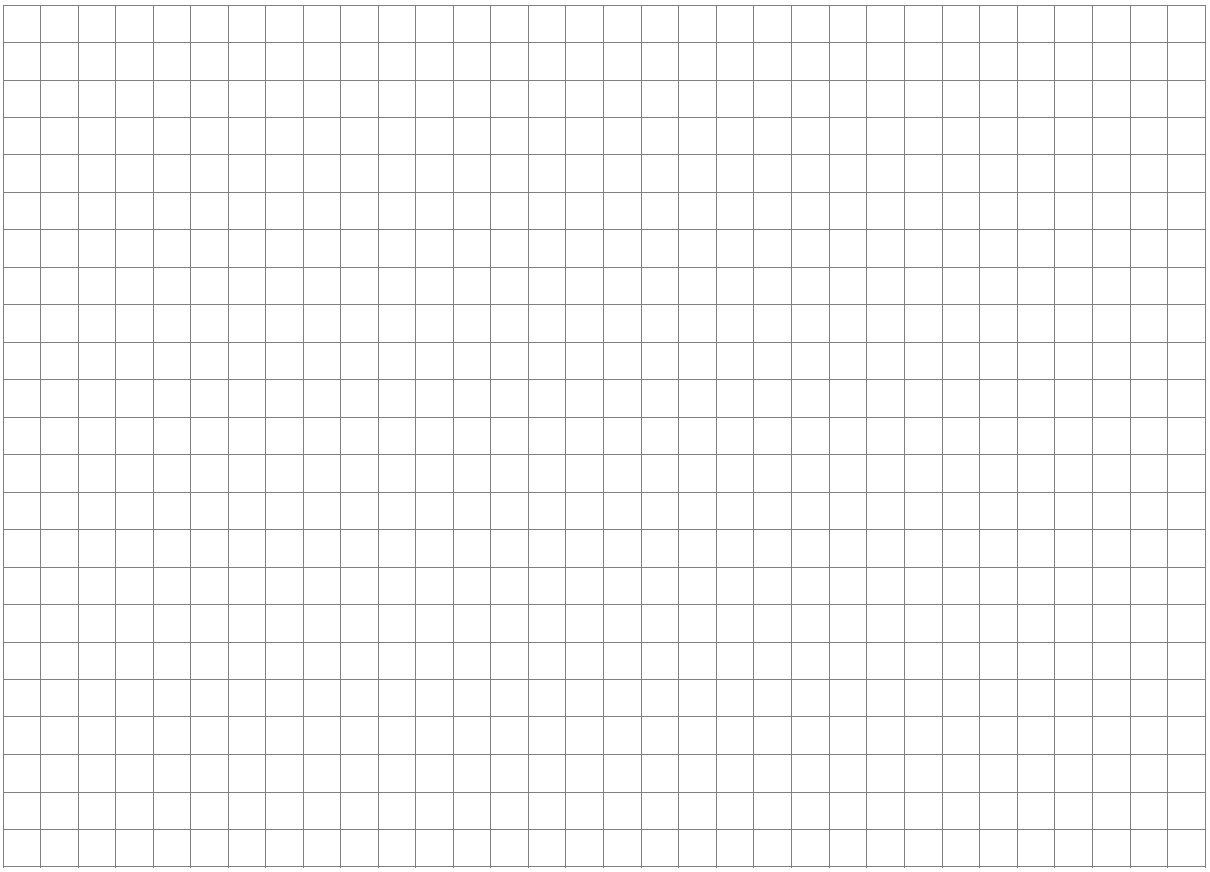
$$2x + 3y - 4z = -11$$
$$-x + 2y + z = -1$$
$$x - y + 3z = 12$$

