



PRE-LEAVING CERTIFICATE EXAMINATION, 2016

MARKING SCHEME

MATHEMATICS

HIGHER LEVEL

35 Finglas Business Park, Tolka Valley Road, Finglas, Dublin 11
T: 01 808 1494, F: 01 836 2739, E: info@examcraft.ie, W: www.examcraft.ie

OVERVIEW OF MARKING SCHEME

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10	0, 2, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

Marking categories for all questions are shown throughout the solutions. In certain cases, typically involving rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, Scale 10C* indicates that 9 marks may be awarded.

PAPER 1

QUESTION 1

(a) Scale 15C (0, 5, 10, 15)

Solve the following simultaneous equations:

$$\begin{aligned}x - y &= 1 \\(x - 5)^2 + (y - 4)^2 &= 32\end{aligned}$$

$$\begin{aligned}y &= x - 1 \\(x - 5)^2 + (x - 5)^2 &= 32 \\(x - 5)^2 &= 16 \\(x - 5) &= \pm 4 \\x = 9 \quad y &= 8 \\x = 1 \quad y &= 0\end{aligned}$$

High Partial Credit (10 Marks)

- Solves with no more than one error

Low Partial Credit (5 Marks)

- Any correct step

(b) Scale 10C (0, 4, 8, 10)

Express the following as a single fraction in its simplest form.

$$\frac{5}{x^2 - 5x - 14} \div \frac{5x + 25}{x^2 - 2x - 35}$$

$$\frac{5}{(x+2)(x-7)} \times \frac{(x-7)(x+5)}{5(x+5)}$$
$$\frac{1}{(x+2)}$$

High Partial Credit (8 Marks)

- Factorises fractions correctly and error in simplifying
- Error in factorising, but simplifies the resulting fractions correctly

Low Partial Credit (4 Marks)

- Any correct step

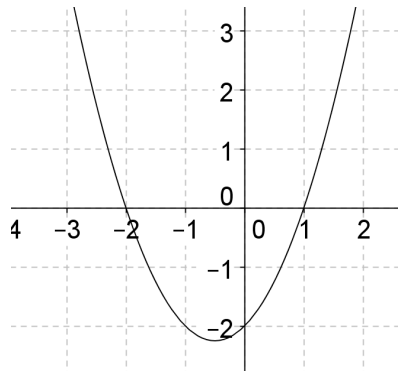
QUESTION 2

(a) Scale 15C (0, 5, 10, 15)

Solve the inequality $\frac{2x+1}{x-1} \leq 1$, $x \neq 1$ for $x \in \mathbb{R}$.

$$\frac{2x+1}{x-1} \leq 1$$
$$(2x+1)(x-1) \leq (x-1)^2$$
$$x^2 + x - 2 \leq 0$$
$$x = 1, x = -2$$

$-2 \leq x < 1$ as $x \neq 1$



High Partial Credit (10 Marks)

- Correct quadratic no solution set
- Error in quadratic but has a relevant correct solution set
- Wrong inequalities used in solution set

Low Partial Credit (5 Marks)

- Multiplies by $(x-1)$

Solve the following equation: $2^{2x+1} - 5(2^x) - 12 = 0$.

$$2^{2x+1} - 5(2^x) - 12 = 0$$

$$2(2^x)^2 - 5(2^x) - 12 = 0$$

$$2y^2 - 2y - 12 = 0$$

$$(2y+3)(y-4) = 0$$

$$y = -\frac{3}{2}, y = 4$$

$$\therefore 2^x = 2^2$$

$$x = 2$$

High Partial Credit (8 Marks)

- Sets up an incorrect quadratic, but completes with solution set
- Correct quadratic, but incorrect solution
- Invalid solution not cancelled/Correct solution not indicated

Low Partial Credit (4 Marks)

- Any correct work with indices

QUESTION 3

(a) Scale 15D (0, 4, 7, 11, 15)

Solve the equation $z^3 - 2z^2 + 5z + 26 = 0$ given that one of the roots is an integer.

$$(-2)^3 - 2(-2)^2 + 5(-2) + 26 = 0$$

$(z + 2)$ is a factor

Long division or other: $(z^2 - 4z + 13)(z + 2)$

$$z = (2 + 3i), (2 - 3i), -2$$

High Partial Credit (11 Marks)

- Error in long division, but continues to end
- Long division correct, but final quadratic not factorised or factorised incorrectly

Mid Partial Credit (7 Marks)

- Finds linear factor

Low Partial Credit (4 Marks)

- Any correct step e.g. any substitution into equation

(b) Scale 10C (0, 4, 8, 10)

Use De Moivre's Theorem to solve the equation $z^3 - 8 = 0$.

$$z^3 = 8$$

$$r = \sqrt{(8)^2} = 8$$

$$\alpha = \tan^{-1} 0 = 0^\circ$$

$$z^3 = [8(\cos(2n\pi) + i \sin(2n\pi))]$$

$$z = [8(\cos(2n\pi) + i \sin(2n\pi))]^{\frac{1}{3}}$$

$$z = \left[8^{\frac{1}{3}} \left(\cos\left(\frac{2n\pi}{3}\right) + i \sin\left(\frac{2n\pi}{3}\right) \right) \right]$$

$$n = 0 \quad z = 2$$

$$n = 1 \quad z = -1 + \sqrt{3}i$$

$$n = 2 \quad z = -1 - \sqrt{3}i$$

High Partial Credit (8 Marks)

- Finds r and θ correctly
- Finds r and θ correctly with one error and continues to end
- Writes in polar form with one error and continues to end

Low Partial Credit (4 Marks)

- Any correct step e.g. plots complex number

QUESTION 4

(a)

Scale 10D

(0, 2, 5, 8, 10)

Prove that $\sqrt{2}$ is an irrational number.

Assume $\sqrt{2}$ is rational.

