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PRE-LEAVING CERTIFICATE EXAMINATION, 2017

MARKING SCHEME

MATHEMATICS

HIGHER LEVEL

35 Finglas Business Park, Tolka Valley Road, Finglas, Dublin 11
T: 01 808 1494, F: 01 836 2739, E: info@examcraft.ie, W: www.examcraft.ie

OVERVIEW OF MARKING SCHEME

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 2, 4, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10 or 0, 3, 7, 10	0, 2, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the body of the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response (no credit)
- correct response (full credit)

B-scales (three categories)

- response of no substantial merit (no credit)
- partially correct response (partial credit)
- correct response (full credit)

C-scales (four categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

D-scales (five categories)

- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

E-scales (six categories)

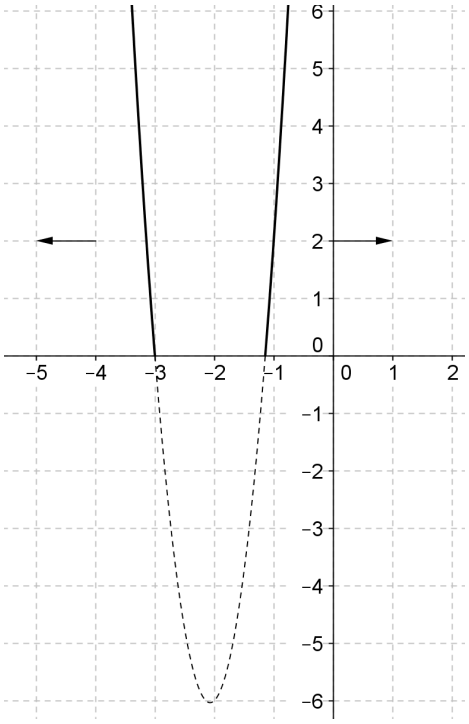
- response of no substantial merit (no credit)
- response with some merit (low partial credit)
- response about half-right (lower middle partial credit)
- response more than half-right (upper middle partial credit)
- almost correct response (high partial credit)
- correct response (full credit)

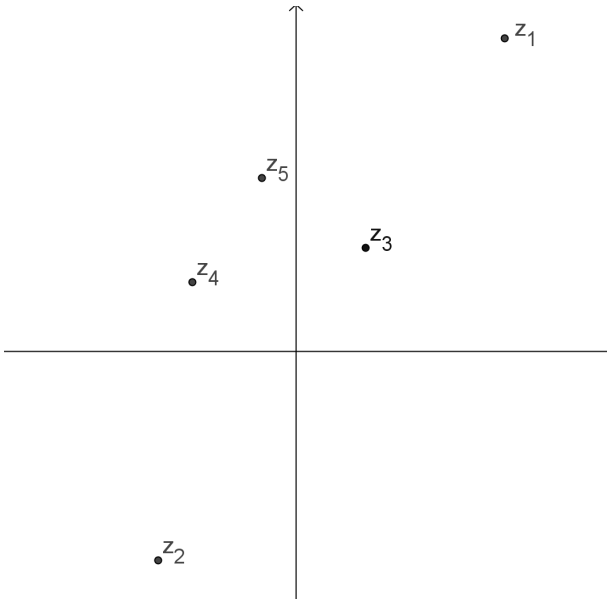
Marking categories for all questions are shown throughout the solutions. In certain cases, typically involving rounding or omission of units, a mark that is one mark below the full-credit mark may also be awarded. Such cases are flagged with an asterisk. Thus, for example, Scale 10C* indicates that 9 marks may be awarded.

PAPER 1

Q1	Model Solution – 25 Marks	Marking Notes
(a)	$(4x)^3-(5)^3$ $(4x-5)(16x^2+20x+25)$	**Scale 5C (0, 2, 4, 5)** **High Partial Credit (4 Marks)** • Factorises the difference of cubes with 1 error in solution **Low Partial Credit (2 Marks)** • Writes $4x$ or 5
(b)	$a+b \geq 2\sqrt{ab}$ $a^2+2ab+b^2 \geq 4ab$ $a^2-2ab+b^2 \geq 0$ $(a-b)^2 \geq 0 \quad \therefore$ True as all square numbers are positive	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Fully correct with no conclusion stated **Low Partial Credit (3 Marks)** • Any correct step
(c)	$x=-3y-3$ $(-3y-3)^2+y^2=13$ $5y^2+9y-2=0$ $(5y-1)(y+2)=0$ $y=-2, \qquad y=\frac{1}{5}$ $x=3, \qquad y=-\frac{18}{5}$	**Scale 10D (0, 2, 5, 8, 10)** **High Partial Credit (8 Marks)** • Solves to find one variable and stops • Finds all variables with errors **Mid Partial Credit (5 Marks)** • Fully correct substitution into quadratic formula but an error in solving equation. **Low Partial Credit (2 Marks)** • Any correct step with linear equation

