



Pre-Leaving Certificate Examination, 2016
Triailscrúdú na hArdteistiméireachta, 2016

Mathematics

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	
---------------	--

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

- (a) Solve the following simultaneous equations:

$$x - y = 1$$

$$(x - 5)^2 + (y - 4)^2 = 32$$

A large grid of squares, approximately 20 columns by 25 rows, intended for students to show their working for Question 1.

- (b)** Express the following as a single fraction in its simplest form.

$$\frac{5}{x^2 - 5x - 14} \div \frac{5x + 25}{x^2 - 2x - 35}$$

Question 2**(25 marks)**

- (a) Solve the inequality $\frac{2x+1}{x-1} \leq 1$, $x \neq 1$ for $x \in \mathbb{R}$.

- (b)** Solve the following equation: $2^{2x+1} - 5(2^x) - 12 = 0$.

A large grid of squares, approximately 20 columns by 30 rows, intended for students to show their work for the problem.

Question 3**(25 marks)**

- (a) Solve the equation $z^3 - 2z^2 + 5z + 26 = 0$ given that one of the roots is an integer.

A large grid of squares, approximately 20 columns by 30 rows, intended for students to show their working for Question 3.

- (b)** Use De Moivre's Theorem to solve the equation $z^3 - 8 = 0$.

A large grid of squares, approximately 20 columns by 30 rows, intended for students to show their work for solving the given equation.

Question 4**(25 marks)**

- (a) Prove that $\sqrt{2}$ is an irrational number.

A large grid of squares, likely provided for working space or drawing, consisting of approximately 20 columns and 30 rows of small squares.

- (b) (i) Write out the general term form of the binomial expansion $\left(x^2 - \frac{1}{x}\right)^{15}$.

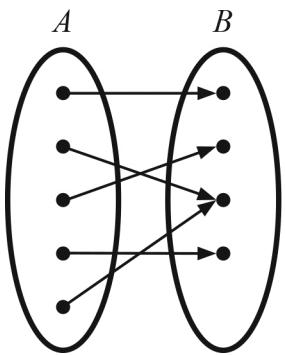
- (ii) Hence, or otherwise, find the value of the term that is independent of x in the expansion.

Question 5

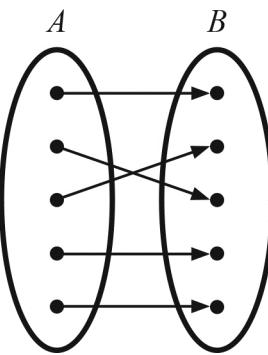
(25 marks)

- (a) Which one of the mapping diagrams below is bijective? Explain your answer fully.

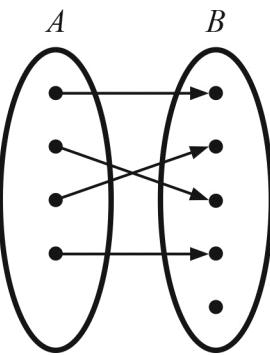
(i)



(ii)



(iii)



(b) Evaluate

(i) $\lim_{n \rightarrow 3} \frac{n^3 - 27}{n - 3}$

(ii) $\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{7x^2 + 2}$.

Question 6**(25 marks)**

- (a) Differentiate $-2x^2 + 3x - 12$ from first principles.

A large grid of squares, approximately 20 columns by 25 rows, intended for考生 to show their working for Question 6.

(b) (i) If $f(x) = e^{\cos x}$, find $f'(x)$ the derivative of $f(x)$.

A large grid of squares, approximately 20 columns by 30 rows, intended for students to show their working for the question.

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx$.

A large grid of squares, approximately 20 columns by 30 rows, intended for考生 to show their working for the question.

Answer **all three** questions from this section.

Question 7

(70 marks)

Mary is 25 years old and intends to retire at the age of 65. She begins to calculate how much she needs in her pension fund. She bases all her calculations on a 4% annual growth rate.

Mary wants to receive a payment of €30,000 at the start of each year for 25 years after her retirement.