Question 3 **(25 marks)**

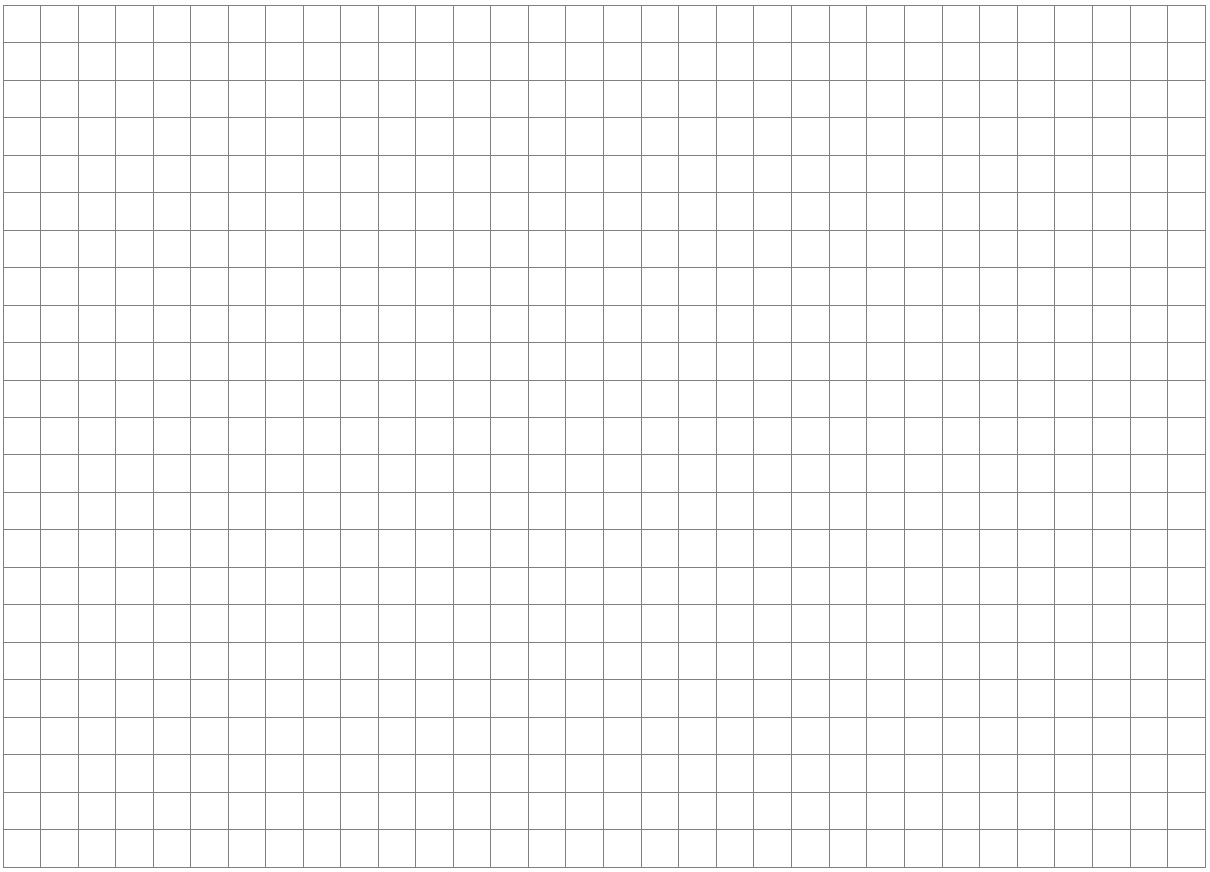
(a) Examine the graph of the polynomial shown. Write down the polynomial for the graph in the form $ax^4 + bx^3 + cx^2 + dx + e$, where $a, b, c, d, e \in \mathbb{R}$.



(b) Investigate if $3x-1$ is a factor of $6x^3-5x^2+4x+5$.



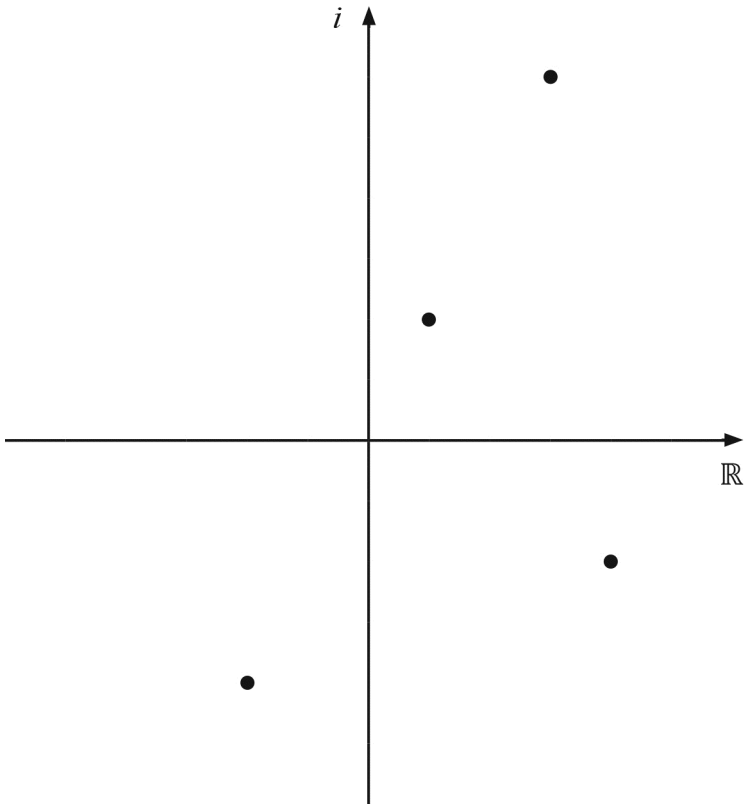
(c) Given that x^2+ax-1 is a factor of $x^3+px^2+qx+r=0$, show that $q=-(ar+1)$.