Then, $\sqrt{2} = \frac{p}{q}$, where p, q are integers, $q \neq 0$

If $\text{HCF}(p, q) \neq 1$, then by dividing p and q by $\text{HCF}(p, q)$, $\sqrt{2} = \frac{a}{b}$

Where $\text{HCF}(a, b) = 1$ (1)

$$\Rightarrow \sqrt{2}b = a$$

$$\Rightarrow 2b^2 = a^2$$

$\Rightarrow a^2$ is divisible by 2

$\Rightarrow a$ is divisible by 2 (2)

$\Rightarrow a = 2c$, where c is an integer

$$\therefore \sqrt{2}b = 2c$$

$$\Rightarrow 2b^2 = 4c^2$$

$$\Rightarrow b^2 = 2c^2$$

$\Rightarrow b^2$ is divisible by 2

$\Rightarrow b$ is divisible by 2 (3)

From (2) and (3), 2 is a common factor of a and b , which contradicts (1)

So, $\sqrt{2}$ is an irrational number

High Partial Credit (8 Marks)

- Reaches (2) correctly

Mid Partial Credit (5 marks)

- Reaches (1) correctly

Low Partial Credit (2 Marks)

- Any correct step

(b) (i)

Scale 5B

(0, 3, 5)

Write out the general term form of the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{15}$.

$$\binom{15}{r} (x^2)^{15-r} \left(-\frac{1}{x}\right)^r$$

Partial Credit (3 Marks)

- Any correct step

(ii)

Scale 10C

(0, 4, 8, 10)

Hence, or otherwise, find the value of the term that is independent of x in the expansion.

$$\binom{15}{r} (x^2)^{15-r} \left(-\frac{1}{x}\right)^r$$
$$\binom{15}{r} x^{30-2r} (-1)^r (x)^{-r}$$
$$x^{30-3r} = x^0$$
$$r = 10$$
$$\binom{15}{10} (x^2)^5 \left(-\frac{1}{x}\right)^{10} = 3003$$

High Partial Credit (8 Marks)

- Sets up general term of expansion correctly
- Incorrect general term, but continues to end to find r
- Expands binomial, but with an error
- Finds correct independent term, but errors if binomial is expanded

Low Partial Credit (4 Marks)

- Any correct part of binomial expansion

QUESTION 5

(a) Scale 5C (0, 2, 4, 5)

Which one of the mapping diagrams below is bijective? Explain your answer fully.

(ii)
It's 1 – 1 and onto

High Partial Credit (4 Marks)

- Correct mapping and one correct reason
- Incorrect mapping both reasons correct

Low Partial Credit (2 Marks)

- Correct mapping named only
- Explains 1-1 or onto for any function

(b) Evaluate

(i) Scale 10C (0, 4, 8, 10)

$$\lim_{n \rightarrow 3} \frac{n^3 - 27}{n - 3}$$

$$\lim_{n \rightarrow 3} \frac{n^3 - 27}{n - 3} = \frac{(n - 3)(n^2 + 3n + 9)}{n - 3}$$
$$\lim_{n \rightarrow 3} (n^2 + 3n + 9)$$
$$3^2 + 3(3) + 9 = 27$$

High Partial Credit (8 Marks)

- Correct factorisation, but error in handling limit
- Factorises incorrectly, but continues to end correctly

Low Partial Credit (4 Marks)

- Attempts to factorise
- Subs 3 into denominator

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{7x^2 + 2}$$

$$\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{7x^2 + 2} = \frac{\frac{2x^2}{x^2} - \frac{3}{x^2}}{\frac{7x^2}{x^2} + \frac{2}{x^2}} = \frac{2}{7}$$

High Partial Credit (8 Marks)

- Error in division of terms, but continues to end
- Divides by x or x^2 but mishandles limit

Low Partial Credit (4 Marks)

- Any correct substitution
- Subs ∞ into formula

QUESTION 6

(a) Scale 10C (0, 4, 8, 10)

Differentiate $-2x^2 + 3x - 12$ from first principles.

$$\begin{aligned} f(x+h) &= -2(x+h)^2 + 3(x+h) - 12 \\ &= -2x^2 - 4xh - 2h^2 + 3x + 3h - 12 \\ f(x+h) - f(x) &= -4xh - 2h^2 + 3h \\ \frac{f(x+h) - f(x)}{h} &= -4x - 2h + 3 \\ \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= -4x - 0 + 3 \\ &= -4x + 3 \end{aligned}$$

High Partial Credit (8 Marks)

- Fully correct but fails to write LHS
- Fully correct LHS but contains one error

Low Partial Credit (4 Marks)

- Differentiates fully correct but not from first principles
- Any correct substitution

(b) (i) Scale 10B (0, 5, 10)

If $f(x) = e^{\cos x}$, find $f'(x)$ the derivative of $f(x)$.

$$f'(x) = -\sin x e^{\cos x}$$

Partial Credit (5 Marks)

- Writes $e^{\cos x}$
- Differentiates $\cos x$

(ii)

Scale 5B

(0, 2, 5)

Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx$.

$$\int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx = \left(-e^{\cos x} \right)_0^{\frac{\pi}{2}}$$
$$(-e^0) - (-e^1)$$
$$-1 + e$$

Partial Credit (2 Marks)

- Writes $e^{\cos x}$
- Integrates $\cos x$

QUESTION 7

Mary is 25 years old and intends to retire at the age of 65. She begins to calculate how much she needs in her pension fund. She bases all her calculations on a 4% annual growth rate.

Mary wants to receive a payment of €30,000 at the start of each year for 25 years after her retirement.

(a) (i) **Scale 10C*** **(0, 4, 8, 10)**

Write down the present value of a future payment of €30,000 in one year's time, correct to the nearest cent.

$$P = \frac{F}{(1+i)^t}$$
$$P = \frac{30000}{(1+0.04)^1}$$
$$P = €28,846.15$$

High Partial Credit (8 Marks)

- Correct substitution, but error in %

Low Partial Credit (4 Marks)

- Write 4% as correct fraction or decimal
- Rearranges formula
- One correct substitution

(ii) **Scale 10C** **(0, 4, 8, 10)**

Write down the present value of a future payment of €30,000 in t years' time.

$$P = \frac{F}{(1+i)^t}$$
$$P = \frac{30000}{(1+0.04)^1}$$
$$P = \frac{30000}{(1.04)^1}$$

**Accept answer from part (i) without incurring error here

**If candidate makes same mistake here as part (a) award 9 marks.

High Partial Credit (8 Marks)

- Correct substitution but error in %

Low Partial Credit (4 Marks)

- Writes 4% as correct fraction or decimal
- Rearranges formula
- One correct substitution

How much, correct to the nearest euro, will Mary require to have in her fund to guarantee this payment?

$$\text{Fund} = \frac{30000}{(1.04)^0} + \frac{30000}{(1.04)^1} + \frac{30000}{(1.04)^2} + \dots + \frac{30000}{(1.04)^{24}}$$

$$a = 30000 \quad r = \frac{1}{1.04}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{25} = \frac{30000 \left(1 - \left(\frac{1}{1.04} \right)^{25} \right)}{1 - \frac{1}{1.04}}$$

$$S_{25} = \text{€}487,409$$

High Partial Credit (15 Marks)

- Identifies a and r correctly
- Some correct substitution into correct formula.