- (a) (i) Write down the present value of a future payment of €30,000 in one year's time, correct to the nearest cent.

- (ii) Write down the present value of a future payment of €30,000 in t years' time.

- (iii) How much, correct to the nearest euro, will Mary require to have in her fund to guarantee this payment?

- (b) Mary decides to invest monthly to provide the pension in part (a).

- (i) In how many months will Mary retire?

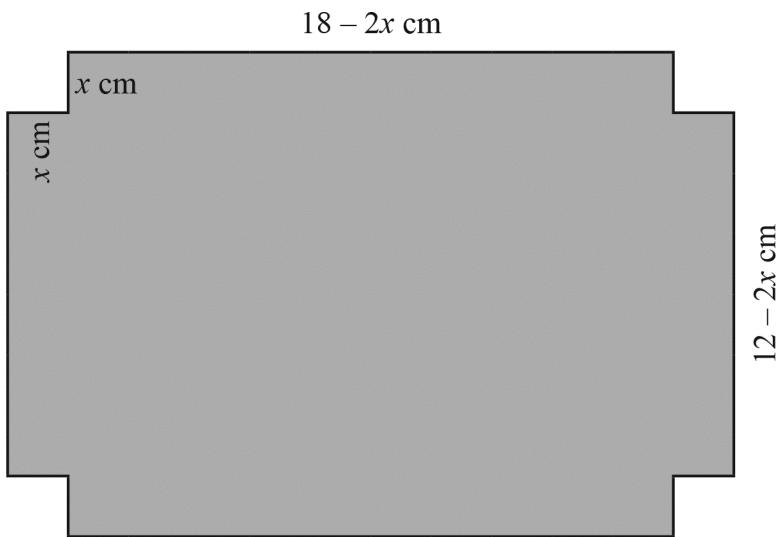
- (ii) Calculate the rate of interest that compounded monthly would be equivalent to an effective annual rate of 4%, correct to 4 decimal places.

- (iii) If Mary makes equal payments of $\text{€}P$ over the next 480 months, what value of P , correct to the nearest euro, will give Mary the retirement fund she requires?

Question 8

(40 marks)

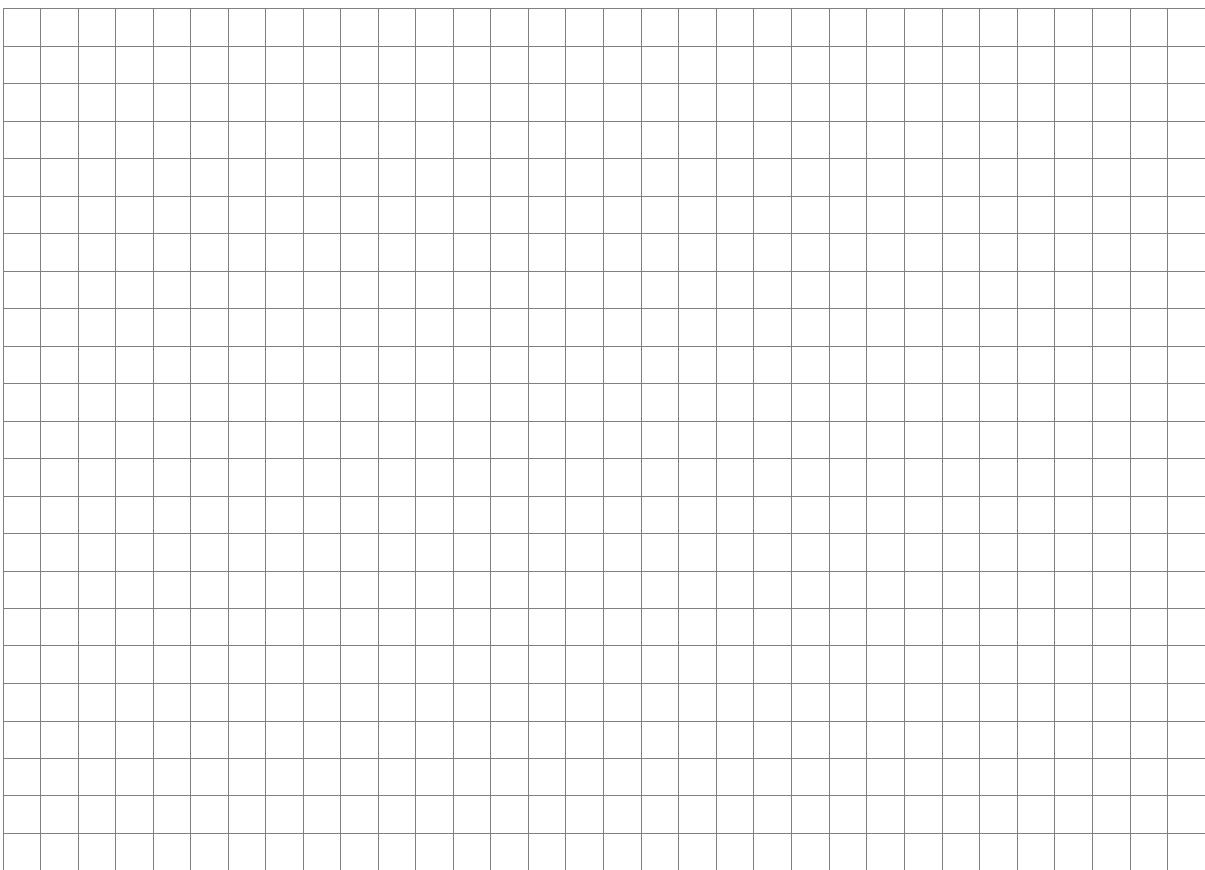
An opened top box of height x cm is to be manufactured from a sheet of cardboard measuring 12 cm by 18 cm as shown.