Question 4 **(25 marks)**

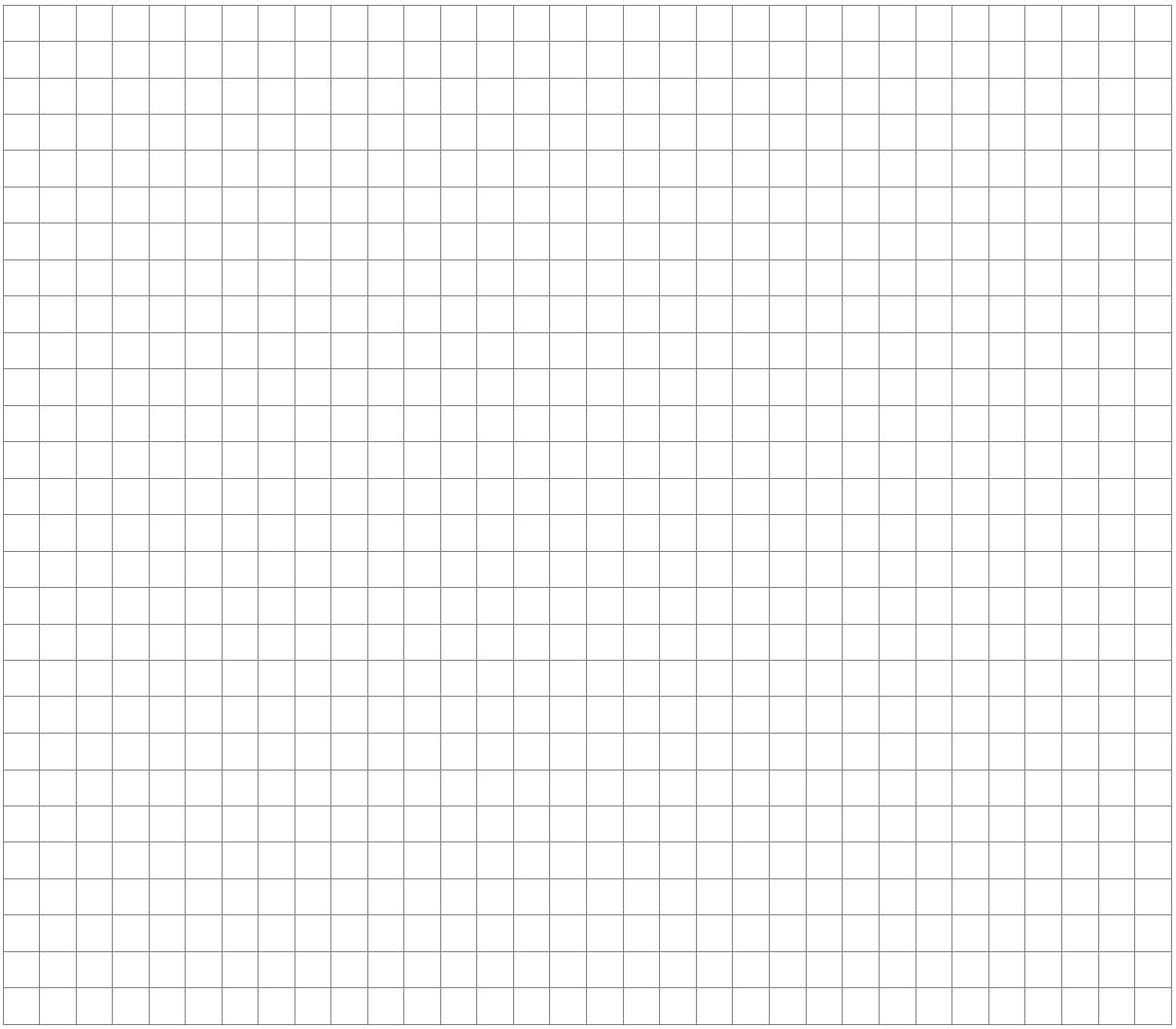
(a) Label each of the complex numbers given the following information.

$z_2 = 3z_1$ $z_4 = iz_3$ $z_3 = -2z_1$



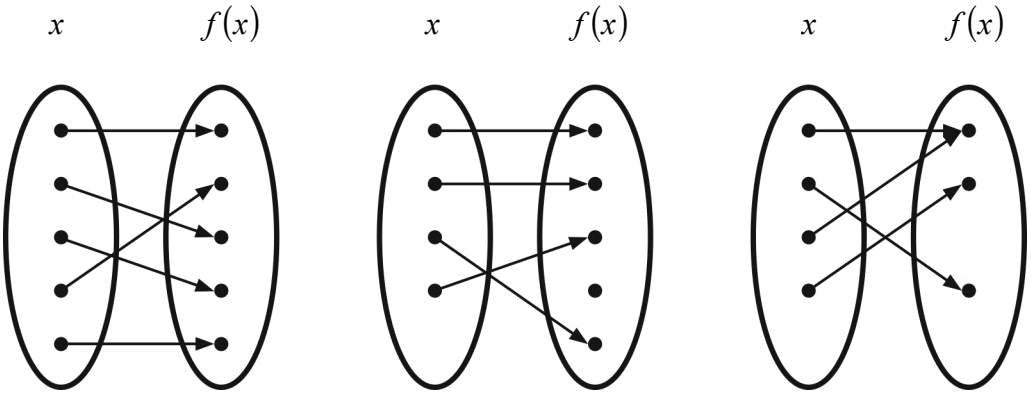
(b) **(i)** Given $\omega = 2 + 2\sqrt{3}i$, write ω in Polar Form.