Mid Partial Credit (10 Marks)

- Identifies a correctly
- Identifies r correctly
- Writes S_n formula

Low Partial Credit (5 Marks)

- Writes 4% as fraction or decimal
- Writes correct formula
- States geometric series

(b) Mary decides to invest monthly to provide the pension an in part (a).

(i) **Scale 5C** **(0, 2, 5)**

In how many months will Mary retire?

$$40 \times 12 = 480 \text{ months}$$

Partial Credit (2 Marks)

- Uses 12 or 25 in any multiplication

(ii) **Scale 10C*** **(0, 4, 8, 10)**

Calculate the rate of interest that compounded monthly would be equivalent to an effective annual rate of 4%, correct to 4 decimal places.

$$F = P(1+i)^t$$
$$1.04 = 1(1+i)^{12}$$
$$\sqrt[12]{1.04} - 1 = i$$
$$i = 0.0033\%$$

High Partial Credit (8 Marks)

- Fully correct substitution but error in solving
- Incorrect substitution but solves correctly

Low Partial Credit (4 Marks)

- Writes 4% as correct fraction or decimal
- Any correct substitution into formula

If Mary makes equal payments of € P over the next 480 months, what value of P , correct to the nearest euro, will give Mary the retirement fund she requires?

$$P(1.0033)^1 + P(1.0033)^2 + P(1.0033)^3 + \dots + P(1.0033)^{480} = \text{€}487,409$$

$$a = 1.0033 \quad r = 1.0033$$

$$S_{480} = \frac{1.0033(1 - (1.0033)^{480})}{1 - 1.0033}$$

$$P\left(\frac{1.0033(1.0033^{480} - 1)}{1.0033 - 1}\right) = \text{€}487,409$$

$$P(1174.079682) = \text{€}487,409$$

$$P = \text{€}415$$

High Partial Credit (11 Marks)

- Identifies a and r correctly
- Some correct substitution into correct formula

Mid Partial Credit (7 Marks)

- Identifies a correctly
- Identifies r correctly
- Writes S_n formula

Low Partial Credit (5 Marks)

- Writes 4% as fraction or decimal
- Writes correct formula
- States geometric series

QUESTION 8

An opened top box of height x cm is to be manufactured from a sheet of cardboard measuring 12 cm by 18 cm as shown.

(a) **Scale 10C** **(0, 4, 8, 10)**

Show that the volume of the box can be written as $V = 4x^3 - 60x^2 + 216x$.

$$V = (x)(18 - 2x)(12 - 2x)$$
$$V = 216x - 60x^2 + 4x^3$$

High Partial Credit (8 Marks)

- One error in multiplication of brackets

Low Partial Credit (4 Marks)

- Any correct multiplication

(b) **Scale 10C** **(0, 4, 8, 10)**

Find the value of x , correct to two decimal places, which will give the maximum volume for the box.

$$\frac{dv}{dx} = 216 - 120x + 12x^2$$

$$x = \frac{-(-120) \pm \sqrt{(-120)^2 - 4(12)(216)}}{2(12)}$$

$$x \neq 7.65 \text{ cm} \quad x = 2.35 \text{ cm}$$

$$\frac{d^2v}{dx^2} = -120 + 24x$$

$$-120 + 24(2.35) = -63.6 \quad \therefore \text{Max}$$

*Accept answer based on earlier work

High Partial Credit (8 Marks)

- Fails to cancel an inappropriate answer
- One error in derivative of formula
- Fails to check second derivative

Low Partial Credit (4 Marks)

- Any correct derivative

(c)

Scale 10C*

(0, 4, 8, 10)

If the company decides to choose a value of x such that $x \in \mathcal{N}$, write down the range of values of x that could be chosen.

$\{1, 2, 3, 4, 5\}$ cm

Can't be 6 cm or larger as $2(6) = 12$ cm so width/length of box would be zero ... or similar

High Partial Credit (8 Marks)

- Two or more correct figures
- Correct statement with only some correct figures
- Incorrect statement with all correct figures

Low Partial Credit (4 Marks)

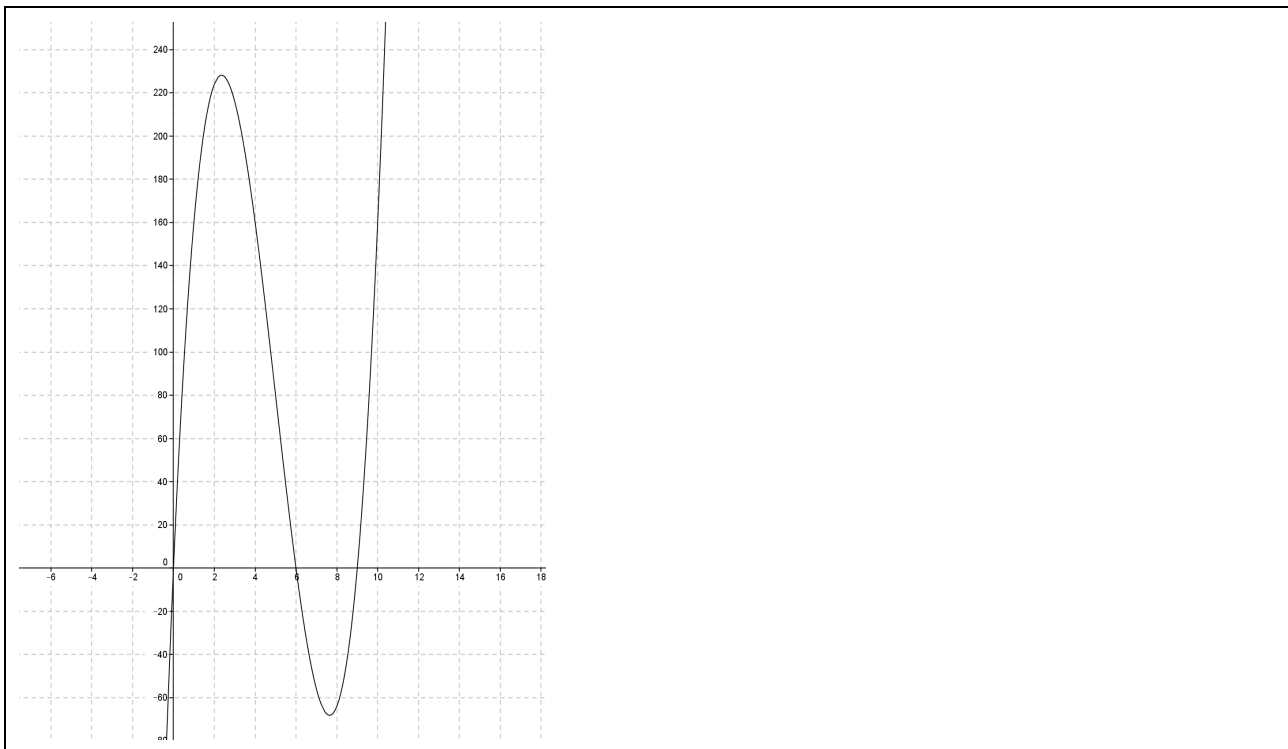
- Any correct figure
- Correct statement

