



**Pre-Leaving Certificate Examination, 2016**  
**Triailscrúdú na hArdteistiméireachta, 2016**

# Mathematics

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	
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Grade
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## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

**You will lose marks if all necessary work is not clearly shown.**

**Answers should include the appropriate units of measurement, where relevant.**

**Answers should be given in simplest form, where relevant.**

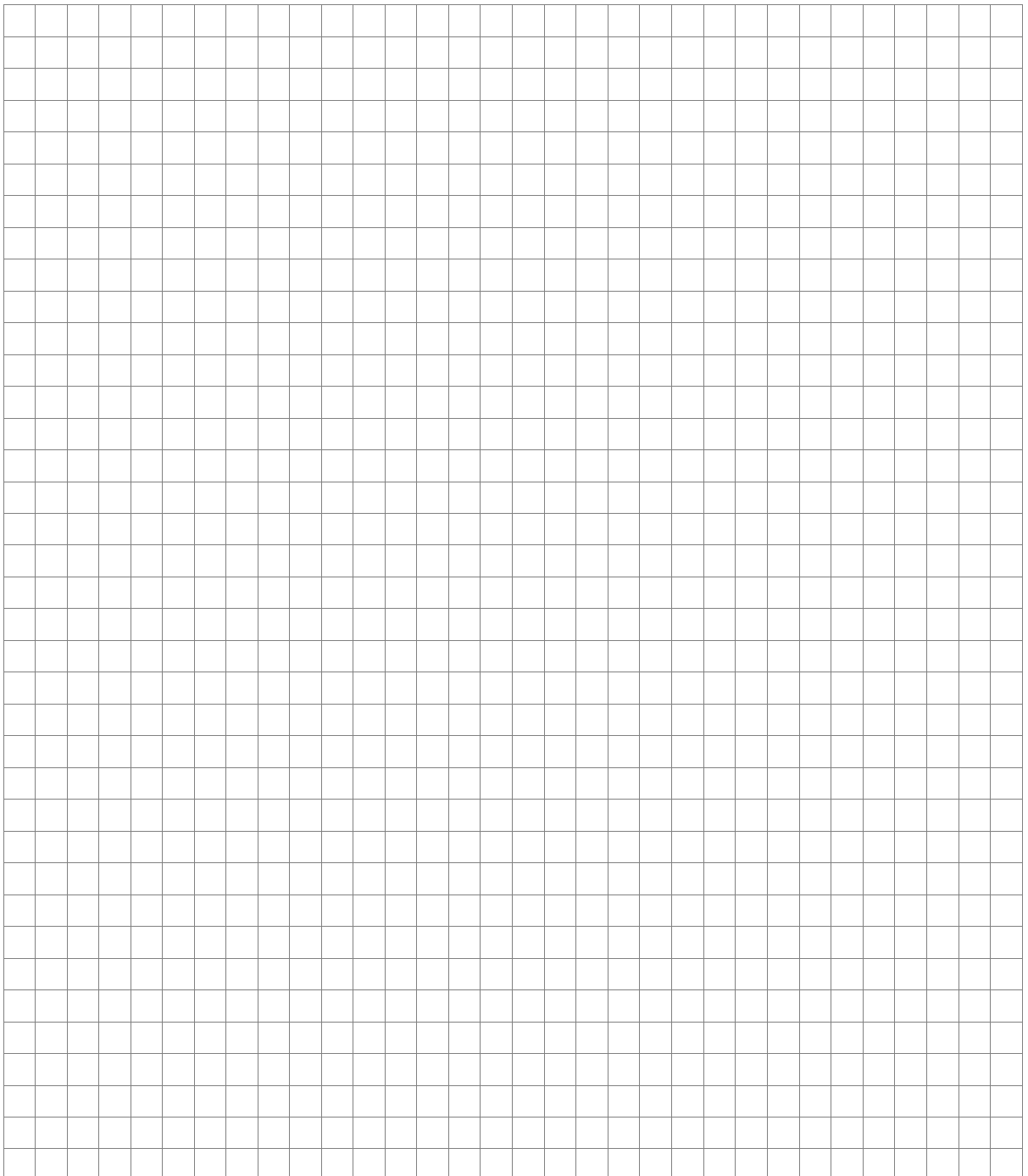
Write down the make and model of your calculator(s) here:

Answer **all six** questions from this section.

**Question 1****(25 marks)**

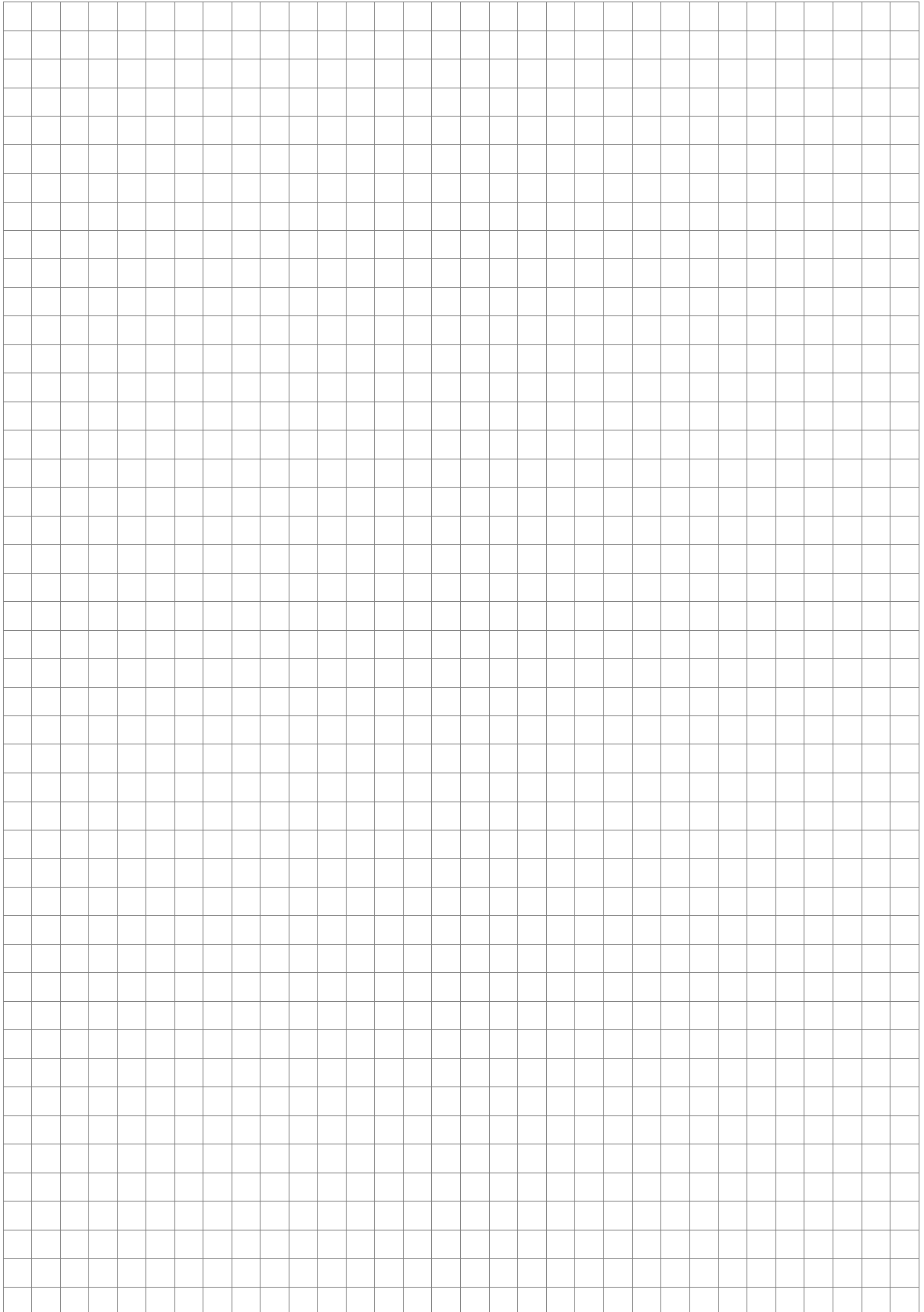
(a) Solve the following simultaneous equations:

