Pre-Leaving Certificate Examination, 2017

Triailscrúdú na hArdteistiméireachta, 2017

Mathematics

Paper 2

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
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5	
6	
7	
8	
9	
Total	

Running total	

Grade

Instructions

There are **two** sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1   **(25 marks)**

(a)   In a quiz, students are asked to solve two logical puzzles. 45% of the students solved
           problem 1. 25% of the students solved both problems. What is the probability that a student
           solved the second problem, given that the student had already solved the first problem?

(b)   The probability of a water pump being faulty is 0.04. A random sample of 12 pumps is
           selected off the production line. Calculate the probability that:

           (i)   only 1 pump is faulty, correct to two decimal places.

           (ii)   no more than 3 pumps are faulty, correct to two decimal places.

Question 2 **(25 marks)**

(a) If $\begin{pmatrix} 15 \\ r+3 \end{pmatrix} = \begin{pmatrix} 15 \\ 4r-13 \end{pmatrix}$, find r given that $r+3 \neq 4r-13$.
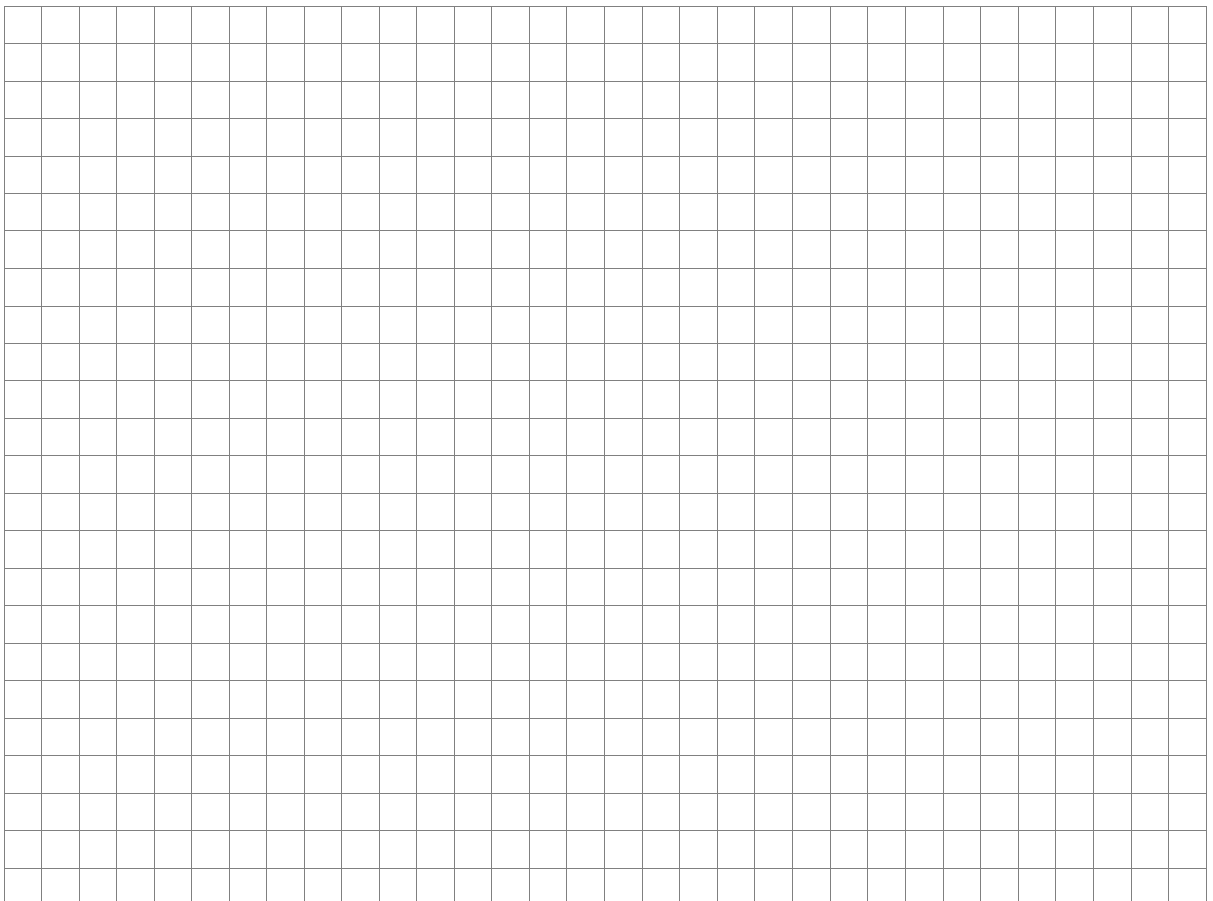
(b) A bag contains 5 yellow, 3 green, 6 blue and 2 black discs. If 4 discs are chosen at random, find the probability that:

(i) the discs are all the same colour.