(d)

Scale 10C

(0, 4, 8, 10)

Using the information in the previous parts, draw a sketch of the volume function of the box. Indicate clearly the turning points and the roots of the curve.



High Partial Credit (8 Marks)

- Maximum/minimum incorrect
- Y intercept incorrect
- Graph not consistent with earlier work

Low Partial Credit (4 Marks)

- Any correct point

QUESTION 9

Carbon-14 is a radio-isotope of the element Carbon that is used in carbon dating. Its decay can be modelled by the function

$$F = Be^{kt}$$

where F is the final mass remaining, in kg, at any time t , in years, and B is the initial amount, in grams, of Carbon-14 and k is the decay constant. The half-life (the time it takes for half of the mass to decay) of Carbon-14 is 5730 years.

(a) **Scale 10C** **(0, 4, 8, 10)**

If the original mass of Carbon-14 is 1kg, show that $k = \frac{\ln(0.5)}{5730}$.

**Note student can take initial amount as any figure but $F = 0.5B$

$$0.5 = 1e^{k(5730)}$$

$$\ln(0.5) = 5730k$$

$$k = \frac{\ln(0.5)}{5730}$$

High Partial Credit (8 Marks)

- Fully correct equation with 1 error in solving
- Correct F and B value chosen with one error

Low Partial Credit (4 Marks)

- Substitutes 5730 into equation and stops
- Incorrect F value chosen but continues to end to find B with one error

A research scientist is presented with a document which contains the writings of a soldier from the Trojan War circa 1250 BC (around 3250 years ago). After testing it is found that the parchment contained 1×10^{-12} g of Carbon-14.

(b) **Scale 15C*** **(0, 5, 10, 15)**

Calculate the original amount of Carbon-14 in the parchment.

$$1 \times 10^{-12} = B e^{\frac{\ln(0.5)}{5730}(3250)}$$

$$B = 1.48 \times 10^{-12} \text{ g}$$

**Allow candidates work from (a) without penalty

High Partial Credit (10 Marks)

- Uses 1×10^{-12} as B in an otherwise fully correct solution
- Correct substitution with error in solving

Low Partial Credit (5 Marks)

- Any correct substitution
- Identifies t as 3250 and stops

(c) **Scale 15C** **(0, 5, 10, 15)**

It is known that parchments from this area usually originally contain in the region of 1.3×10^{-12} g of Carbon-14. Taking the final amount as 1×10^{-12} g calculate the age of the document and explain if you believe it to be genuine or a fake.

$$1 \times 10^{-12} = 1.3 \times 10^{-12} e^{\frac{\ln(0.5)}{5730}(t)}$$

$$\frac{5730 \ln\left(\frac{1 \times 10^{-12}}{1.3 \times 10^{-12}}\right)}{\ln(0.5)} = t$$

$$t = 2169 \text{ years}$$

∴ Not original, but is a fake

**Allow candidates work from (a) and (b) without penalty

High Partial Credit (10 Marks)

- Incorrect substitution but solves correctly
- Correct substitution with error in solving
- Fully correct but incorrect or no conclusion

Low Partial Credit (5 Marks)

- Any correct substitution

PAPER 2

QUESTION 1

- (a) In a biology experiment, a group of sixth year students planted 50 cress seeds. 37 of the seeds germinated.

(i) **Scale 5B*** **(0, 2, 5)**

Calculate the probability that 6 out of 10 seeds germinate, correct to four decimal places.

$$\binom{10}{6} \left(\frac{37}{50}\right)^6 \left(\frac{13}{50}\right)^4 = 0.1576$$

*Accept decimal or percentage

Partial Credit (2 Marks)

- Any correct probability written

(ii) **Scale 10C** **(0, 4, 8, 10)**

Calculate the probability that the eighth seed will be the fifth seed to germinate.

$$\binom{7}{4} \left(\frac{37}{50}\right)^4 \left(\frac{13}{50}\right)^3 \left(\frac{37}{50}\right)$$

0.1365

**Accept answer based on part (i)

High Partial Credit (8 Marks)

- Binomial set up correctly with one error
- One error in setting up of binomial but calculated correctly

Low Partial Credit (4 Marks)

- Any correct probability

A bag contains 3 red marbles, 4 blue marbles and x green marbles. Given that the probability of choosing 2 green marbles is $\frac{5}{26}$ calculate the number of marbles in the bag.

$$\frac{\binom{x}{2}}{\binom{x+7}{2}} = \frac{\frac{(x)(x-1)}{2!}}{\frac{(x+7)(x+6)}{2!}}$$

$$\frac{(x)(x-1)}{(x+7)(x+6)} = \frac{5}{26}$$

$$26(x)(x-1) = 5(x+7)(x+6)$$

$$3x^3 - 13x - 30 = 0$$

$$(3x+5)(x-6) = 0$$

$$x = 6$$

$$\text{Total} = 13$$

**Accept fraction without combinations

High Partial Credit (8 Marks)

- Fully correct trial and error
- Finds $x = 6$ and stops
- Sets up quadratic
- One error in fractions but continues to end

Mid Partial Credit (5 Marks)

- Sets up one combination correctly
- Sets up fractions correctly

Low Partial Credit (2 Marks)

- Any correct step
- Attempts trial and error

QUESTION 2

- (a) The weights of the players, W , at a football match are normally distributed with a mean of 79 kg and a standard deviation of 3 kg.