$$\begin{aligned}x - y &= 1 \\(x - 5)^2 + (y - 4)^2 &= 32\end{aligned}$$



(b) Express the following as a single fraction in its simplest form.

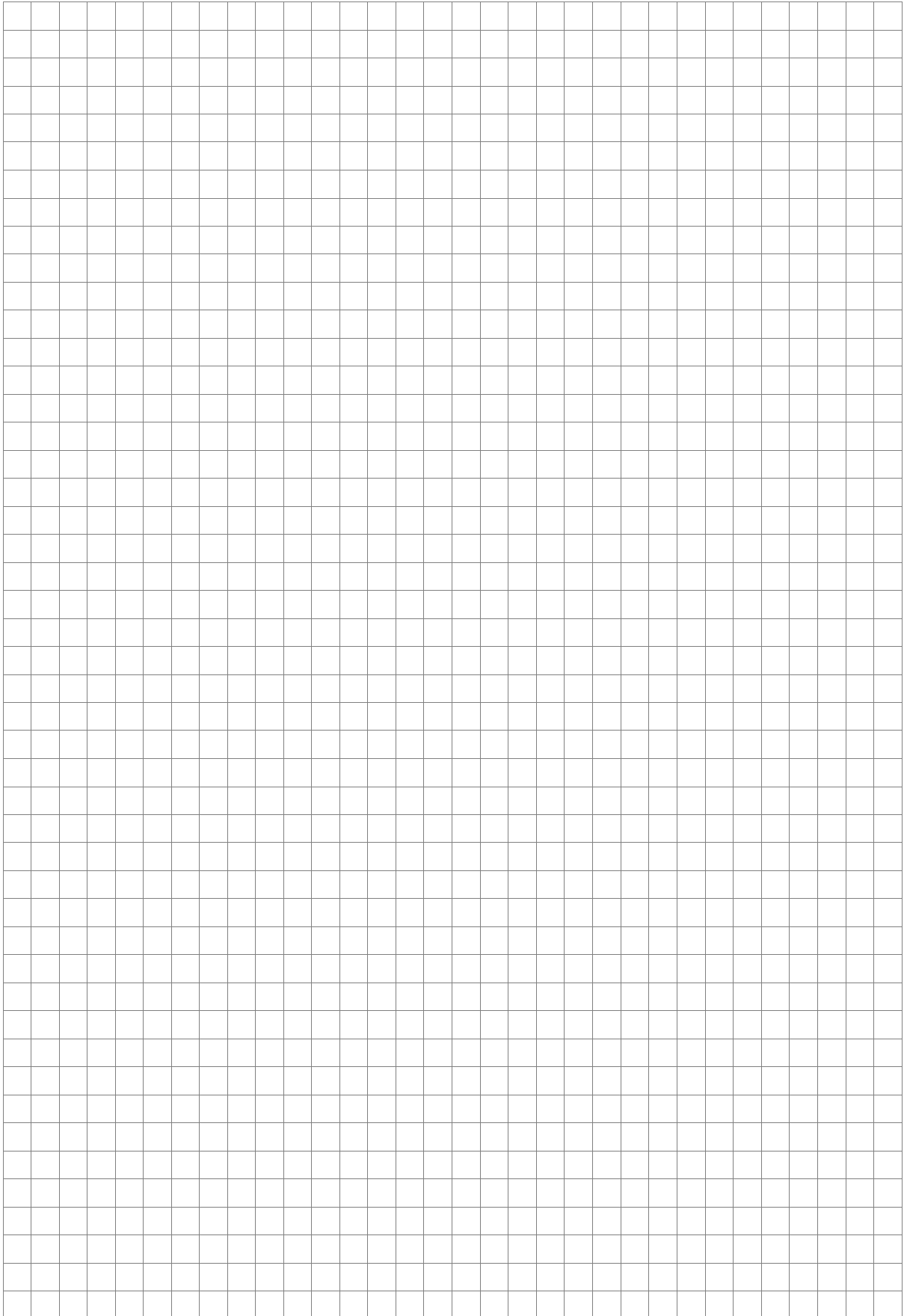
$$\frac{5}{x^2 - 5x - 14} \div \frac{5x + 25}{x^2 - 2x - 35}$$