(ii) Hence solve the equation $z^4 = \omega$.



Question 5 **(25 marks)**

(a) Identify the following functions as injective, surjective or bijective.



(b) The growth of a certain species of bacteria can be modelled with the equation $N = Re^{kt}$.
A science teacher puts 100 bacteria into nutrient agar plates (agar will act as a food source for the bacteria). Five hours later, there are 600 bacteria.

(i) Calculate the value of k correct to four decimal places.

(ii) After how many hours will the number of bacteria in the plate be 12,000?

(c) Solve for x correct to two decimal places:

$$3^{2x+1} - 10\left(3^{x}\right) - 8 = 0$$

Question 6 **(25 marks)**

(a) Find:

(i) $\int (2x^2 + 4x - 2)dx$.

(ii) $\int (\sin 3x - \cos 2x)dx$.

(b) Evaluate $\int_1^3 \left(\frac{x^3 - 8}{x - 2} \right)$.

(c) Find the area enclosed by the x-axis and the curve $y = -x^2 + 2x + 3$.

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(60 marks)**

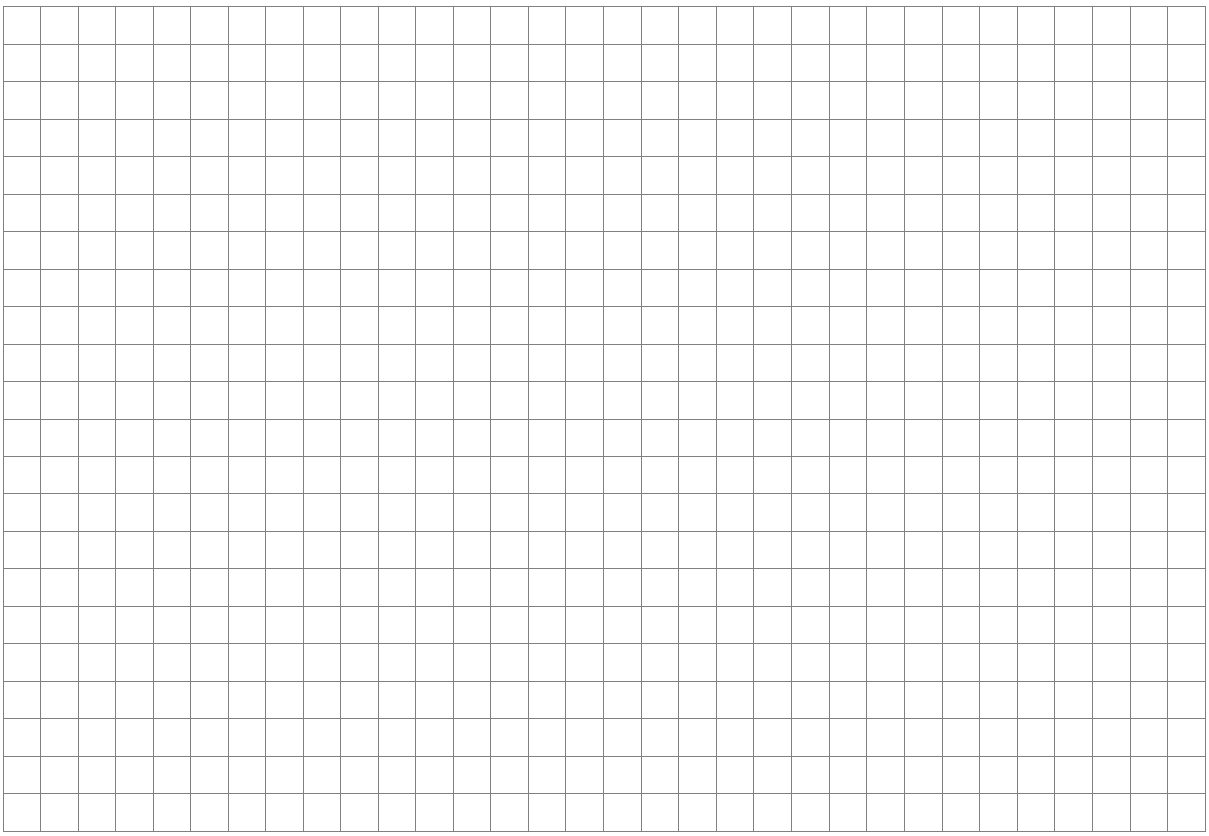
(a) John wishes to borrow €X from his bank over n years. Interest of r% APR will be applied to the loan. He will make an annual repayment of €A. Show the amount owing at the end of the first year, P_1, can be written as:

$$P_1 = X(1+r)A$$

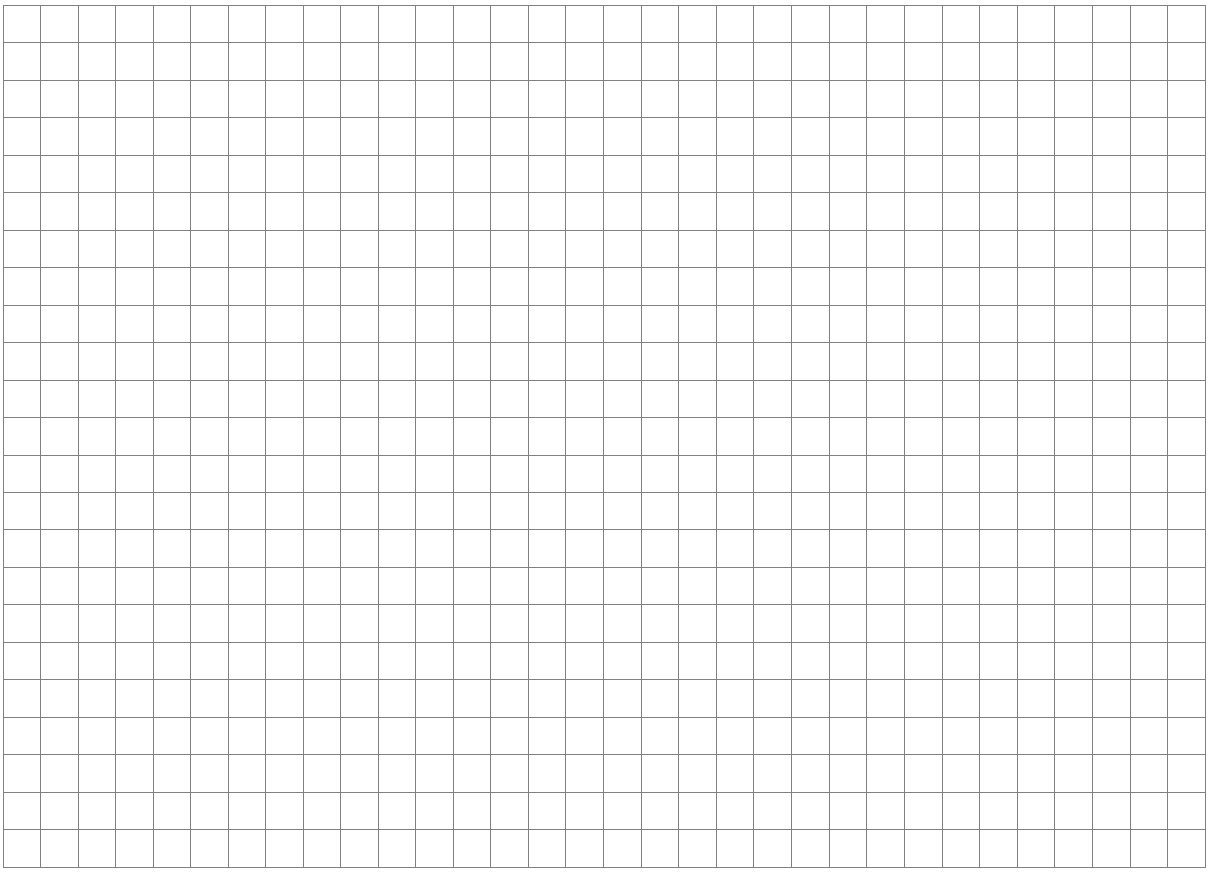
(b) If $P_n = 0$ after the final year of the mortgage, show that the annual repayment A can be written as $A = \dfrac{Xr(1+r)^n}{(1+r)^n - 1}$.

(c) John borrows €150,000 over 20 years at 4% compound interest per annum.

(i) Calculate the monthly interest rate correct to five significant figures.



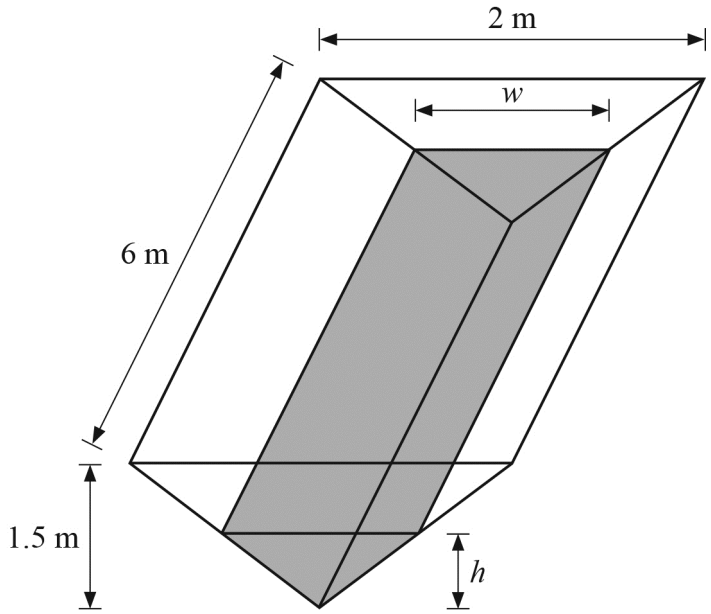
(ii) Calculate John's monthly repayment correct to the nearest euro.