(i) Scale 5B (0, 2, 5)

Find $P(W \leq 80.1 \text{ kg})$.

$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{80.1 - 79}{3} = 0.366 \approx 0.37$$
$$= 64.43\%$$

Partial Credit (2 Marks)

- Any correct step

(ii) Scale 10D (0, 2, 5, 8, 10)

Find $P(75.5 \text{ kg} \leq W \leq 81 \text{ kg})$.

$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{75.5 - 79}{3} = -1.17$$
$$= 87.9\%$$
$$= 1 - 87.9$$
$$12.1\%$$
$$z = \frac{x - \mu}{\sigma}$$
$$z = \frac{81 - 79}{3} = 0.67$$
$$= 74.86\%$$
$$\therefore 74.86 - 12.1 = 62.76\%$$

High Partial Credit (8 Marks)

- Both probabilities correct
- One probability incorrect but finished correctly
- Both probabilities worked out with consistent error and finished correctly to end

Mid Partial Credit (5 Marks)

- Works out 1 probability correctly
- Works out both probabilities with one error or a consistent error in both

Low Partial Credit (2 Marks)

- Any correct step e.g. writes down formula

The attendance at a GAA match depends on the weather. The probability of a large crowd attending is 0.9. The probability of large crowd attending if it is raining is 0.3. The probability of it raining on match day is 0.2. Calculate the probability of a large crowd attending the match given it is raining.

Let R be the event of rain and L be the event of a large crowd

$$P(L \setminus R) = 0.3P(R) = 0.2$$

$$P(L \setminus R) = \frac{P(R \cap L)}{P(R)}$$

$$P(R \cap L) = (0.3)(0.2) = 0.06$$

High Partial Credit (8 Marks)

- Correct substitution into formula, but one error in solving

Low Partial Credit (4 Marks)

- Writes probability of no rain or small crowd
- Writes formula and stops

QUESTION 3

The line L passes through the point $(6, 4)$ and has a slope of m where $m > 0$.

(a) **Scale 5C** **(0, 2, 4, 5)**

Write down the equation of the line L , in terms of m , in the form $ax + by + c = 0$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4 &= m(x - 6) \qquad mx - y + (4 - 6m) = 0 \end{aligned}$$

High Partial Credit (4 Marks)

- Any correct substitution into formula

Low Partial Credit (2 Marks)

- Writes formula $y - y_1 = m(x - x_1)$ and stops

(b) Find, in terms of m , the co-ordinates of the points where L cuts:

(i) **Scale 5C** **(0, 2, 4, 5)**

The x -axis.

$$\begin{aligned} mx - 0 + (4 - 6m) &= 0 \\ x &= \frac{6m - 4}{m} \quad \therefore \left(\frac{6m - 4}{m}, 0 \right) \end{aligned}$$

High Partial Credit (4 Marks)

- Point not given

Low Partial Credit (2 Marks)

- Write x or y equal to zero
- Attempts to solve for x

(ii) **Scale 5C** **(0, 2, 4, 5)**

The y -axis.

$$\begin{aligned} 0 - y + (4 - 6m) &= 0 \\ y &= 4 - 6m \quad \therefore (0, 4 - 6m) \end{aligned}$$

High Partial Credit (4 Marks)

- Point not given

Low Partial Credit (2 Marks)

- Write x or y equal to zero
- Attempts to solve for x

The line L forms a triangle of area 6 square units with the axes. Find the value of m .

$$\text{Area} = \frac{1}{2}bh$$

$$\frac{1}{2}\left(\frac{6m-4}{m}\right)(6m-4) = 49$$

$$18m^2 + 25m + 8 = 0$$

$$(9m+8)(2m+1) = 0$$

$$m = -\frac{8}{9} \quad m = -\frac{1}{2}$$

**Allow work based on candidate's earlier work

High Partial Credit (8 Marks)

- Slopes found, but invalid slope not cancelled off

Mid Partial Credit (5 Marks)

- Sets up equation correctly
- Any correct substitution into formula

Low Partial Credit (2 Marks)

- Correct formula

QUESTION 4

(a) Scale 10C (0, 4, 8, 10)

Find the centre and radius of the circle $C_1 : x^2 + y^2 - 4x + 6y - 21 = 0$.

$$(x-2)^2 + (y+3)^2 = 34$$

Centre $(2, -3)$, Radius $= \sqrt{34}$

High Partial Credit (8 Marks)

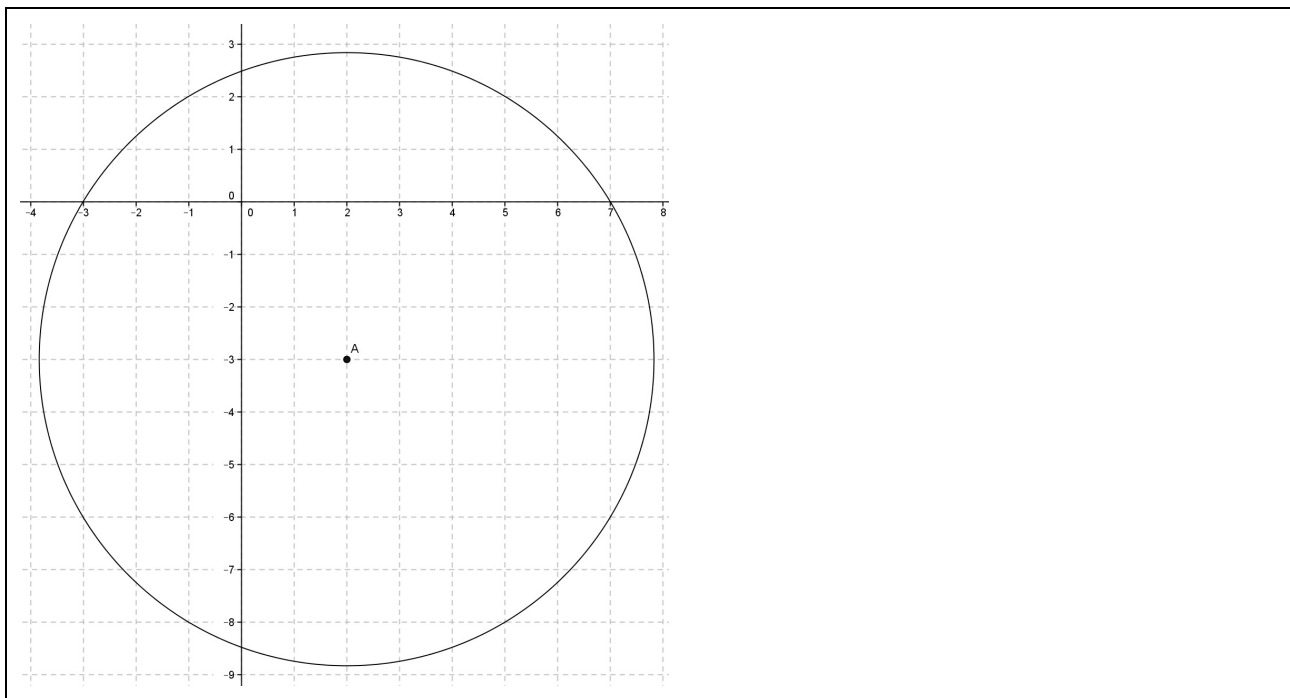
- Finds centre correctly
- Finds radius correctly