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$px+\frac{1}{x}=5$ $px^2+1=5x$ $px^2-5x+1=0$	**Scale 5C (0, 2, 4, 5)** **High Partial Credit (4 Marks)** • Substitutes x in correctly but quadratic not in correct form **Low Partial Credit (2 Marks)** • Substitutes in for x • Writes $\frac{1}{5^y}$
(b) (i)	$px^2-5x+1=0$ $b^2-4ac=0$ $(-5)^2-4(p)(1)=0$ $4p=25$ $p=\frac{25}{4}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Fully correct substitution into discriminant **Low Partial Credit (3 Marks)** • Writes discriminant • Identifies a, b, c • Partially correct substitution into formula
(b) (ii)	$\frac{25}{4}(5^y)+5^{-y}=5$ $25x^2-20x+4=0$ $(5x-2)(5x-2)=0$ $x=\frac{2}{5}$ $5^y=\frac{2}{5}$ $y=\log_5\frac{2}{5}$ $y=-0.57$	**Scale 10D* (0, 2, 5, 8, 10)** **High Partial Credit (8 Marks)** • Sets up equation to solve y but solves with an error **Mid Partial Credit (5 Marks)** • Solves for x but not for y **Low Partial Credit (2 Marks)** • Subs in value for p and stops • Writes $\frac{1}{5^y}$ • Substitutes x for 5^y and stops • Forms fully correct equation

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$\frac{7-2x}{x+3} \le 5,$ $(7-2x)(x+3) \le 5(x+3)^2$ $-2x^2 + x + 21 = 5x^2 + 30x + 45$ $7x^2 + 29x + 24 = 0$ $(7x+8)(x+3) = 0$ $x = -\frac{8}{7} \quad x = -3$ $\left\{x \le -3 \quad \cup \quad x \ge -\frac{8}{7}\right\}$  	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Sets up correct quadratic but solves with error • Incorrect quadratic solved incorrectly with solution set **Low Partial Credit (3 Marks)** • Any correct step
(b)	$7^1 - 1 = 6$ which is divisible by 3 Thus true for $n = 1$ Assume true for $n = k$ $7^k - 1 = 3A$ $\therefore 7^k = 3A + 1$ Prove true for $n = k + 1$ $7^{k+1} - 1$ $7^1 7^k - 1$ $7(3A+1) - 1$ $21A + 6$ $3(7A + 2)$ Thus true for $n = k + 1$ assuming $n = k$ is true Hence true for all $n \in \mathbb{N}$	**Scale 15C (0, 4, 7, 11, 15)** **High Partial Credit (11 Marks)** • "$n = k + 1$" correct with no statement or incorrect statement **Mid Partial Credit (7 Marks)** • "Assume true for $n = k$" step correct **Low Partial Credit (4 Marks)** • $n = 1$ proven to be true

Q4	Model Solution – 25 Marks	Marking Notes
(a)		**<u>Scale 5C (0, 2, 4, 5)</u>** **High Partial Credit (4 Marks)** • 3 correct **Low Partial Credit (2 Marks)** • 1 correct
(b)	$\lvert z_1\rvert = \sqrt{(2)^2 + (-5)^2} = \sqrt{29}$ $\lvert z_2\rvert = \sqrt{(-1)^2 + (-3)^2} = \sqrt{10}$ $\frac{z_1}{z_2} = \frac{(2-5i)(-1+3i)}{(-1-3i)(-1+3i)} = \frac{13}{10} + \frac{11}{10}i$ $\left\lvert \frac{z_1}{z_2}\right\rvert = \sqrt{\left(\frac{13}{10}\right)^2 + \left(\frac{11}{10}\right)^2} = \frac{\sqrt{290}}{10}$ $\frac{\sqrt{29}}{\sqrt{10}} = \frac{\sqrt{290}}{10}$ $\frac{\sqrt{290}}{10} = \frac{\sqrt{290}}{10}$ $\therefore$ true	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Finds $\left\lvert \frac{z_1}{z_2}\right\rvert$ • Finds $\lvert z_1\rvert$ and $\lvert z_2\rvert$ correctly **Low Partial Credit (3 Marks)** • Finds $\lvert z_1\rvert$ • Finds $\lvert z_2\rvert$
(c)	$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$ $(\cos\theta + i\sin\theta)^3 = c^3 + 3ic^2s - 3cs^2 - is^3$ $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$ $\cos 3\theta = \cos^3\theta - 3\cos\theta(1 - \cos^2\theta)$ $\cos 3\theta = \cos^3\theta - 3\cos\theta + 3\cos^3\theta$ $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$	**<u>Scale 10D (0, 2, 5, 8, 10)</u>** **High Partial Credit (8 Marks)** • Sets up correct equation **Mid Partial Credit (5 Marks)** • Expands brackets correctly **Low Partial Credit (2 Marks)** • Writes $(\cos\theta + i\sin\theta)^3$

<table>
<tr><th>Q5</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td colspan="2"><u>Scale 5C (0, 2, 4, 5)</u>