- (a) Show that the volume of the box can be written as $V = 4x^3 - 60x^2 + 216x$.



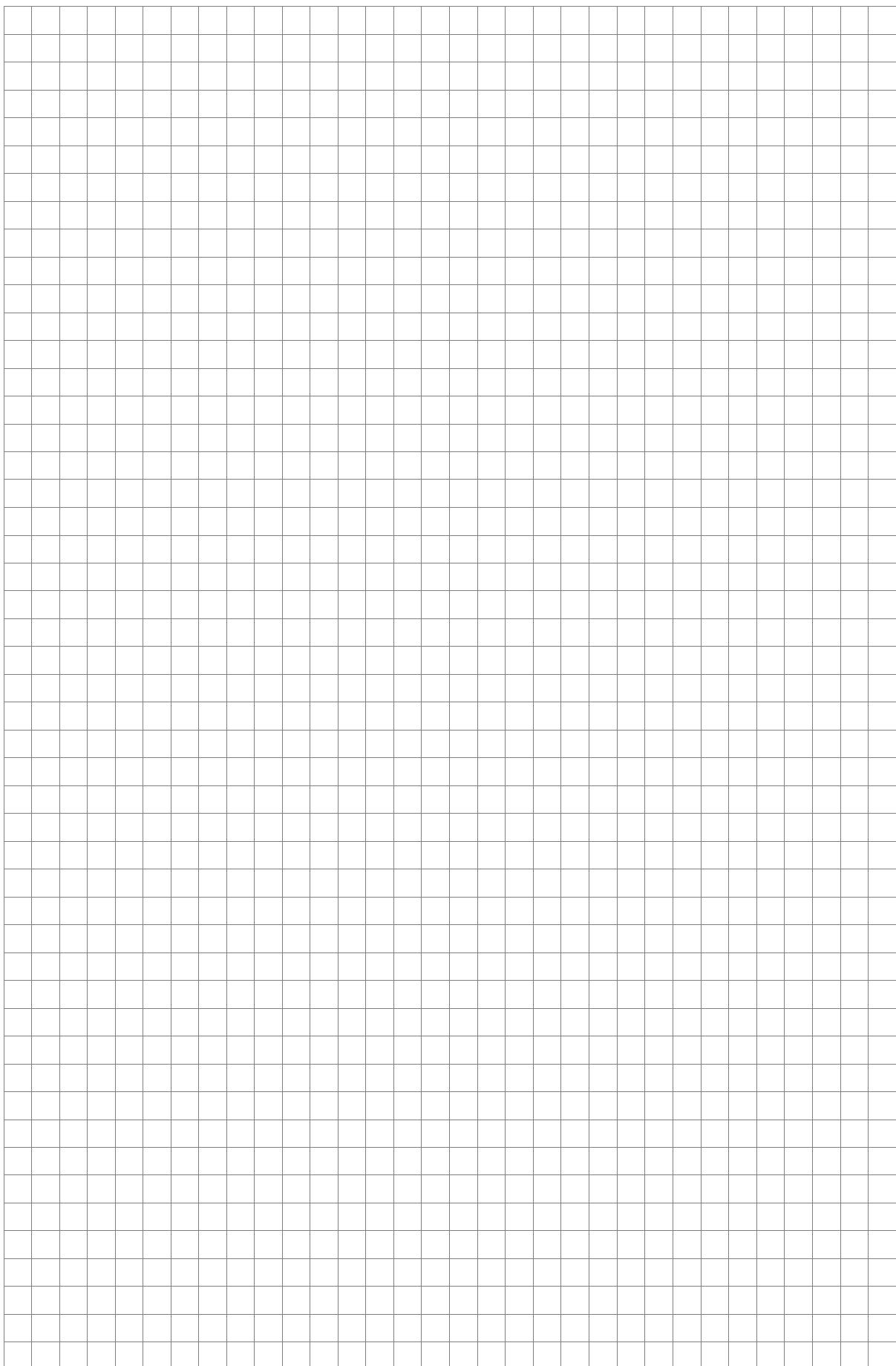
- (b)** Find the value of x , correct to two decimal places, which will give the maximum volume for the box.