Low Partial Credit (4 Marks)

- Any correct step in rearranging formula or completing squares

(b) Scale 5B (0, 2, 5)

Construct the circle C_1 .



**Allow centre and radius from (a)

Partial Credit (2 Marks)

- Any correct point such as centre plotted
- Any circle constructed

A second circle C_2 , has radius length half of C_1 and touches C_1 internally at the point $(5, 2)$. Find the equation of the circle C_2 .

$$\text{Mid} = \left(\frac{5+2}{2}, \frac{2-3}{2} \right) = (3.5, -0.5)$$

$$\text{Radius} = \sqrt{(5-3.5)^2 + (2+0.5)^2} = \sqrt{\frac{17}{2}}$$

$$C_2 : (x-3.5)^2 + (y+0.5)^2 = 8.5$$

High Partial Credit (8 Marks)

- Finds centre correctly
- Finds radius correctly
- Plots C_2 on part (b) and continues to end correctly

Low Partial Credit (4 Marks)

- Attempts to find midpoint
- Attempts to find distance
- Plots C_2 on part (b) and stops

QUESTION 5

(a) In the triangle XYZ , $|\angle XYZ| = 90^\circ$ and $|YZ| = p$.

(i) Scale 5B (0, 2, 5)

Show that $|WY| = p \tan(45^\circ - \alpha)$.

$$\tan(45^\circ - \alpha) \quad |WY| = p \tan(45^\circ - \alpha)$$

Partial Credit (2 Marks)

- Writes SOHCAHTOA or part of

(ii) Scale 10C (0, 2, 5, 8, 10)

Hence, or otherwise, show that $|XW| = 2p \tan 2\alpha$.

$$|XY| = p \tan(45^\circ + \alpha)$$

$$|XY| = |XW| + |WY|$$

$$|XW| = p \tan(45^\circ + \alpha) - p \tan(45^\circ - \alpha)$$

$$|XW| = p \left(\frac{\tan 45^\circ + \tan \alpha}{1 - \tan 45^\circ \tan \alpha} - \frac{\tan 45^\circ - \tan \alpha}{1 + \tan 45^\circ \tan \alpha} \right)$$

Let $\tan \alpha = t$

$$|XW| = p \left(\frac{1+t}{1-t} - \frac{1-t}{1+t} \right)$$

$$|XW| = p \left(\frac{4t}{1-t^2} \right)$$

$$|XW| = 2p \left(\frac{2t}{1-t^2} \right)$$

$$|XW| = 2p(\tan 2\alpha)$$

High Partial Credit (8 Marks)

- Answer not in correct form

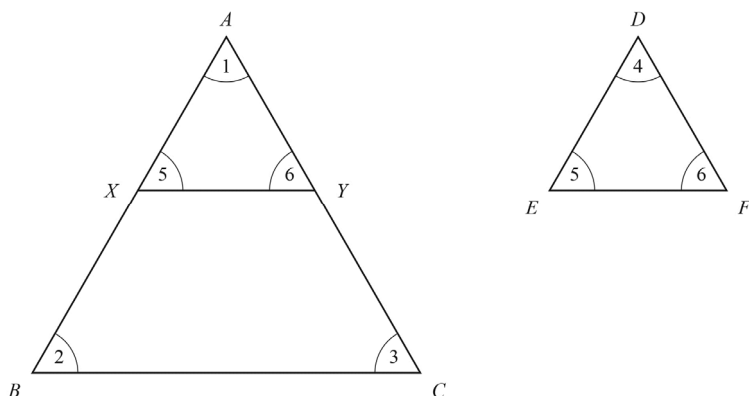
Mid Partial Credit (5 Marks)

- Writes $|XY| = p \tan(45^\circ + \alpha) - p \tan(45^\circ - \alpha)$
- Writes $|XY| = p \left(\frac{\tan 45^\circ + \tan \alpha}{1 - \tan 45^\circ \tan \alpha} - \frac{\tan 45^\circ - \tan \alpha}{1 + \tan 45^\circ \tan \alpha} \right)$

Low Partial Credit (2 Marks)

- Writes SOHCAHTOA or part of
- Writes $|XY| = |XW| + |WY|$

Prove that if two triangles ABC and DEF are similar, then their sides are proportional in order. $\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|} = \frac{|BC|}{|EF|}$



Given: The triangles ABC and DEF in which $|\angle 1| = |\angle 4|$, $|\angle 2| = |\angle 5|$ and $|\angle 3| = |\angle 6|$

To prove: $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$

Construciton: Mark the point X on $|AB|$ such that $|AX| = |DE|$
Mark the point Y on $|AC|$ such that $|AY| = |DF|$

Proof: Join XY
The triangles AXY and DEF are congruent ... SAS
 $\therefore |\angle AXY| = |\angle DEF| = |\angle 5|$... Corresponding angles

$$\therefore |\angle AXY| = |\angle ABC|$$

$$\therefore XY \parallel BC$$

$\frac{|AB|}{|AX|} = \frac{|AC|}{|AY|}$... A line parallel to one side divides the other side in the same ratio (Theorem 12)

$$\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$$

Similarly, it can be proven that $\therefore \frac{|AB|}{|DE|} = \frac{|BC|}{|EF|}$

$$\therefore \frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$$

**Four steps: Given, To Prove, Construction and Proof

High Partial Credit (8 Marks)