High Partial Credit (4 Marks)
• At least 4 correct values

Low Partial Credit (2 Marks)
• One correct value</td></tr>
<tr><td></td><td colspan="2">
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>$g(x)$</td><td>$2e^{0.1(0)}$</td><td>$2e^{0.1(1)}$</td><td>$2e^{0.1(2)}$</td><td>$2e^{0.1(3)}$</td><td>$2e^{0.1(4)}$</td><td>$2e^{0.1(5)}$</td><td>$2e^{0.1(6)}$</td><td>$2e^{0.1(7)}$</td><td>$2e^{0.1(8)}$</td></tr>
</table>
</td></tr>
<tr><td>(a)
(ii)</td><td colspan="2"><u>Scale 5C (0, 2, 4, 5)</u>

High Partial Credit (4 Marks)
• Fully correct substitution with error in calculation

Low Partial Credit (2 Marks)
• Any correct substitution</td></tr>
<tr><td></td><td colspan="2">$A=\frac{1}{2}\left(2e^{0.1(0)}+2e^{0.1(8)}\left(2e^{0.1(1)}+2e^{0.1(2)}+2e^{0.1(3)}+2e^{0.1(4)}+2e^{0.1(5)}+2e^{0.1(6)}+2e^{0.1(7)}\right)\right)$
$A=24.5$ square units</td></tr>
<tr><td>(a)
(iii)</td><td>$\int_0^8 2e^{0.1x}$
$20e^{0.8}-20e^{0}=24.5$</td><td><u>Scale 5C (0, 2, 4, 5)</u>

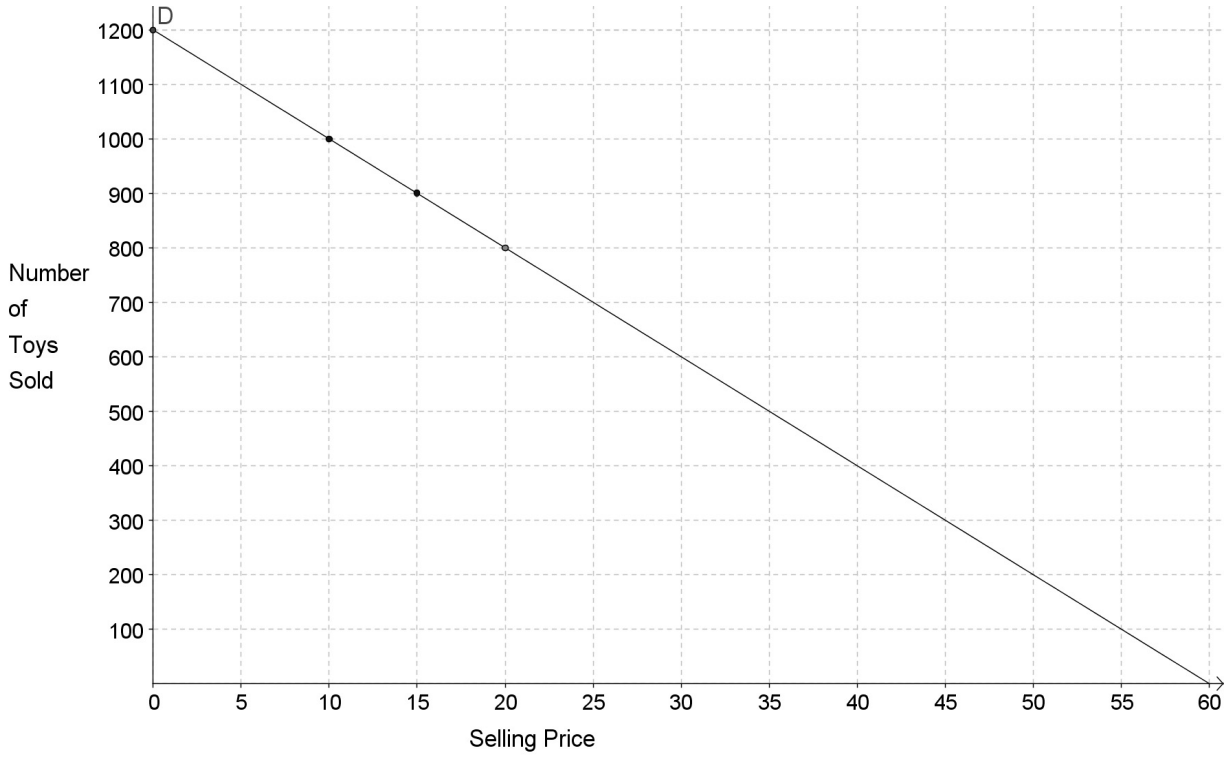
High Partial Credit (4 Marks)
• Error in calculation for a correct integral
• Incorrect integration (not oversimplified) with fully correct substitution

