Question 2 **(25 marks)**

(a) Find the range of values of x for which $|x-4| \geq 2$, where $x \in \mathbb{R}$.

Sq both sides

$(|x-4|)^2 \geq (2)^2$

$x^2 - 8x + 16 \geq 4$

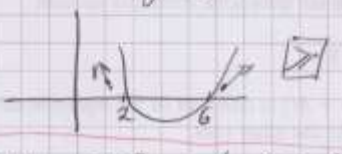
[0, 3, 7, 10]

$x^2 - 8x + 12 \geq 0$

$(x-2)(x-6) = 0$

$x = 2 \quad x = 6$

Sketch graph



Sol: $x \leq 2$ and $x \geq 6 \quad x \in R$

(b) Solve the simultaneous equations:

$$x^2 + xy + 2y^2 = 4$$
$$2x + 3y = -1.$$

$2x + 3y = -1$

$2x = -1 - 3y$

$x = \dfrac{-1-3y}{2}$

Sub in:

$\left(\dfrac{-1-3y}{2}\right)^2 + \left(\dfrac{-1-3y}{2}\right)y + 2y^2 = 4$

[0, 5, 10, 15]

$\dfrac{1 + 6y + 9y^2}{4} - \dfrac{y - 3y^2}{2} + 2y^2 = 4 \quad (\times 4)$

$1 + 6y + 9y^2 - 2y - 6y^2 + 8y^2 = 16$

$11y^2 + 4y - 15 = 0$

$(11y + 15)(y - 1) = 0$

$y = -15/11 \quad y = 1$

Find x $\quad x = \dfrac{-1-3y}{2}$

@ $y = -15/11 \quad x = \dfrac{-1-3\left(\frac{-15}{11}\right)}{2} = 17/11$

@ $y = 1 \quad x = \dfrac{-1-3(1)}{2} = -2$

Sols

$\left(17/11, \dfrac{-15}{11}\right)$

$(-2, 1)$

Question 3 **(25 marks)**

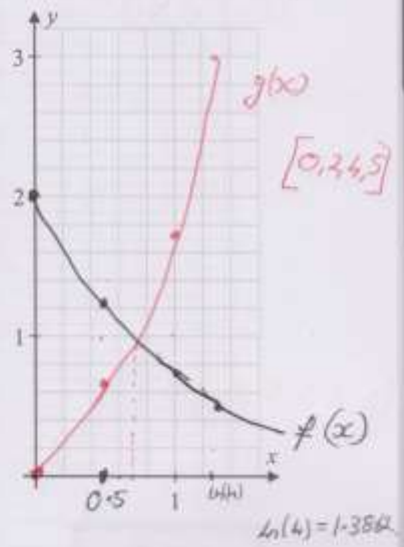
(a) **(i)** $f(x) = \frac{2}{e^x}$ and $g(x) = e^x - 1$, where $x \in \mathbb{R}$.

Complete the table below. Write your values correct to two decimal places where necessary.

[0,2,4,5]

x	0	0·5	1	ln(4)
$f(x) = \frac{2}{e^x}$	$\frac{2}{e^0} = \boxed{2}$	$\frac{2}{e^{0.5}} = \boxed{1{\cdot}21}$	$\frac{2}{e^1} = \boxed{0{\cdot}74}$	$\frac{2}{e^{\ln 4}} = \boxed{0{\cdot}5}$
$g(x) = e^x - 1$	$e^0 - 1 = \boxed{0}$	$e^{0.5} - 1 = \boxed{0{\cdot}65}$	$e^1 - 1 = \boxed{1{\cdot}72}$	$e^{\ln 4} - 1 = \boxed{3}$

(ii) In the grid on the right, use the table to draw the graphs of $f(x)$ and $g(x)$ in the domain $0 \le x \le \ln(4)$. Label each graph clearly.



$\ln(4) = 1{\cdot}386$

(iii) Use your graphs to estimate the value of x for which $f(x) = g(x)$.

$\boxed{x = 0{\cdot}7}$

[0,2,5]

(b) Solve $f(x) = g(x)$ using algebra.

$\frac{2}{e^x} = e^x - 1 \quad (\times e^x)$

$2 = e^x . e^x - 1 . e^x$

$2 = e^{2x} - e^x$

$0 = e^{2x} - e^x - 2$

Let $e^x = y$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2 \quad y = -1$

sub back in:

$y = 2$

$e^x = 2$

$\ln e^x = \ln 2$

$x \ln e = \ln 2$

$x = \ln 2$

$\boxed{x \approx 0{\cdot}69 \rightarrow \text{Sd.}}$

$y = -1$

$e^x = -1$

$\ln e^x = \ln(-1)$ invalid!