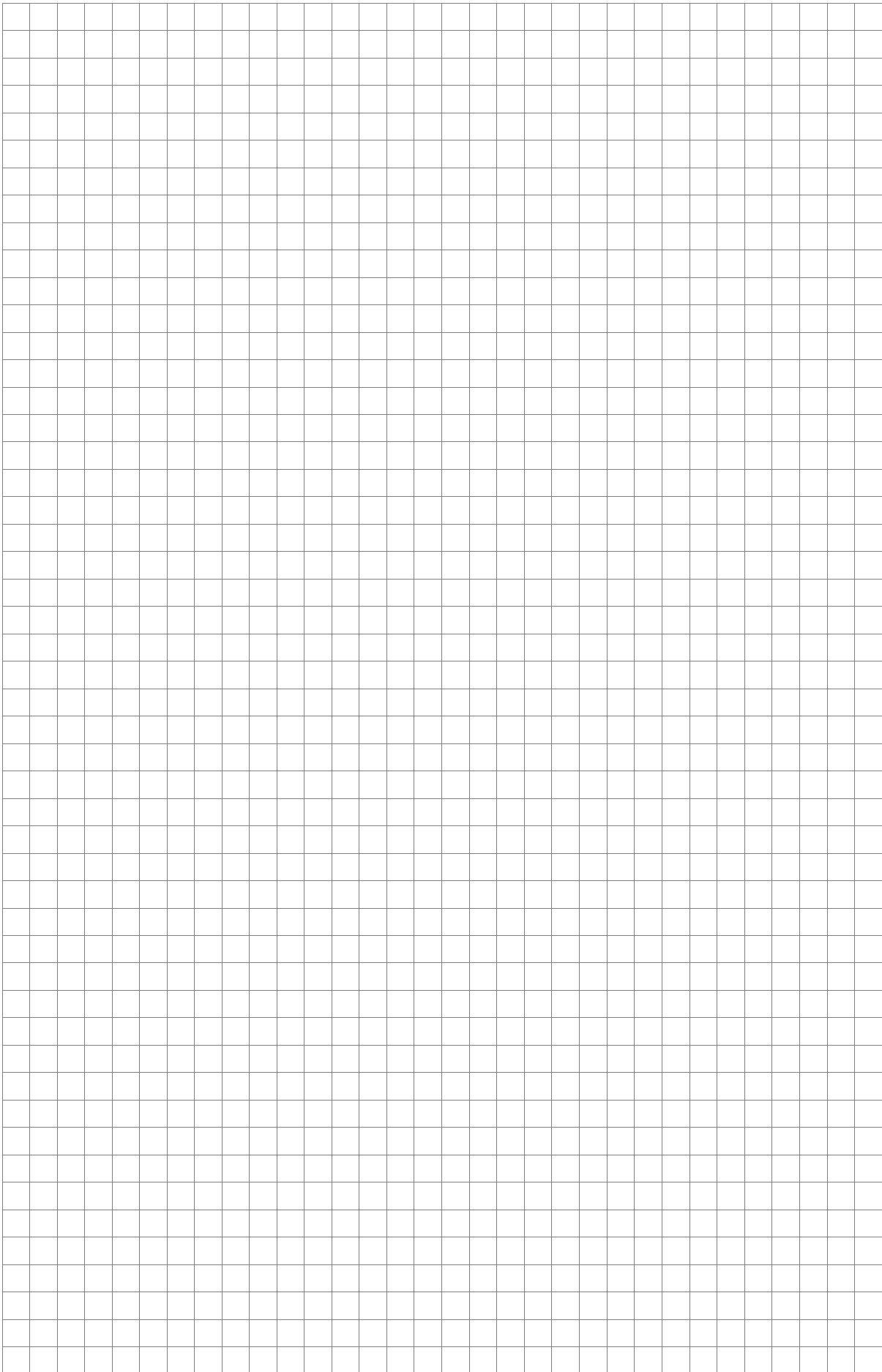
**Question 2**

**(25 marks)**

- (a) Solve the inequality  $\frac{2x+1}{x-1} \leq 1, x \neq 1$  for  $x \in \mathbb{R}$ .



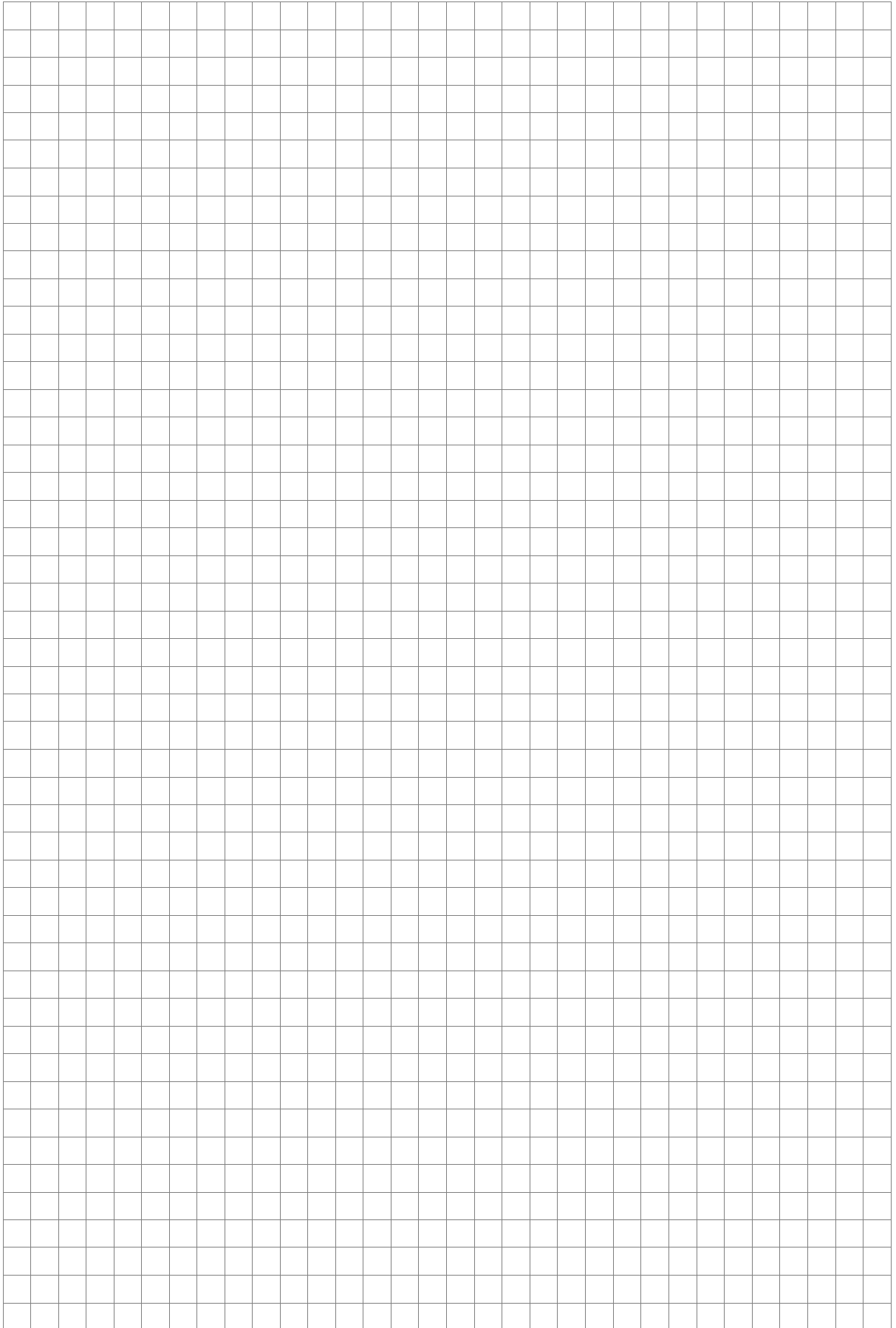
(b) Solve the following equation:  $2^{2x+1} - 5(2^x) - 12 = 0$ .