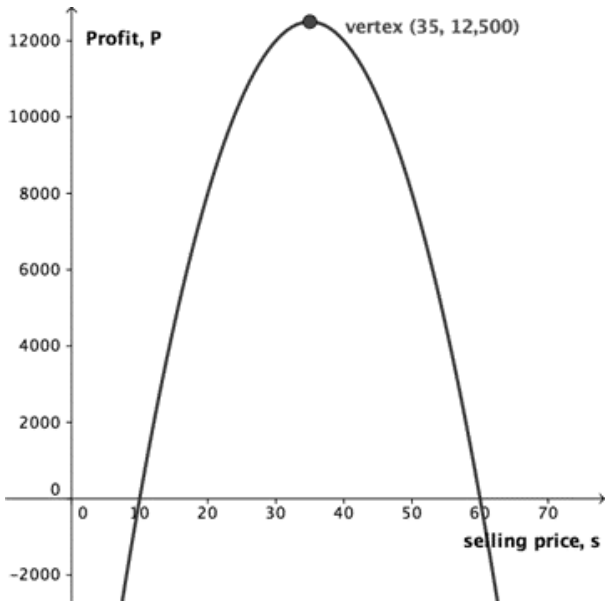
Low Partial Credit (2 Marks)
• Integrates correctly</td></tr>
<tr><td>(b)</td><td>$f(x+h)=2x^2+4xh+2h^2+3x+3h+3$
$f(x+h)-f(x)=4xh+2h^2+3h$
$\frac{f(x+h)-f(x)}{h}=4x+2h+3$
$\lim_{h\to\infty}\frac{f(x+h)-f(x)}{h}=4x+3$</td><td><u>Scale 10C (0, 3, 7, 10)</u>

High Partial Credit (7 Marks)
• Correct procedure with no more than one error

Low Partial Credit (3 Marks)
• Any correct step</td></tr>
</table>

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$y = x^2 \sin(3x+5)$ $\frac{dy}{dx} = x^2(\cos(3x+5))(3) + \sin(3x+5)(2x)$ $\frac{dy}{dx} = 3x^2 \cos(3x+5) + 2x\sin(3x+5)$	**Scale 5C (0, 2, 4, 5)** **High Partial Credit (4 Marks)** • One error in solution **Low Partial Credit (2 Marks)** • Any correct step
(b)	$\cos^{-1}\frac{2}{x^2} = \cos^{-1}\frac{2x^{-2}}{1}$ $\frac{dy}{dx} = -\frac{1}{\sqrt{1-(2x^{-2})^2}} \times (-4x)^{-3}$ $\frac{dy}{dx} = -\frac{-4x^{-3}}{\sqrt{1-4x^{-4}}}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Substitutes into formula correctly but error in solution **Low Partial Credit (3 Marks)** • Any correct step
(c)	$\frac{dy}{dx} = 3x^4e^{3x} + 4x^3e^{3x}$ $3x^4e^{3x} + 4x^3e^{3x} - 3(x^4e^{3x}) = \frac{4x^4e^{3x}}{x}$ $4x^3e^{3x} = 4x^3e^{3x}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Sets up equation correctly but error in solution **Low Partial Credit (3 Marks)** • Any correct differential • Incorrect equation set up

Q7	Model Solution – 75 Marks	Marking Notes
(a) (i)	It is a linear relationship. As the price increases by €5 the number of toys sold decreases by 100.	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Partially correct statement
(a) (ii)	**Scale 10B (0, 5, 10)** **Partial Credit (5 Marks)** • Any point plotted correctly	
	 *Accept answer based on part **(a)**	
(a) (iii)	1,200 *Accept answer based on graph	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Incorrect point from graph
(a) (iv)	$n = -20x + 1{,}200$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Incorrect slope but equation found • Incorrect y-intercept but equation found **Low Partial Credit (3 Marks)** • Finds slope • Any correct substitution into slope formula • Attempts to find rise over run

(b) (i)	Total Revenue = $n\,x$ and Production Costs = $10n$ $P = n\,x - 10n$	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Two partially correct equations with final equation found • Two correct equations with no final equation found **Low Partial Credit (3 Marks)** • One equation correct
(b) (ii)	$P = nx - 10n$ $n = -20x + 1{,}200$ $P = (-20x + 1{,}200)x - 10(-20x + 1{,}200)$ $P = -20x^2 + 1{,}400x - 12{,}000$	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Correct substitution but error in forming equation **Low Partial Credit (3 Marks)** • Any correct substitution into profit
(b) (iii)	$\frac{dP}{dx} = -40x + 1{,}400$ $40x = 1{,}400$ $x = €35$	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Correct differential and stops • Correct differential with error in solving **Low Partial Credit (3 Marks)** • Partially correct differential
(b) (iv)	$P = -20(35)^2 + 1{,}400(35) - 12{,}000$ $P = €12{,}500$	**<u>Scale 5B (0, 2, 5)</u>** **Partial Credit (2 Marks)** • Partially correct substitution into profit formula
(b) (v)		**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Partially correct sketch **Low Partial Credit (3 Marks)** • Any correct point plotted