- 3 correct steps

Mid Partial Credit (5 Marks)

- 2 correct steps

Low Partial Credit (2 Marks)

- Any correct step

QUESTION 6

(a) Scale 10D (0, 2, 5, 8, 10)

The area of a sector of a circle of radius 9 cm is 13.5 cm^2 . Find, in radians, the measure of the acute angle in the sector.

$$A = \frac{1}{2} r^2 \theta$$

$$\frac{1}{2} (9)^2 \theta = 13.5$$

$$\theta = \frac{1}{3} \text{ rads}$$

High Partial Credit (8 Marks)

- Subs into formula correctly
- Fully correct answer in degrees of 19.1°

Mid Partial Credit (5 Marks)

- Any correct substitution in correct formula
- Fully correct substitution in non radian formula for area of a circle

Low Partial Credit (2 Marks)

- Writes formula for area of a circle and stops
- Incorrect area formula used

(b) Scale 5C (0, 2, 4, 5)

Show that $(\cos \alpha + \sin \alpha)^2 = 1 + \sin 2\alpha$.

$$(\cos \alpha + \sin \alpha)^2 = 1 + \sin 2\alpha$$

$$\cos^2 \alpha + 2 \cos \alpha \sin \alpha + \sin^2 \alpha = 1 + \sin 2\alpha$$

$$1 + 2 \cos \alpha \sin \alpha = 1 + \sin 2\alpha$$

$$1 + \sin 2\alpha = 1 + \sin 2\alpha$$

High Partial Credit (4 Marks)

- Fully correct multiplication of brackets

Low Partial Credit (2 Marks)

- Any correct multiplication

Solve the equation $\cos 3\theta = \frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 360^\circ$.

$$\cos 3\theta = \frac{1}{\sqrt{2}}$$

$$3\theta = \cos^{-1} \frac{1}{\sqrt{2}}$$

$$3\theta = 45^\circ$$

$$3\theta = 45^\circ, 405^\circ, 765^\circ$$

$$3\theta = 315^\circ, 675^\circ, 1035^\circ$$

$$\theta = 15^\circ, 135^\circ, 255^\circ$$

$$\theta = 105^\circ, 225^\circ, 345^\circ$$

High Partial Credit (8 Marks)

- Finds at least 4 correct solutions

Mid Partial Credit (5 Marks)

- Finds 45° and 315°

Low Partial Credit (2 Marks)

- Finds 45° and stops

QUESTION 7

(a) Scale 10C (0, 4, 8, 10)

Discuss any three advantages/disadvantages of the sample chosen by Red C.

- Good sample size
- Good spread of sample (countrywide)
- Not a simple random sample
- Half mobile and half landlines excludes people with no phones. More likely to get older people on landlines
- People might have lied on phone and not been over 18

High Partial Credit (8 Marks)

- Two correct points

Low Partial Credit (4 Marks)

- One correct point

(b) Scale 5B (0, 2, 5)

How many likely voters from the sample will support Fianna Fáil?

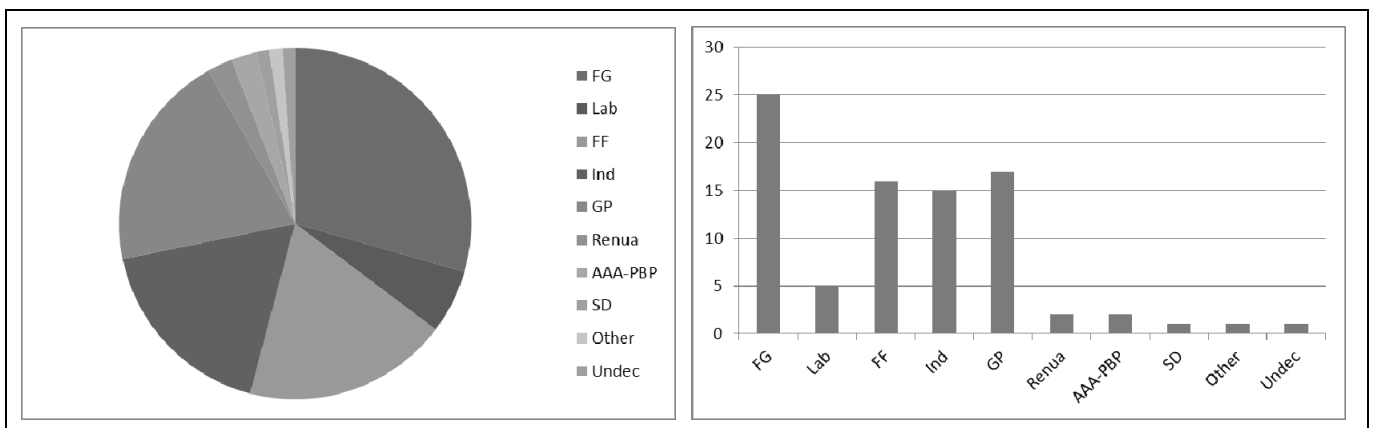
$1004 \times 17\% = 170$ or 171 voters

Partial Credit (2 Marks)

- Any correct step

(c) Scale 10C (0, 4, 8, 10)

Display the number of likely voters on a suitable graphical display.



High Partial Credit (8 Marks)

- Incorrect suitable display

Low Partial Credit (4 Marks)

- Unsuitable display e.g. line plot

- (d) In a poll carried out by a rival marketing and research company, a sample of x voters was surveyed. 60% of the sample claimed that they voted in the last general election. A 95% confidence interval for the proportion of voters who said they voted was:

$$0.55706 \leq p \leq 0.64294$$

(i) Scale 15C (0, 5, 10, 15)

Calculate the number of voters who were surveyed.

$$\begin{aligned} \hat{p} + E &= 0.64294 \\ 0.6 + 1.96 \sqrt{\frac{(0.6)(0.4)}{n}} &= 0.64294 \\ \frac{0.24}{n} &= \left(\frac{0.6429 - 0.6}{1.96} \right)^2 \\ n &= 500 \text{ voters} \end{aligned}$$

**Accept 501 voters

High Partial Credit (10 Marks)

- 1.96 not used or incorrect figure used but continues correctly to end
- Fully correct substitution with errors in solving

Low Partial Credit (5 Marks)