- (c)** If the company decides to choose a value of x such that $x \in \mathbb{N}$, write down the range of values of x that could be chosen.



- (d) Using the information in the previous parts, draw a sketch of the volume function of the box. Indicate clearly the turning points and the roots of the curve.



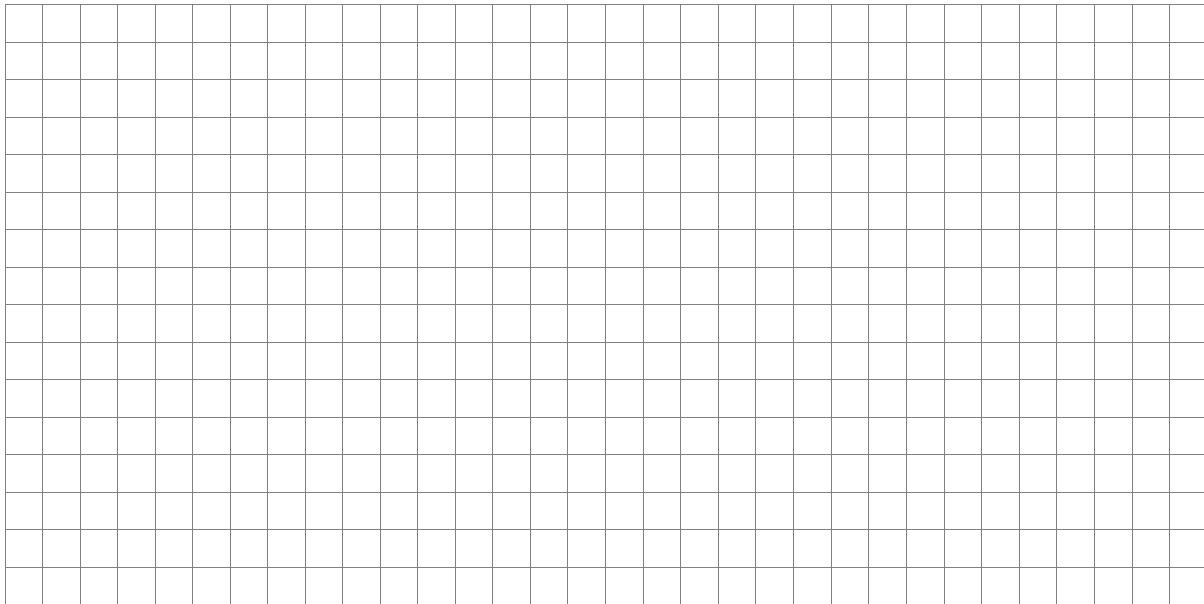
Question 9**(40 marks)**

Carbon-14 is a radio-isotope of the element Carbon that is used in carbon dating. Its decay can be modelled by the function