Q8	Model Solution – 40 Marks	Marking Notes
(a)	$\pi r^2 h = 98\pi$ $r^2 h = 98$ $h = \frac{98}{r^2}$ cm	**Scale 10C* (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Forms correct equation but h not in correct form **Low Partial Credit (3 Marks)** • Any correct step
(b)	$S = 2\pi r^2 + 2\pi rh$ $S = 2\pi r^2 + 2\pi r\left(\frac{98}{r^2}\right)$ $S = 2\pi\left(\frac{r^3 + 98}{r}\right)$ cm^2	**Scale 10C* (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Correct SA formula with h subbed in • Not in correct form **Low Partial Credit (3 Marks)** • Total SA formula written • Any correct step
(c)	$S = 2\pi r^2 + 196\pi r^{-1}$ $\frac{dS}{dr} = 4\pi r - 196\pi r^{-2}$ $4r^3 = 196$ $r = 3.7\ cm$	**Scale 10C* (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • One fully correct differential with equation solved with no error • Fully correct differential but equation solved with error **Low Partial Credit (3 Marks)** • Any correct step
(d)	$S = 2\pi r^2 + 2\pi rh$ $S = 2\pi(3.7)^2 + 2\pi(3.7)\left(\frac{98}{3.7^2}\right)$ $S = 252.44\ cm^2$	**Scale 10C* (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Fully correct substitution but error in solution • Incorrect substitution but calculated correctly **Low Partial Credit (3 Marks)** • Any correct step

<table>
<tr><th>Q9</th><th>Model Solution – 35 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$1.025 = 1(1+i)^{12}$
$\sqrt[12]{1.025} - 1 = i$
$i = 0.20598\%$</td><td><u>Scale 10C (0, 3, 7, 10)</u>

High Partial Credit (7 Marks)
• Sets up correct equation but error in solving
• Error in equation but solves to end

Low Partial Credit (3 Marks)
• Any correct step</td></tr>
<tr><td rowspan="2">(b)</td><td colspan="2"><u>Scale 15C (0, 5, 10, 15)</u>

High Partial Credit (10 Marks)
• Sets up geometric series correctly
• Incorrect geometric series but continues to end with no further error

Low Partial Credit (5 Marks)
• Any correct step</td></tr>
<tr><td colspan="2">$15{,}000 = A + A(1.0020598) + A(1.0020598)^2 + \ldots + A(1.0020598)^{71}$
$15{,}000 = A\left(\frac{1\left(1-(1.0020598)^{72}\right)}{1-1.0020598}\right)$
$A = €193.48$</td></tr>
<tr><td>(c)</td><td>$A = 12{,}000\left(\frac{0.06(1+0.06)^3}{(1+0.06)^3-1}\right)$
$A = €4{,}489$</td><td><u>Scale 10C (0, 3, 7, 10)</u>

High Partial Credit (7 Marks)
• Fully correct substitution but error in calculation
• Partially correct substitution but calculates with no further error

Low Partial Credit (3 Marks)
• Any correct substitution</td></tr>
</table>

PAPER 2

<table>
<tr><th>Q1</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$P(1) = 0.45 \qquad P(1 \text{ and } 2) = 0.25$
$P(2 \setminus 1) = \frac{P(2 \cap 1)}{P(1)} = \frac{0.25}{0.45} = \frac{5}{9}$</td><td><u>Scale 10C (0, 3, 7, 10)</u>

High Partial Credit (7 Marks)
• Correct substitution into formula

Low Partial Credit (3 Marks)
• Identifies either probability correctly
• Writes conditional formula</td></tr>
<tr><td>(b)
(i)</td><td>$\binom{12}{1}(0.04)^1(0.96)^{11}$
0.31</td><td><u>Scale 5B* (0, 2, 5)</u>

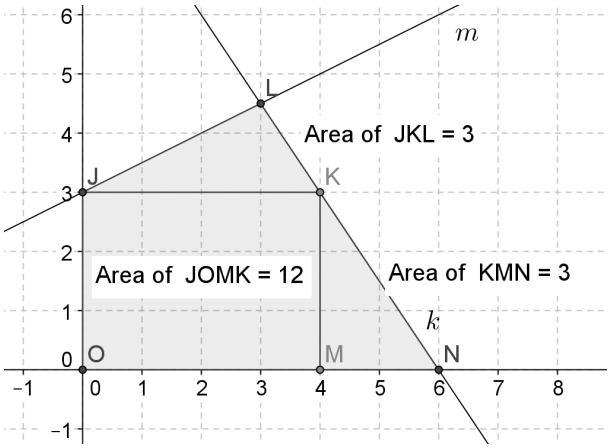
Partial Credit (2 Marks)
• Any correct step</td></tr>
<tr><td rowspan="2">(b)
(ii)</td><td colspan="2"><u>Scale 10D (0, 3, 7, 10)</u>

High Partial Credit (7 Marks)
• Works out two correct
• Fully correct substitution
• Correct but error in calculation

Low Partial Credit (3 Marks)
• Any correct step</td></tr>
<tr><td colspan="2">$\left[\binom{12}{0}(0.04)^0(0.96)^{12} + \binom{12}{1}(0.04)^1(0.96)^{11} + \binom{12}{2}(0.04)^2(0.96)^{10} + \binom{12}{3}(0.04)^3(0.96)^9\right]$
$= 99.90\%$</td></tr>
</table>