(iii) After 12 years, John decides to pay off the remainder of the loan in one lump sum. Calculate the remaining balance on the loan correct to the nearest euro.

Question 8 **(50 marks)**

A water trough is in the shape of a prism as shown.



(a) Calculate the internal volume of the water trough.

(b) Write an equation to represent the volume (V) of water in the tank at any time t in terms of h and w.

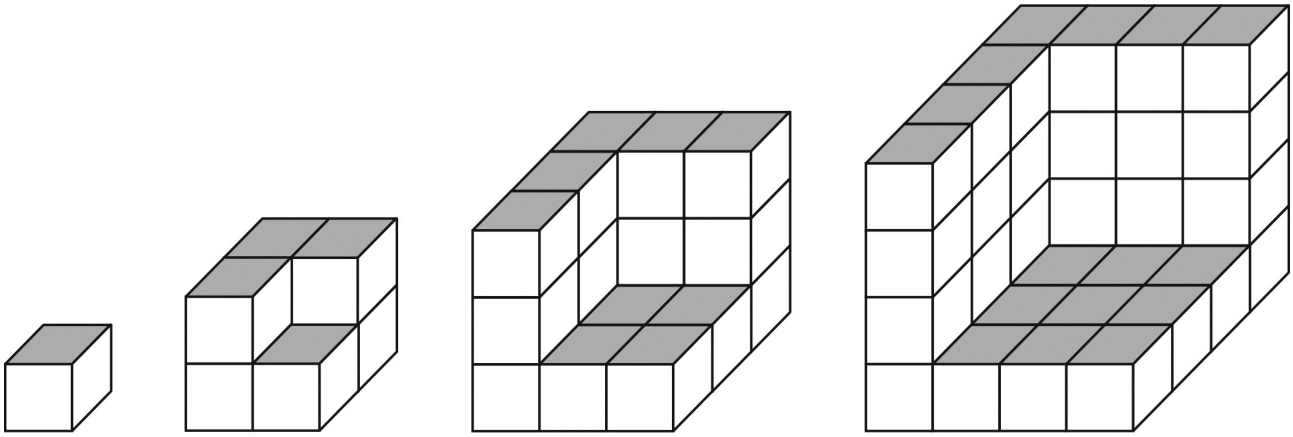
(c) Show that $w = \frac{4}{3}h$ metres.

(d) Show that the volume of the water trough can be expressed as: $V = 4h^2$.

(e) If water is being pumped in at a constant rate of 5 m^3/s, at what rate is the height of the water changing when the water has a height of 120 cm?

Question 9 **(40 marks)**

Paul is using wooden blocks to build different shapes as shown.