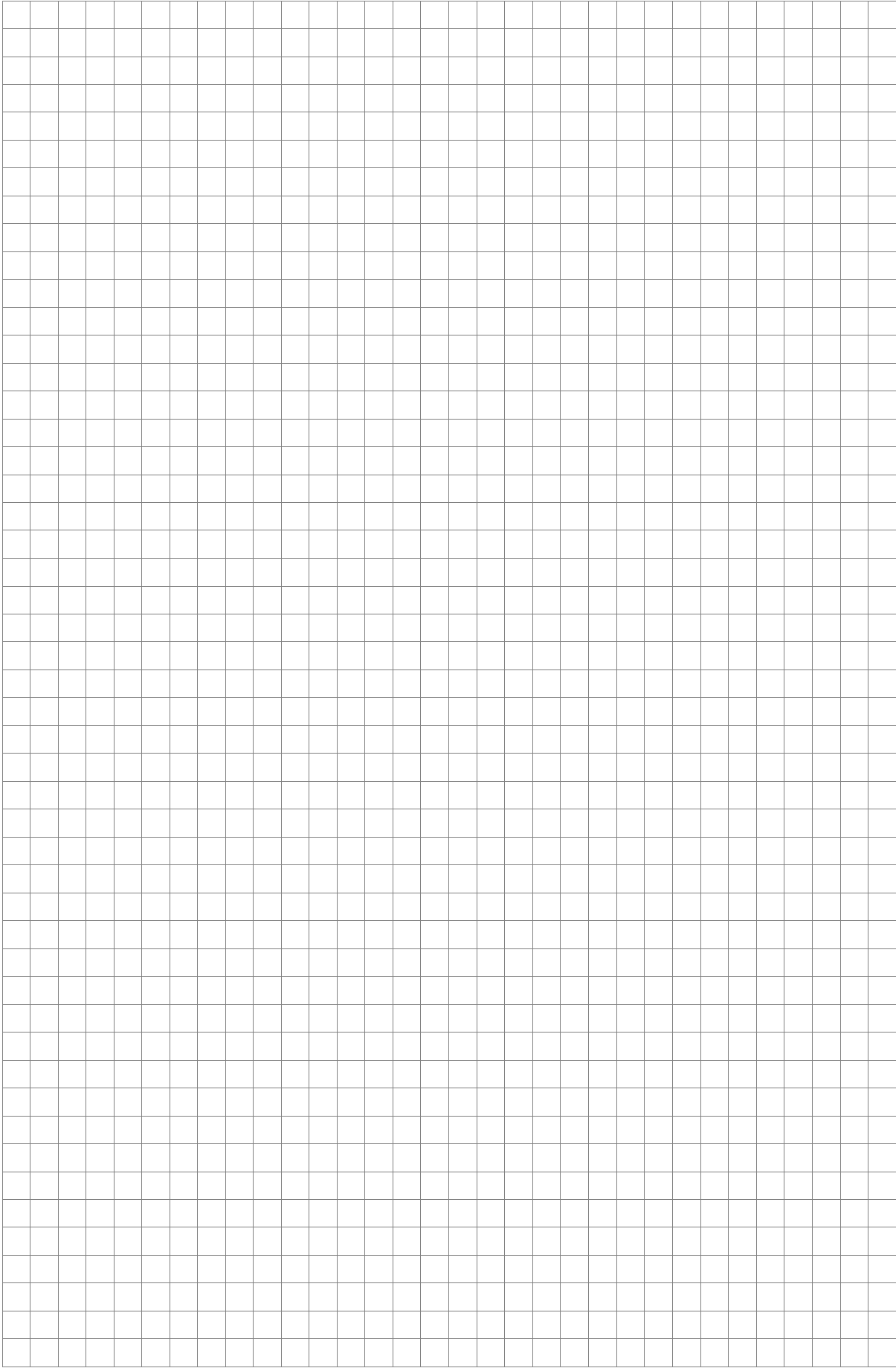
**Question 3**

**(25 marks)**

- (a) Solve the equation  $z^3 - 2z^2 + 5z + 26 = 0$  given that one of the roots is an integer.



(b) Use De Moivre's Theorem to solve the equation  $z^3 - 8 = 0$ .