Q2	Model Solution – 25 Marks	Marking Notes
(a)	$\binom{15}{r+3}=\binom{15}{15-(r+3)}=\binom{15}{4r-13}$ $15-(r+3)=4r-13$ $25=5r$ $r=5$	**<u>Scale 10D (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Sets up equation $15-(r+3)=4r-13$ **Low Partial Credit (3 Marks)** • Some work with $15-(r+3)$ or $15-(4r-13)$
(b) (i)	$\frac{5C4+6C4}{16C4}=\frac{1}{91}$ *Accept decimal or percentage	**<u>Scale 5B (0, 2, 5)</u>** **Partial Credit (2 Marks)** • Any correct step
(b) (ii)	$\frac{5C1\times 6C1\times 3C1\times 2C1}{16C4}=\frac{9}{91}$	**<u>Scale 5B* (0, 2, 5)</u>** **Partial Credit (2 Marks)** • Any correct step
(b) (iii)	$\frac{2C2\times 14C2}{16C4}=\frac{1}{20}$	**<u>Scale 5B* (0, 2, 5)</u>** **Partial Credit (2 Marks)** • Any correct step

Q3	Model Solution – 25 Marks	Marking Notes
(a)	Centre: $(5, 4)$ $r = \sqrt{25+16-21} = 2\sqrt{5}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Radius correct • Centre correct **Low Partial Credit (3 Marks)** • Radius or centre partially correct • Partially correct in finding centre or radius
(b) (i)	$y - y_1 = m(x - x_1)$ $y + 2 = x(x - 3)$ $mx - y - (3m + 2) = 0$	**Scale 5C (0, 2, 5)** **Partial Credit (2 Marks)** • Any correct step
(b) (ii)	$\frac{\lvert m(5) - 1(4) - (3m + 2)\rvert}{\sqrt{m^2 + 1}} = 2\sqrt{5}$ $\lvert 2m - 6\rvert = 2\sqrt{5} \times \sqrt{m^2 + 1}$ $4m^2 - 24m + 36 = 4(5)(m^2 + 1)$ $4m^2 - 24m + 36 = 20m^2 + 20$ $0 = 16m^2 + 24m - 16$ $0 = 2m^2 + 3m - 2$ $(2m - 1)(m + 2) = 0$ $m = \frac{1}{2}, \quad m = -2$ $\therefore -2x - y + 4 = 0, \quad x - 2y - 7 = 0$ **Allow work based on candidates earlier work	**Scale 10D (0, 2, 5, 8, 10)** **High Partial Credit (8 Marks)** • Slopes found but equations not found • Error in slopes but finds correct equations for 2 tangents **Mid Partial Credit (5 Marks)** • Sets up equation correctly • Any correct substitution into formula **Low Partial Credit (2 Marks)** • Any correct step

Q4	Model Solution – 25 Marks	Marking Notes
(a) (i)	$k:\quad 3x+2y=18$ $m:\quad -x+2y=6$	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • One correct • Two partially correct **Low Partial Credit (3 Marks)** • Any correct step
(a) (ii)	 Area = 18 square units	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • One correct • Two found but with errors **Low Partial Credit (3 Marks)** • Any correct step
(b)	$\tan\theta=\dfrac{(-1.5)-(0.5)}{1+(-1.5)(0.5)}$ $\tan\theta=-8$ $\theta=83°$	**<u>Scale 5C (0, 2, 4, 5)</u>** **High Partial Credit (4 Marks)** • Incorrect slopes but fully substituted and calculated without further error • Correct slopes fully substituted **Low Partial Credit (2 Marks)** • One correct slope

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$3\theta = \tan^{-1} 1$ $3\theta = 45°$ $3\theta = 45°, 225°, 405°, 585°, 765°, 945°$ $\theta = 15°, 75°, 135°, 195°, 225°, 315°$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Finds at least 2 correct values for 3θ **Low Partial Credit (3 Marks)** • Any correct step
(b)	$17^2 = 15^2 + x^2$ $x = 8$ $\sin 2\beta = 2\sin\beta\cos\beta$ $\sin 2\beta = 2\left(\frac{8}{17}\right)\left(\frac{15}{17}\right) = \frac{240}{289}$	**Scale 5C (0, 2, 4, 5)** **High Partial Credit (4 Marks)** • Finds hypotenuse • Writes $\sin 2\beta = 2\sin\beta\cos\beta$ **Low Partial Credit (2 Marks)** • Any correct step
(c) (i)	$\cos 2\theta = \cos(\theta + \theta)$ $\cos(\theta + \theta) = \cos\theta + \cos\theta - \sin\theta\sin\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Any correct step
(c) (ii)	$\cos 2\theta = \cos^2\theta - \sin^2\theta$ $\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$ $\cos 2\theta = \cos^2\theta - 1 + \cos^2\theta$ $\cos 2\theta = 2\cos^2\theta - 1$	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Any correct step