[0,3,7,10]

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(b) Given $\log_a 2 = p$ and $\log_a 3 = q$, where $a > 0$, write each of the following in terms of p and q:

(i) $\log_a \frac{8}{3}$

$\log_a 8 - \log_a 3$

$\log_a (2)^3 - \log_a 3$

$3\log_a 2 - \log_a 3$

$\boxed{3p - q}$

[0, 2, 4, 5]

(ii) $\log_a \frac{9a^2}{16}$.

$\log_a 9a^2 - \log_a 16$

$\log_a 9 + \log_a a^2 - \log_a 2^4$

$\log_a 3^2 + \log_a a^2 - \log_a 2^4$

$2\log_a 3 + 2\log_a a - 4\log_a 2$

$2q + 2(1) - 4p.$

$\boxed{2q + 2 - 4p}$

[0, 2, 3, 4, 5]

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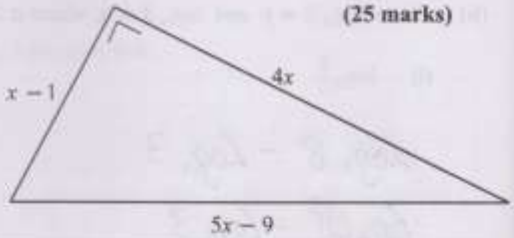
Question 5 **(25 marks)**

(a) (i) The lengths of the sides of a right-angled triangle are given by the expressions $x - 1$, $4x$, and $5x - 9$, as shown in the diagram. Find the value of x.



$(5x-9)^2 = (x-1)^2 + (4x)^2$ (Pythagoras)

$25x^2 - 90x + 81 = x^2 - 2x + 1 + 16x^2$

$8x^2 - 88x + 80 = 0$ $(\div 8)$

$x^2 - 11x + 10 = 0$

$(x - 10)(x - 1) = 0$

$\boxed{x = 10 \text{ Sol!}}$ $x = 1$ Invalid as side $x-1$ would be zero!

[0, 2, 5, 8, 10]

(ii) Verify, with this value of x, that the lengths of the sides of the triangle above form a pythagorean triple.

[0, 2, 5]

at $x = 10$

$x - 1 \Rightarrow 10 - 1 = 9$

$4x \Rightarrow 4(10) = 40$

$5x - 9 \Rightarrow 5(10) - 9 = 41$

$41^2 = 40^2 + 9^2$

$1681 = 1600 + 81$

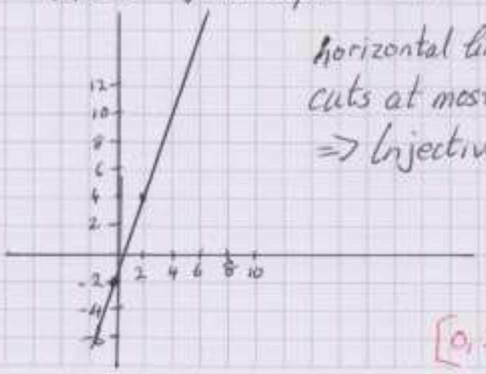
$1681 = 1681$

True is a Pythagorean Triple.

(b) **(i)** Show that $f(x) = 3x - 2$, where $x \in \mathbb{R}$, is an injective function.

$y = 3x - 2$ is a line

$m = 3$ $\quad$ y intercept $= -2$.



horizontal line test:
cuts at most once
$\Rightarrow$ Injective

[0, 2, 5]

OR

If $f(a) = f(b)$
or Then $a = b$
$\Rightarrow$ Injective.

$f(a) = 3a - 2$

$f(b) = 3b - 2$

$f(a) = f(b)$

$3a - 2 = 3b - 2$

$3a = 3b$

$a = b$

$\therefore f(x)$ is Injective

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(ii) Given that $f(x) = 3x - 2$, where $x \in \mathbb{R}$, find a formula for f^{-1}, the inverse function of f. Show your work.

$y = 3x - 2$.

$y + 2 = 3x$

$\dfrac{y+2}{3} = x$

$\Rightarrow \boxed{f^{-1}(x) = \dfrac{x+2}{3}}$

[0, 2, 5]

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