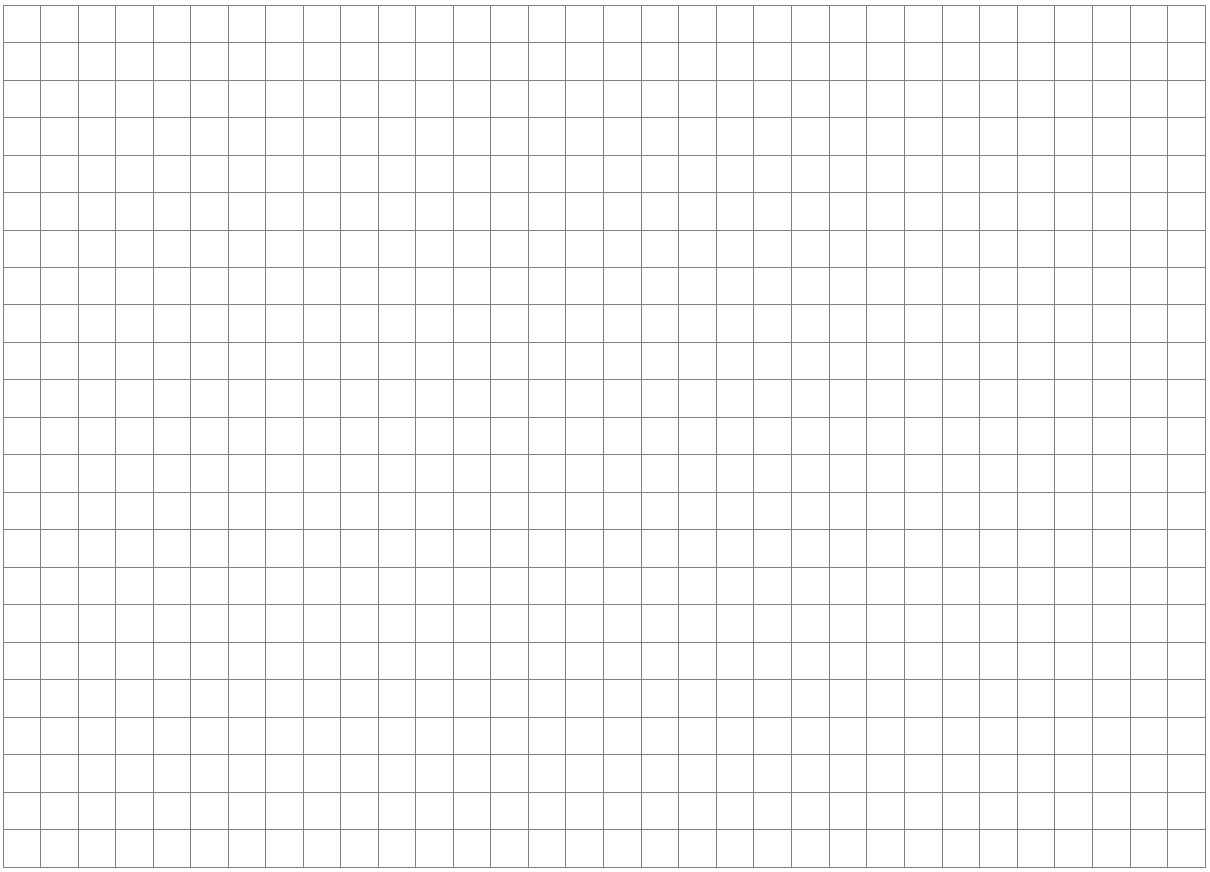
1 2 3 4

(a) Complete the following table

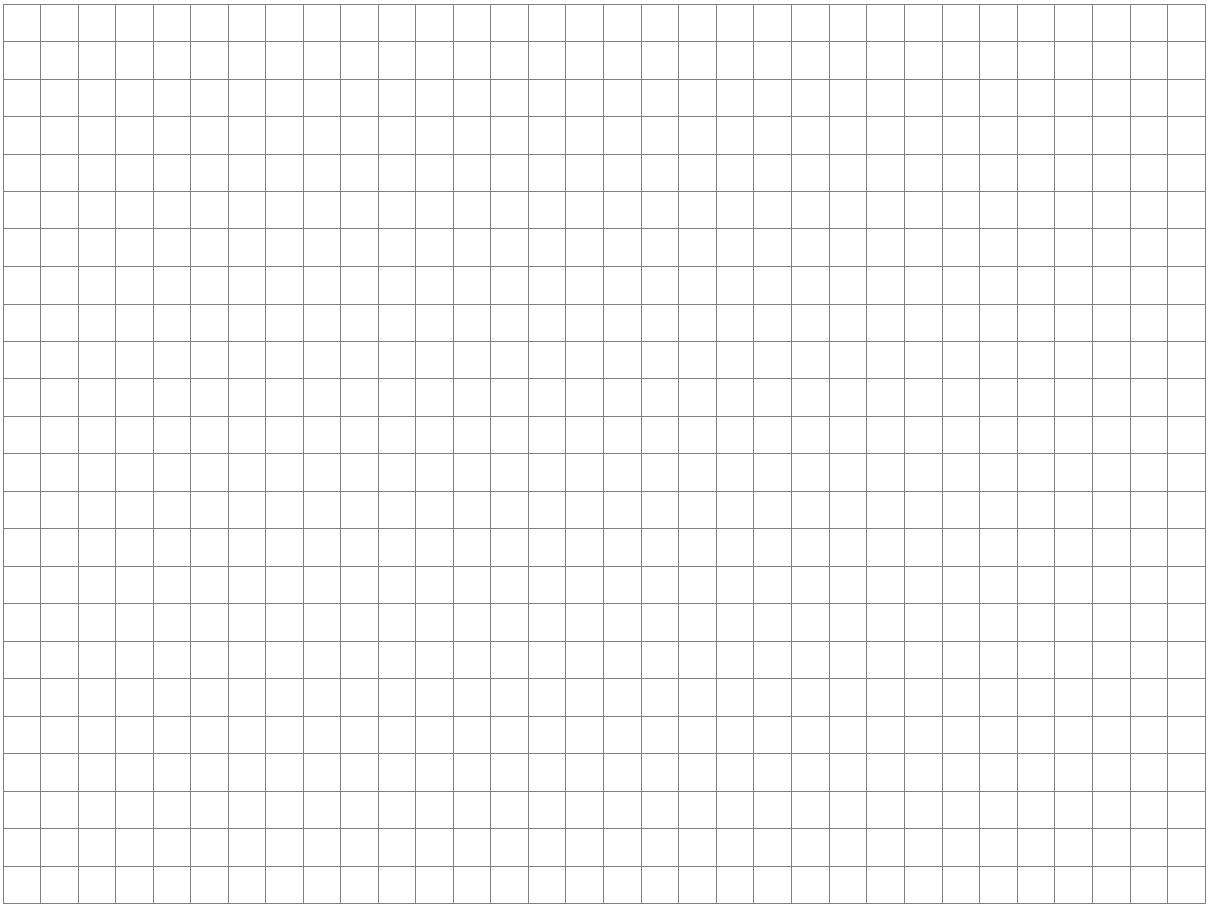
Shape	1	2	3	4
Number of blocks				

(b) Describe how the number of blocks is changing.

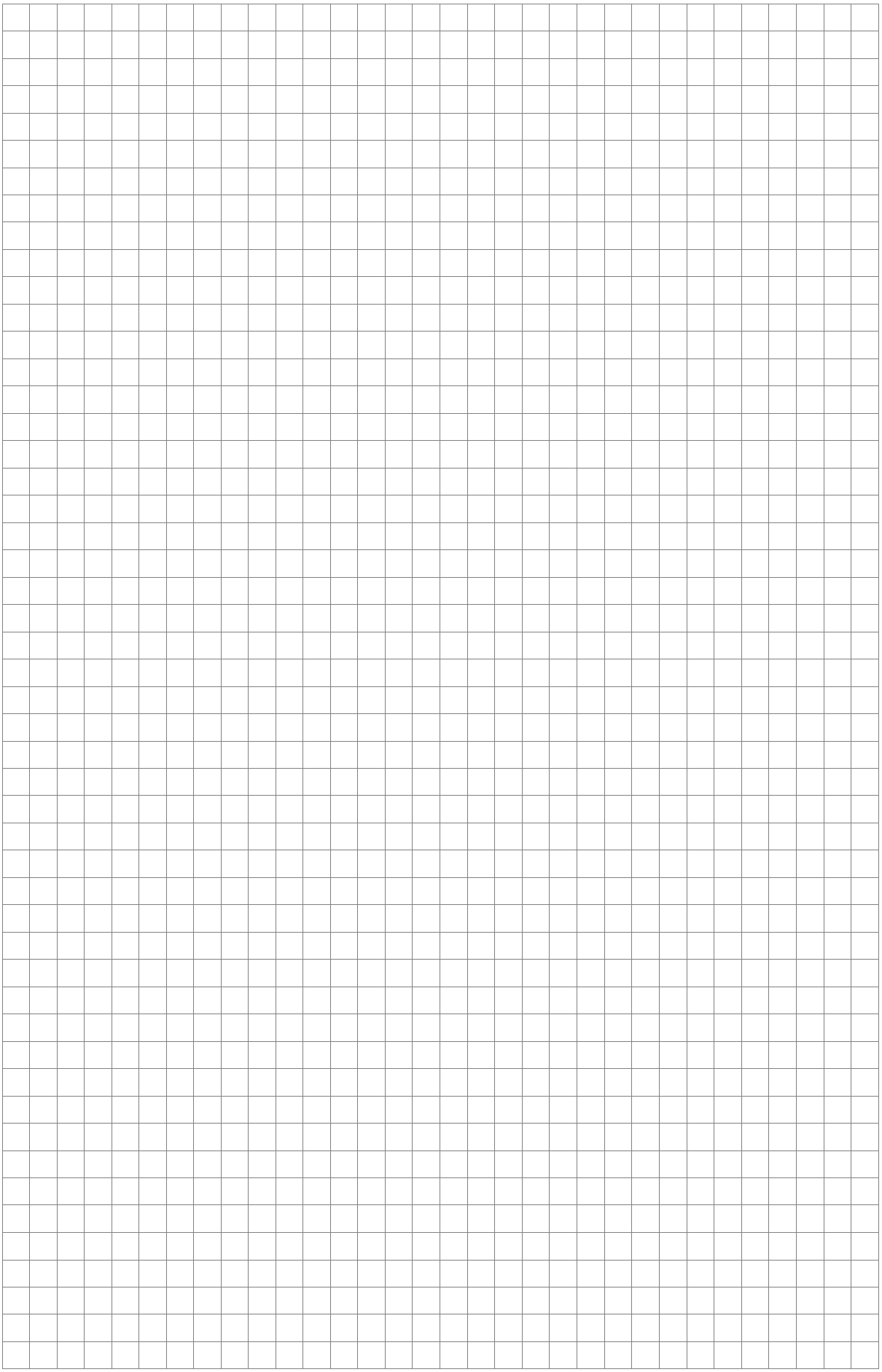
(c) How many blocks would be in the 5^{th} shape?



(d) Write an expression in n for the number of blocks in the n^{th} pattern in the sequence.



(e) What shape will require 397 blocks to build it?



You may use this page for extra work.