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$\sin\theta = \frac{h}{l}$ $h = l\sin\theta$	**<u>Scale 5B (0, 2, 5)</u>** **Partial Credit (2 Marks)** • Any correct step (check diagram)
(b) (i)	Area = 2 triangles + rectangle $A = 2\left(\frac{1}{2}\right)(l\cos\theta)(l\sin\theta) + (l)(l\sin\theta)$ $A = l^2\cos\theta\sin\theta + l^2\sin\theta$ $A = l^2\sin\theta(1+\cos\theta)$	**<u>Scale 10C (0, 3, 7, 10)</u>** **High Partial Credit (7 Marks)** • Fully correct formula for the area but error in finishing **Low Partial Credit (3 Marks)** • Any correct step
(b) (ii)	$\frac{dA}{d\theta} = l^2\cos^2\theta + (\sin\theta)(-l^2\sin\theta) + l^2\cos\theta$ $= l^2\cos^2\theta - l^2\sin^2\theta + l^2\cos\theta$ $= l^2\cos^2\theta - l^2(1-\cos^2\theta) + l^2\cos\theta$ $\therefore l^2(2\cos^2\theta + \cos\theta - 1) = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\cos\theta = \frac{1}{2} \quad \cos\theta \neq -1$ $\theta = \frac{\pi}{3}$	**<u>Scale 10D (0, 2, 5, 8, 10)</u>** **High Partial Credit (8 Marks)** • Correct differential but error in solving equation • Incorrect differential but solves with no further error (no over simplification allowed) **Mid Partial Credit (5 Marks)** • Correct differentiation and stops • Partially correct differentiation but continues **Low Partial Credit (2 Marks)** • Any correct step

Q7	Model Solution – 60 Marks	Marking Notes
(a)	$\lvert OK\rvert = R - h$ Radius minus the depth h from top of the tank.	**Scale 5A (0, 5)**
(b)	$\lvert KB\rvert^2 = R^2 - (R-h)^2$ $\lvert KB\rvert^2 = R^2 - (R^2 - 2Rh + h^2)$ $\lvert KB\rvert = \sqrt{2Rh - h^2}$ $\text{Area}\,\Delta OBK = \frac{1}{2}(R-h)\sqrt{2Rh-h^2}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Sets up equation using Pythagoras correctly **Low Partial Credit (3 Marks)** • Any correct step
(c)	$A = R^2\theta$ $\cos\theta = \frac{R-h}{R}$ $\theta = \cos^{-1}\left(\frac{R-h}{R}\right)$ $\therefore A = R^2\cos^{-1}\left(\frac{R-h}{R}\right)$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Finds $\cos\theta$ • Uses incorrect area formula in otherwise correct solution **Low Partial Credit (3 Marks)** • Any correct step
(d)	$V = l\,(\text{Area Circle} - \text{Area Sector} + \text{Area}\,\Delta)$ $V = l\left(\pi R^2 + 2\left(\frac{1}{2}(R-h)\sqrt{2Rh-h^2}\right) - R^2\cos^{-1}\left(\frac{R-h}{R}\right)\right)$ $\text{Volume} = l\left(\pi R^2 + (R-h)\sqrt{2Rh-h^2} - R^2\cos^{-1}\left(\frac{R-h}{R}\right)\right)$	**Scale 10D (0, 2, 5, 8, 10)** **High Partial Credit (8 Marks)** • Fully correct substitution with errors in final form **Mid Partial Credit (5 Marks)** • Sets up word equation partially correct • Gets some substitution correct into formula **Low Partial Credit (2 Marks)** • Any correct step
(e) (i)	$V = \pi(0.75)^2(2.5)$ $V = \frac{45}{32}\pi\ m^3$	**Scale 5B* (0, 2, 5)** **Partial Credit (2 Marks)** • Any correct substitution into formula
(e)	**Scale 5C* (0, 2, 4, 5)**	

(ii)

High Partial Credit (4 Marks)
- Fully correct substitution with errors in solving

Low Partial Credit (2 Marks)
- Any correct substitution into formula

$$V = 2.5\left(\pi(0.75)^2 + 2\left(\frac{1}{2}(0.75-0.6)\sqrt{2(0.75)(0.6)-(0.6)^2}\right) - (0.75)^2\cos^{-1}\left(\frac{0.75-0.6}{0.75}\right)\right)$$

$$V = 2.5\left(\frac{9}{16}\pi + 0.11022 - 0.77\right)$$

$$V = 2.77\ m^3$$

(f)

$$l\left(\pi R^2 + (R-h)\sqrt{2Rh-h^2} - R^2\cos^{-1}\left(\frac{R-h}{R}\right)\right) = 99.5$$

$$\pi R^2 + (R-h)\sqrt{2Rh-h^2} - R^2\cos^{-1}\left(\frac{R-h}{R}\right) = \frac{199}{16}$$

$$\pi R^2 + \left(\frac{R-h}{R}\right)\sqrt{2R\left(\frac{R}{2}\right)-\left(\frac{R}{2}\right)^2} - R^2\cos^{-1}\left(\frac{R-\frac{R}{2}}{R}\right) = \frac{199}{16}$$

$$\pi R^2 + \left(\frac{R}{2}\right)\sqrt{R^2-\frac{R^2}{4}} - R^2\cos^{-1}\left(\frac{\frac{R}{2}}{R}\right) = \frac{199}{16}$$

$$\pi R^2 + \left(\frac{R}{2}\right)\sqrt{\frac{3R^2}{4}} - R^2\cos^{-1}\left(\frac{1}{2}\right) = \frac{199}{16}$$