- Any correct substitution into formula
- Writes $\hat{p} + E = 0.64294$

(ii) Scale 15C (0, 5, 10, 15)

How many voters would have to be sampled in order to cut the margin of error by 25% at the 95% confidence interval?

$$\begin{aligned} 0.04294 \times 0.75 &= 0.032205 & \frac{1}{\sqrt{500}} \times 0.75 &= 0.033541 \\ 1.96 \sqrt{\frac{(0.6)(0.4)}{n}} &= 0.032205 & \frac{1}{\sqrt{n}} &= 0.033541 \\ \frac{0.24}{n} &= \left(\frac{0.032205}{1.96} \right)^2 & \text{OR} & \frac{1}{0.033541} &= \sqrt{n} \\ n &= 888 \text{ voters} & & n &= 888.88 \text{ or } 889 \end{aligned}$$

**Accept 889 for full marks

High Partial Credit (10 Marks)

- 1.96 not used or incorrect figure used, but continues correctly to end
- Fully correct substitution with errors in solving

Low Partial Credit (5 Marks)

- Any correct substitution into formula
- Calculates 75% or 25%

QUESTION 8

A veterinary team are performing an operation on a large mammal in a zoo. They note that under anaesthetic the animal's temperature varies sinusoidally over time. A maximum temperature of 48°C is reached 15 minutes after they begin the procedure. The minimum temperature of 24°C occurs 30 minutes later during the procedure.

They wish to be able to predict the variance in the temperature of the animal by using a suitable graph.

(a) Using the information above:

(i) **Scale 5C** **(0, 2, 4, 5)**

Write down the temperature of the animal at the start of the procedure.

$$(48 - 24) \div 2 = 12$$
$$24 + 12 = 36^{\circ}\text{C}$$

High Partial Credit (4 Marks)

- Finds 12

Low Partial Credit (2 Marks)

- Writes 48 or 24

(ii) **Scale 5A** **(0, 5)**

Hence, or, otherwise, write down the equation of the midway line for the sinusoidal curve.

$$y = 36$$

**Accept answer based on candidate's part (i)

(iii) **Scale 5A** **(0, 5)**

Write down the maximum height of the graph above the midway line.

$$12^{\circ}\text{C}$$

**Accept answer based on earlier work

(iv) **Scale 5B** **(0, 2, 5)**

Write down the period of the graph.

1 hour or 60 min

Low Partial Credit (2 Marks)

- Some work with 15 minutes or 30 minutes

(b) The graph of the animal's temperature over time can be represented as the function:

$$f(t) = a + b \sin(ct)$$

(i) **Scale 10C** **(0, 4, 8, 10)**

Using the information in part (a) Write down the function $f(t)$.

$$f(t) = 36 + 12 \sin\left(\frac{\pi}{30}t\right)$$

**Accept answer based on candidate's earlier work

High Partial Credit (8 Marks)

- Two from a , b , c correct

Low Partial Credit (4 Marks)

- a , b or c correct

(ii) **Scale 15C** **(0, 5, 10, 15)**

Draw the function $f(t)$ over the period of the two hour operation.



High Partial Credit (10 Marks)

- 3 of 5 critical points on curve drawn

Low Partial Credit (5 Marks)

- Any point plotted correctly
- $y = 105$ drawn or marked

(iii) **Scale 5B** **(0, 2, 5)**

What is the animal's temperature after 26 minutes?

$$36 + 12 \sin\left(\frac{\pi}{30}(26)\right) = 40.88^\circ\text{C} \pm 1^\circ\text{C}$$

Partial Credit (2 Marks)

- Any correct work on graph towards finding temperature

QUESTION 9

Three cogs in a machine are designed as shown. The centres of the cogs are connected with metal bars. A belt runs on the outside of the three cogs. The larger cogs both have radius $3\sqrt{2}$ cm.

(a) **Scale 10C*** **(0, 4, 8, 10)**

Find, in surd form, the radius of the small cog.

$$\begin{aligned}(6\sqrt{2})^2 &= (x + 3\sqrt{2})^2 + (x + 3\sqrt{2})^2 \\ x^2 + 6\sqrt{2}x - 18 &= 0 \\ x &= (6 - 3\sqrt{2}) \text{ cm}\end{aligned}$$

High Partial Credit (8 Marks)

- Correct equation set up
- Incorrect equation but solved to find radius of small cog

Low Partial Credit (4 Marks)

- Any correct work on diagram
- Adds two large radii

(b) **Scale 10C*** **(0, 4, 8, 10)**

$$\begin{aligned}(6)^2 &= (6\sqrt{2} - 6)^2 + x^2 \\ 36 &= 72 - 72\sqrt{2} + 36 + x^2 \\ x &= 5.46 \text{ cm}\end{aligned}$$

High Partial Credit (8 Marks)

- Correct equation set up
- Two correct sides of triangle indicated on diagram

Low Partial Credit (4 Marks)

- Any correct work on diagram
- Any correct step in finding total length of any side of triangle

(c)

Scale 15C*

(0, 5, 10, 15)

Calculate the length of the belt that runs over the cogs, correct to two decimal places.

$$\text{Length} = 4\pi(3\sqrt{2})\left(\frac{157}{360}\right) + 2\pi(6 - 3\sqrt{2})\left(\frac{46}{360}\right) + 2(5.46) + 2(3\sqrt{2})$$

$$\text{Length} = 44.07 \text{ cm}$$

High Partial Credit (10 Marks)

- Two sectors found correctly
- One sector and $2(5.46)$ calculated

Low Partial Credit (5 Marks)

- Any one sector length found correctly
- Attempt at finding sector length
- Converts to radians
- Multiplies 5.46 by 2 and stops

(d)

Scale 10C*

(0, 4, 8, 10)

The cogs are lubricated by oil, which is stored in the shaded region of the mechanism. If the container is 3 cm in depth find the volume of oil it contains.

$$\text{Area} = \frac{1}{2}(6)(6) - 2\pi(3\sqrt{2})^2\left(\frac{45}{360}\right) - \pi(6 - 3\sqrt{2})^2\left(\frac{90}{360}\right)$$

$$\text{Area} = 1.44$$

$$\text{Volume} = 1.44 \times 3 = 4.32 \text{ cm}^3$$

High Partial Credit (8 Marks)

- Area of triangle found and one sector found
- Two sectors found

Low Partial Credit (4 Marks)

- Any correct formula
- Any correct substitution into correct formula

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