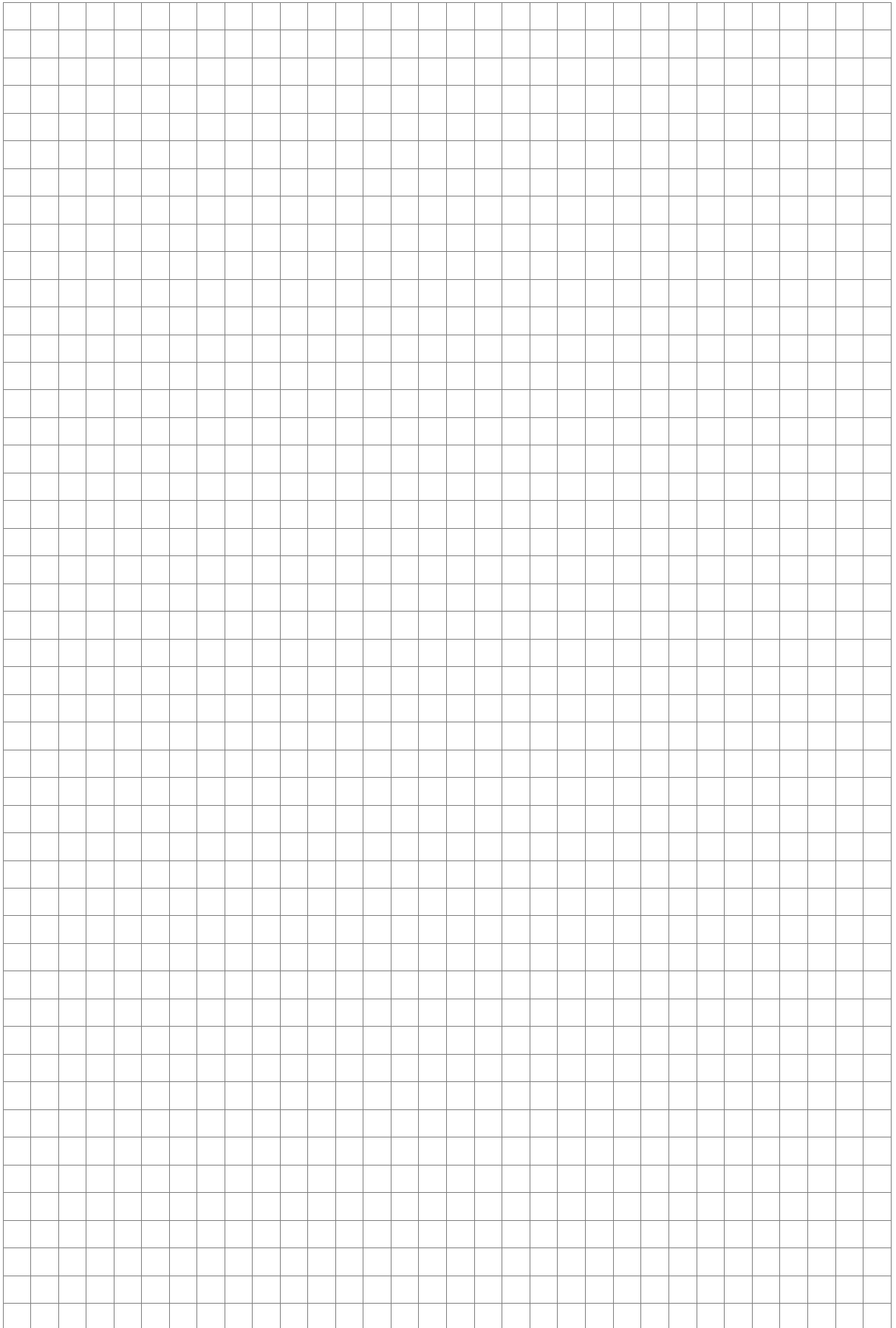




**Question 4**

**(25 marks)**

- (a) Prove that  $\sqrt{2}$  is an irrational number.

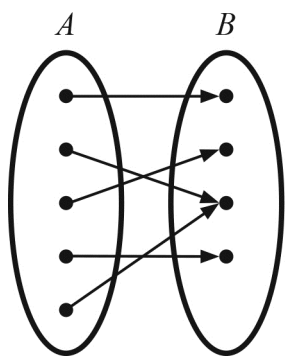
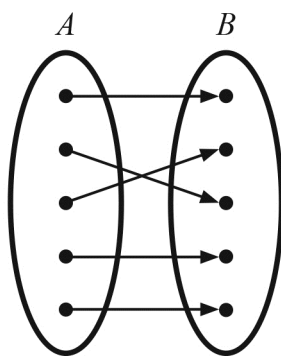
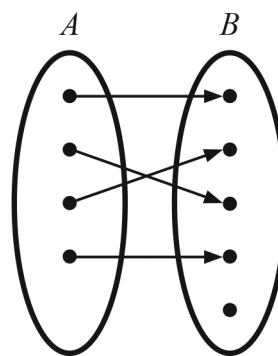


- (b) (i) Write out the general term form of the binomial expansion  $\left(x^2 - \frac{1}{x}\right)^{15}$ .



- (ii) Hence, or otherwise, find the value of the term that is independent of  $x$  in the expansion.



**Question 5****(25 marks)****(a)** Which one of the mapping diagrams below is bijective? Explain your answer fully.**(i)****(ii)****(iii)**

A large grid area for writing the explanation.

(b) Evaluate

(i)  $\lim_{n \rightarrow 3} \frac{n^3 - 27}{n - 3}$



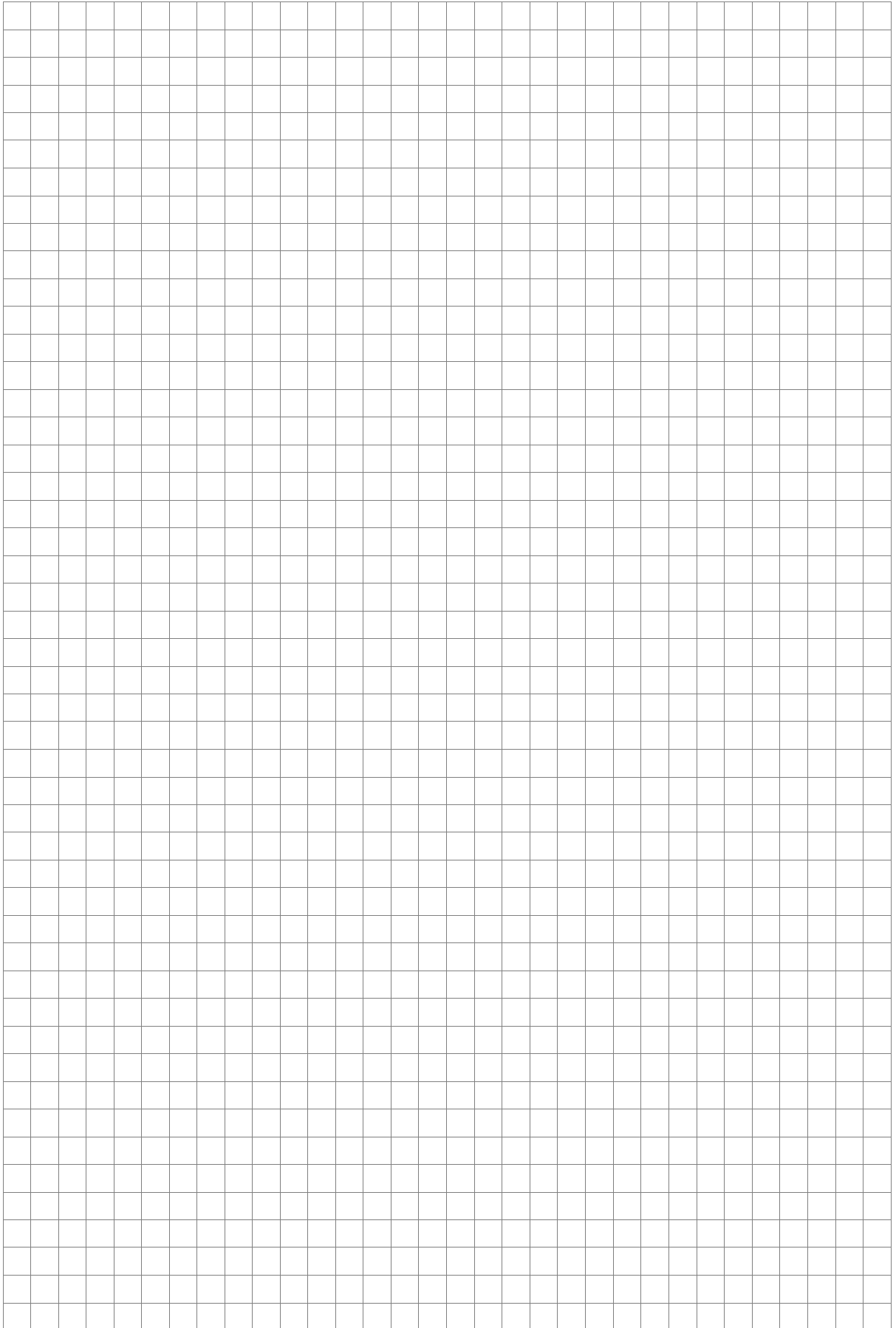
(ii)  $\lim_{x \rightarrow \infty} \frac{2x^2 - 3}{7x^2 + 2}$