$$\pi R^2 + \left(\frac{R}{2}\right)(R)\sqrt{\frac{3}{4}} - R^2\cos^{-1}\left(\frac{1}{2}\right) = \frac{199}{16} *****$$

$$R^2\left(\pi + \frac{1}{2}\sqrt{\frac{3}{4}} - \cos^{-1}\left(\frac{1}{2}\right)\right) = \frac{199}{16}$$

$$R = \sqrt{\frac{\frac{199}{16}}{\pi + \frac{1}{2}\sqrt{\frac{3}{4}} - \cos^{-1}\left(\frac{1}{2}\right)}}$$

$$R = 2.2\ m$$

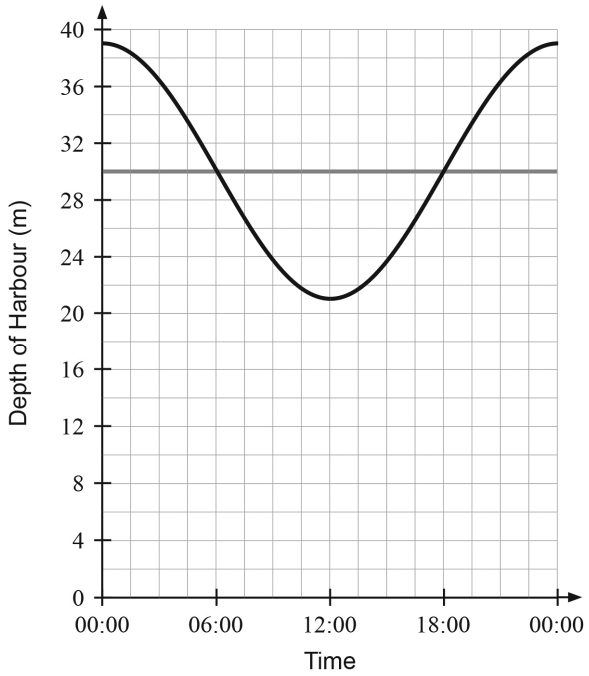
Scale 15D* (0, 4, 7, 11, 15)

High Partial Credit (11 Marks)
- Reaches ***** in solution

Mid Partial Credit (7 Marks)
- Sets up correct equation

Low Partial Credit (4 Marks)
- Any correct substitution into formula

Q8	Model Solution – 55 Marks	Marking Notes
(a)	Any sampling method (cluster, quota, stratified, random sampling) with correct explanation.	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Cluster with no explanation or incorrect explanation
(b)	**(i)** $\bar{x} = €1373.33$ **Accept correct answer without work **(ii)** $\sigma = €318.19$ **Accept correct answer without work	**Scale 15C (0, 5, 10, 15)** **High Partial Credit (10 Marks)** • Mean or standard deviation correct with or without workings **Low Partial Credit (5 Marks)** • Mean or standard deviation partially correct with workings shown
(c)	$[1373.33 - 2(318.19), 1373.33 + 2(318.19)]$ $[€736.95, €2009.71]$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Calculate 1 standard deviation on upper and lower bound • Either upper or lower correct **Low Partial Credit (3 Marks)** • States or implies $\pm\sigma$
(d)	Any 6 areas. Suitable display e.g. side by side bar chart Graph to be correct to the eye. Graphs shows increase from 2014 to 2015	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Display with 2014 and 2015 shown but not correct to eye • 2014, 2015 shown on separate displays **Low Partial Credit (3 Marks)** • Partially correct display • Unsuitable display
(e)	Yes. It clearly shows rents in 2015 are higher than in 2014 **Accept answer based on candidates earlier work	**Scale 5B (0, 2, 5)** **Partial Credit (2 Marks)** • Correct answer with incorrect or no statement
(f)	$1050 - 1.96\frac{390}{\sqrt{50}} \le \mu \le 1050 + 1.96\frac{390}{\sqrt{50}}$ $€942 \le \mu \le €1158$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • $\frac{\sigma}{\sqrt{n}}$ calculated **Low Partial Credit (3 Marks)** • Any correct step • $\frac{1}{\sqrt{n}}$

Q9	Model Solution – 35 Marks	Marking Notes
(a) (i)	$y = 30$ m	**Scale 5B* (0, 2, 5)** **Partial Credit (2 Marks)** • Any correct step e.g. works with 10 and 50
(a) (ii)	24 hours	**Scale 5A (0, 5)**
(b)	30 is midway ... shifts graph vertically 9 is max/min depth (amplitude) cos function as high tide starts $\frac{2\pi}{24} = \frac{\pi}{12}$	**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Two correct explanations **Low Partial Credit (3 Marks)** • One partially correct explanation
(c)		**Scale 10C (0, 3, 7, 10)** **High Partial Credit (7 Marks)** • Plots a fully correct sine function • Cosine function plotted with one error **Low Partial Credit (3 Marks)** • Any correct point of function plotted • Plots an incorrect sine function
(d)	$f(t) = 30 - 9\cos\left(\frac{\pi}{12}t\right)$ Coefficient on cos becomes negative	**Scale 5A (0, 5)**

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