$$F = Be^{kt}$$

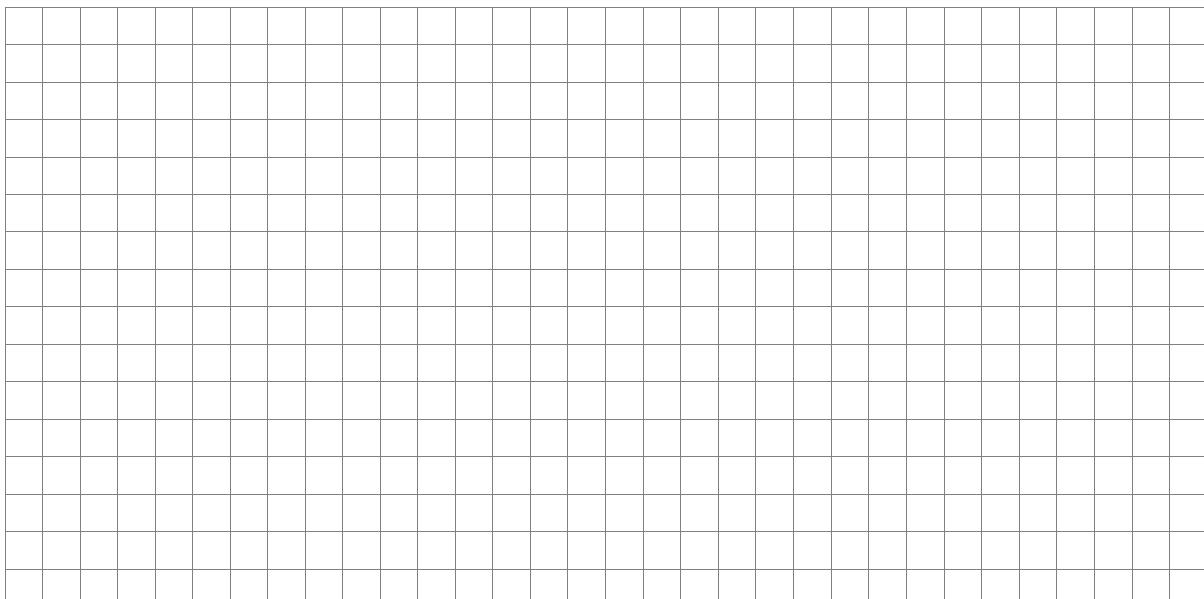
where F is the final mass remaining, in kg, at any time t , in years, and B is the initial amount, in grams, of Carbon-14 and k is the decay constant. The half-life (the time it takes for half of the mass to decay) of Carbon-14 is 5730 years.

- (a) If the original mass of Carbon-14 is 1kg, show that $k = \frac{\ln(0.5)}{5730}$.



A research scientist is presented with a document which contains the writings of a soldier from the Trojan War circa 1250 BC (around 3250 years ago). After testing it is found that the parchment contained $1 \times 10^{-12} g$ of Carbon-14.

- (b) Calculate the original amount of Carbon-14 in the parchment.



- (c) It is known that parchments from this area usually contain in the region of $1.3 \times 10^{-12} \text{ g}$ of Carbon-14. Taking the final amount as $1 \times 10^{-12} \text{ g}$ calculate the age of the document and explain if you believe it to be genuine or a fake.

A large grid of squares, approximately 20 columns by 30 rows, intended for students to show their working for the calculation required in the question.

You may use this page for extra work.