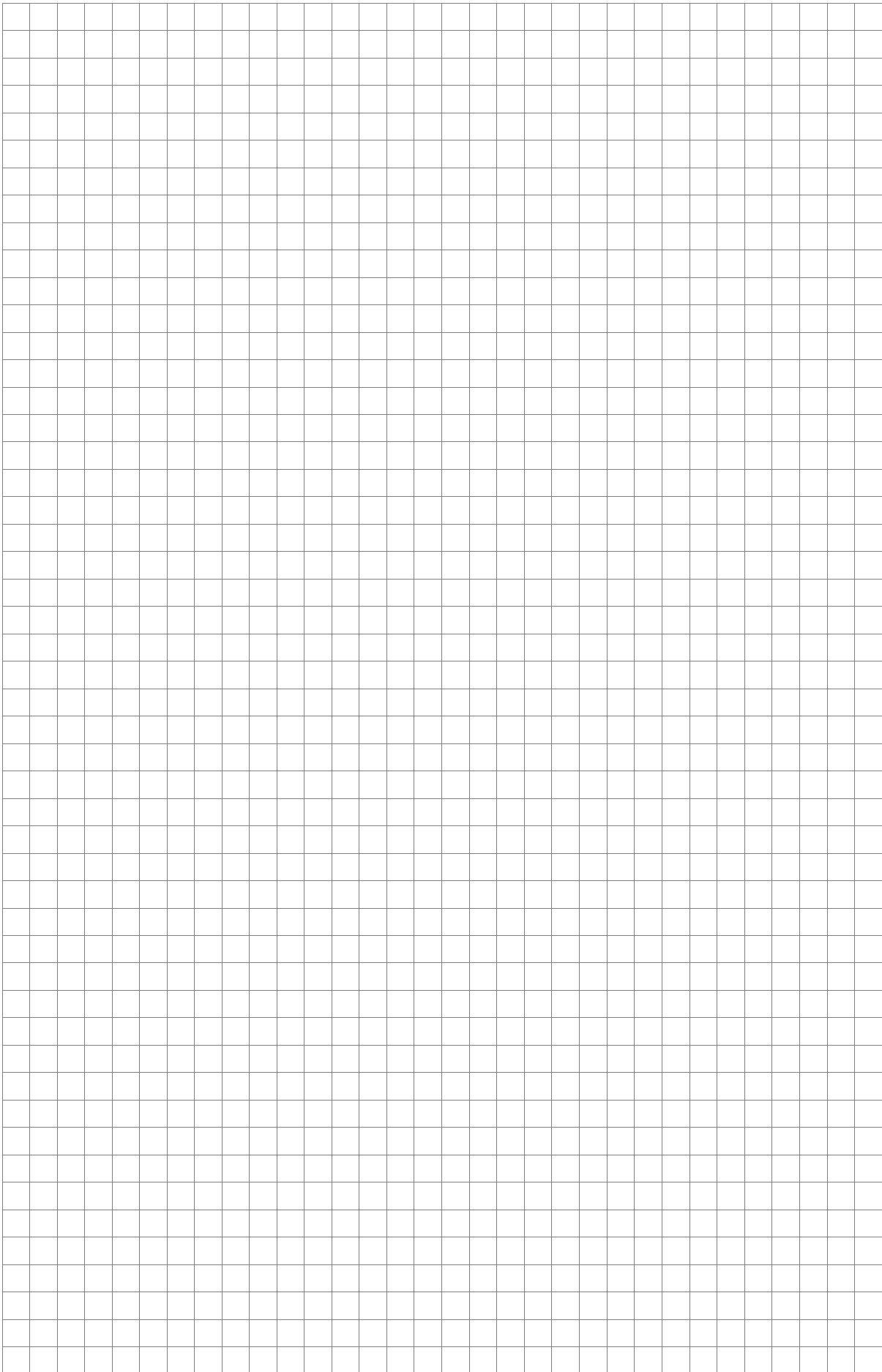
**Question 6**

**(25 marks)**

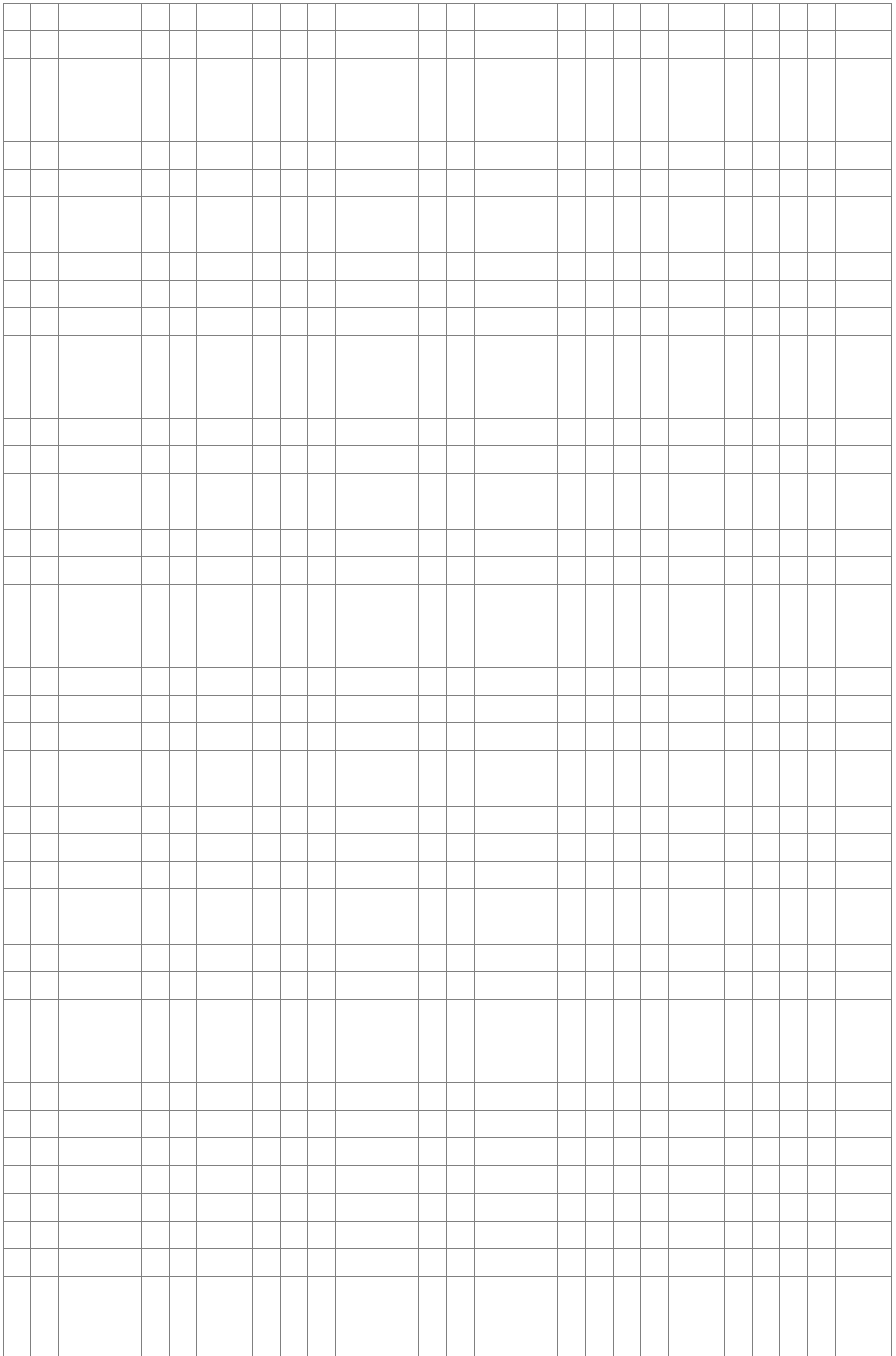
- (a) Differentiate  $-2x^2 + 3x - 12$  from first principles.



(b) (i) If  $f(x) = e^{\cos x}$ , find  $f'(x)$  the derivative of  $f(x)$ .



(ii) Hence, or otherwise, evaluate  $\int_0^{\frac{\pi}{2}} \sin x e^{\cos x} dx$ .









- (ii) Calculate the rate of interest that compounded monthly would be equivalent to an effective annual rate of 4%, correct to 4 decimal places.

A large grid for working out the solution to question (ii). The grid consists of 30 columns and 25 rows of small squares.

- (iii) If Mary makes equal payments of  $€P$  over the next 480 months, what value of  $P$ , correct to the nearest euro, will give Mary the retirement fund she requires?

A large grid for working out the solution to question (iii). The grid consists of 30 columns and 25 rows of small squares.



- (b) Find the value of  $x$ , correct to two decimal places, which will give the maximum volume for the box.

- (c) If the company decides to choose a value of  $x$  such that  $x \in \mathcal{N}$ , write down the range of values of  $x$  that could be chosen.

- (d) Using the information in the previous parts, draw a sketch of the volume function of the box. Indicate clearly the turning points and the roots of the curve.



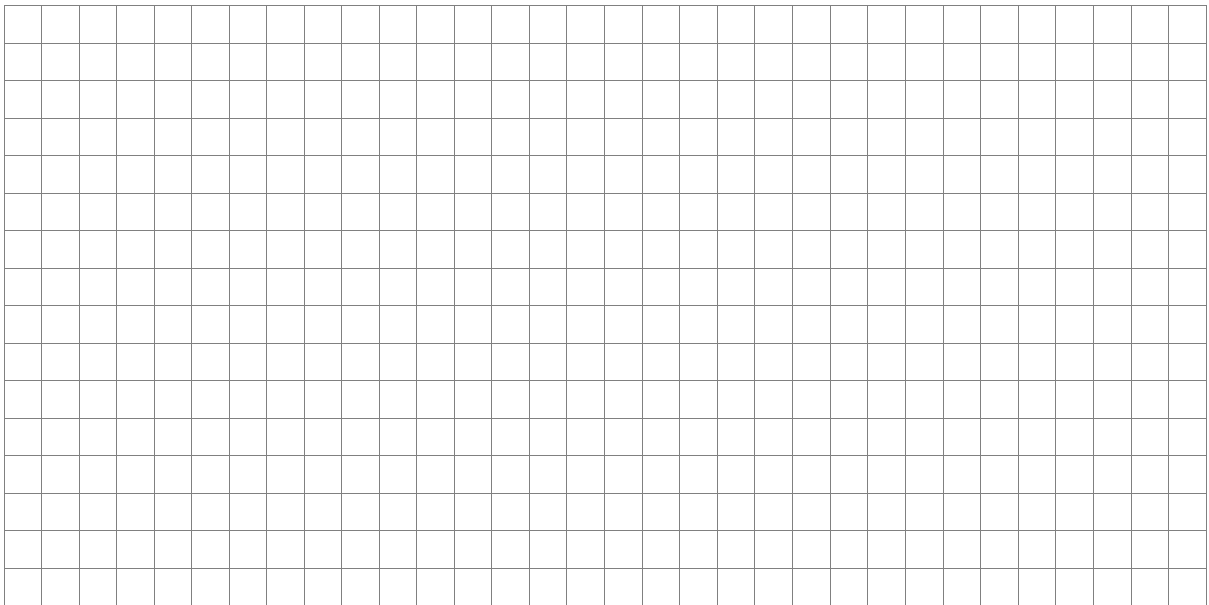
**Question 9****(40 marks)**

Carbon-14 is a radio-isotope of the element Carbon that is used in carbon dating. Its decay can be modelled by the function

$$F = Be^{kt}$$

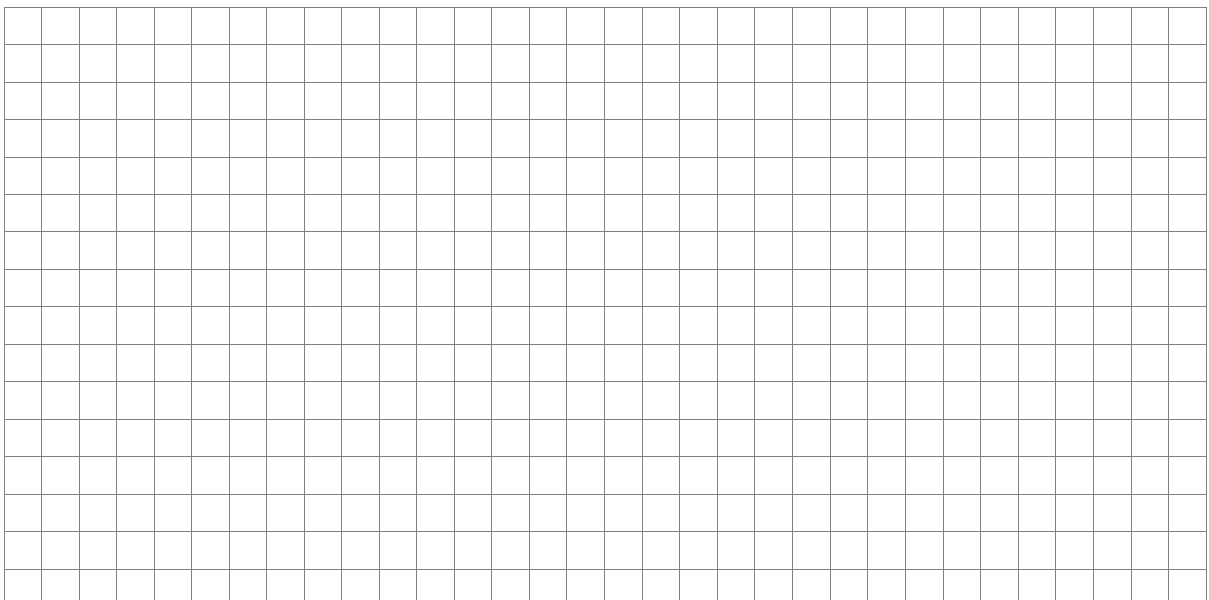
where  $F$  is the final mass remaining, in kg, at any time  $t$ , in years, and  $B$  is the initial amount, in grams, of Carbon-14 and  $k$  is the decay constant. The half-life (the time it takes for half of the mass to decay) of Carbon-14 is 5730 years.

- (a) If the original mass of Carbon-14 is 1kg, show that  $k = \frac{\ln(0.5)}{5730}$ .

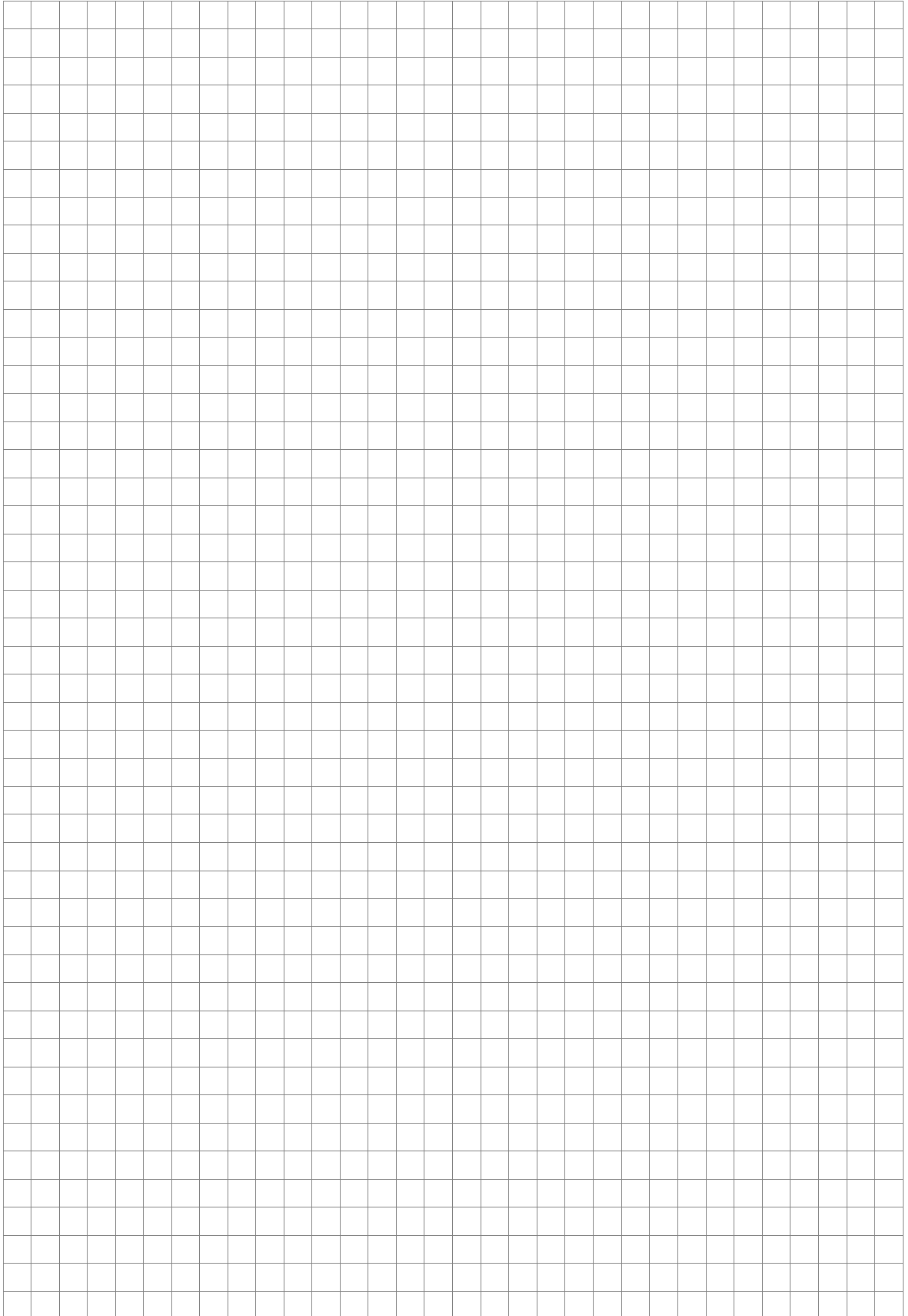


A research scientist is presented with a document which contains the writings of a soldier from the Trojan War circa 1250 BC (around 3250 years ago). After testing it is found that the parchment contained  $1 \times 10^{-12}$  g of Carbon-14.

- (b) Calculate the original amount of Carbon-14 in the parchment.



- (c) It is known that parchments from this area usually contain in the region of  $1.3 \times 10^{-12} \text{ g}$  of Carbon-14. Taking the final amount as  $1 \times 10^{-12} \text{ g}$  calculate the age of the document and explain if you believe it to be genuine or a fake.



**You may use this page for extra work.**