(ii) the discs are all different colours.
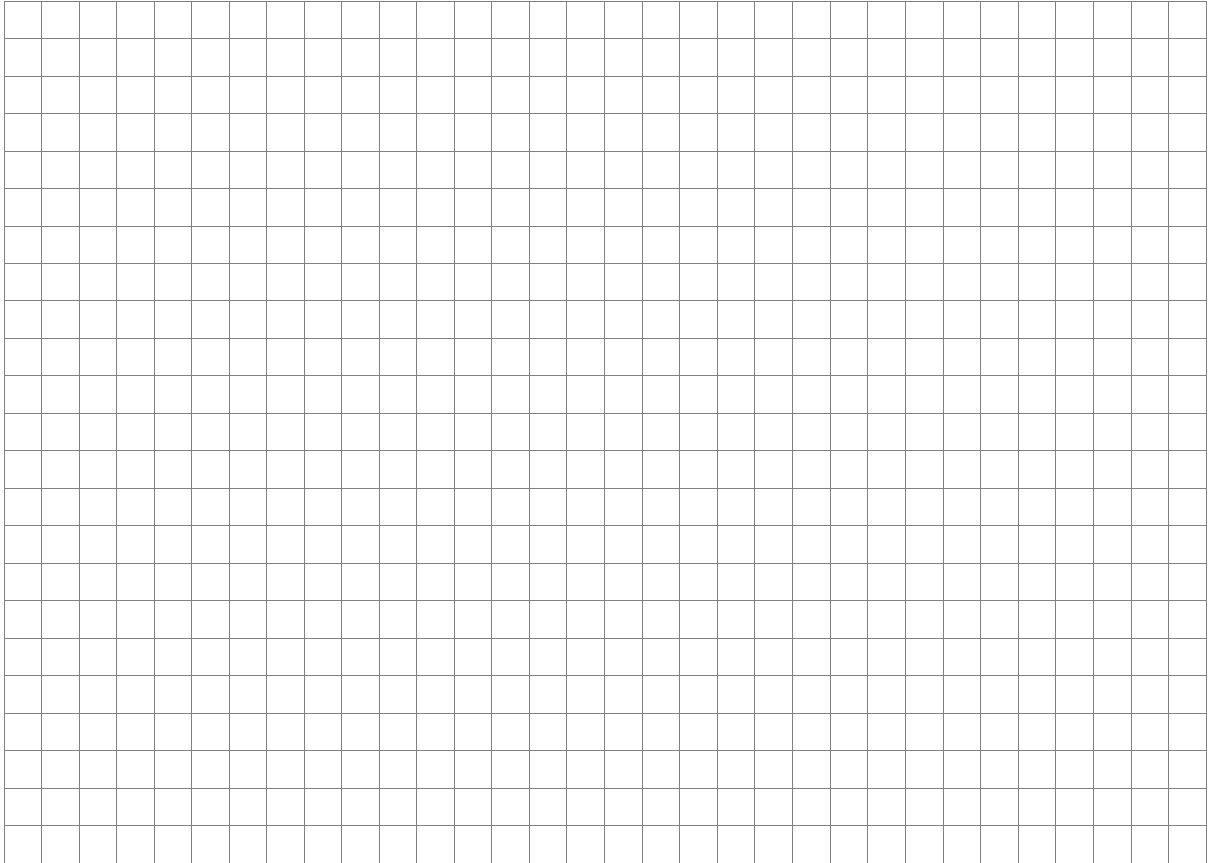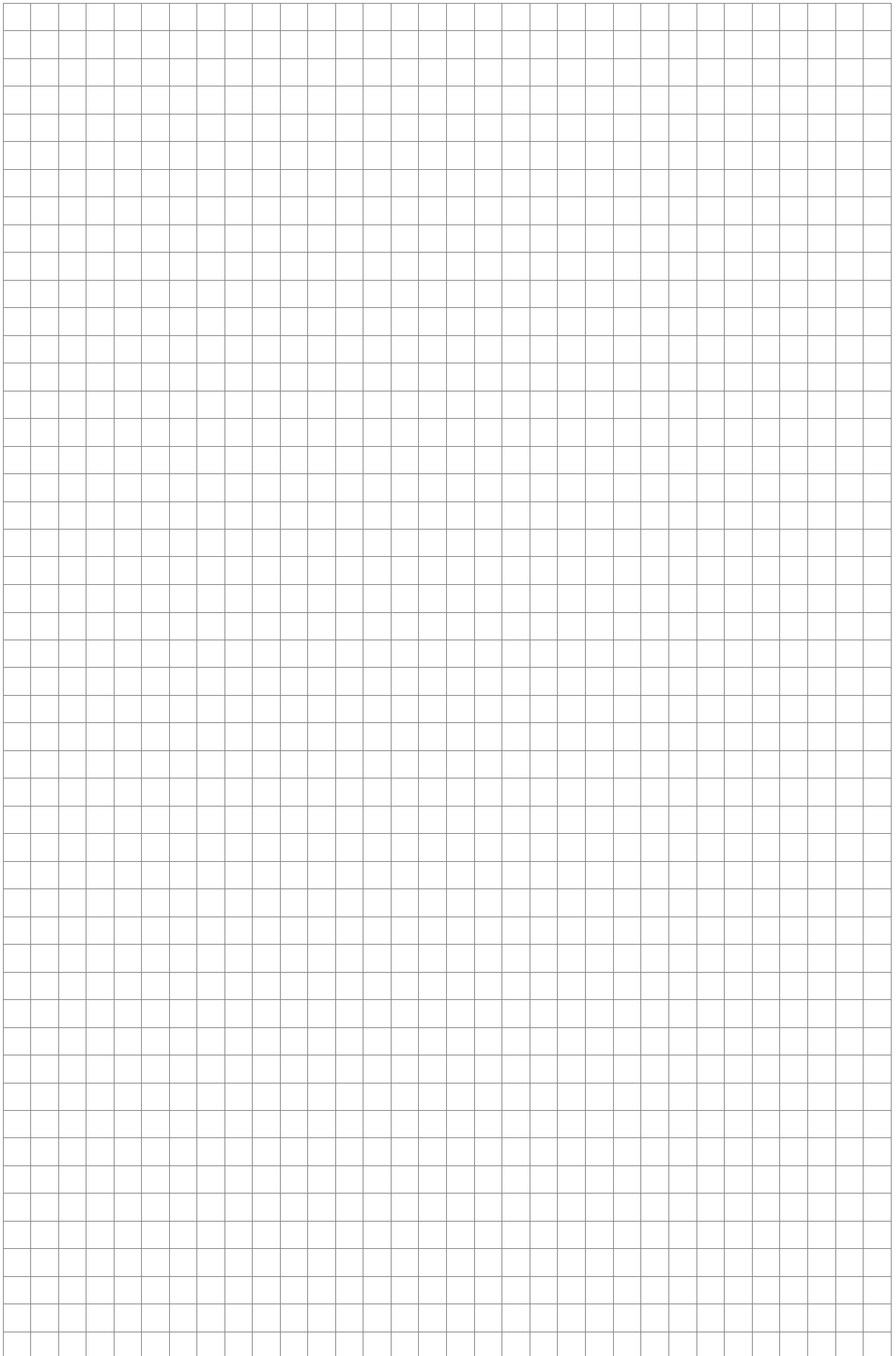
(iii) two of the discs are black.

Question 3 (25 marks)

A circle has equation $c : x^2 + y^2 - 10x - 8y = -21$.

(a) Write down the centre and radius of the circle.

(b) **(i)** Show that the equation of any line through the point $(3, -2)$ can be written as $mx - y - (3m + 2) = 0$.

(ii) Hence, or otherwise, find the equations of two tangents that can be drawn from the point $(3, -2)$ to the circle c.
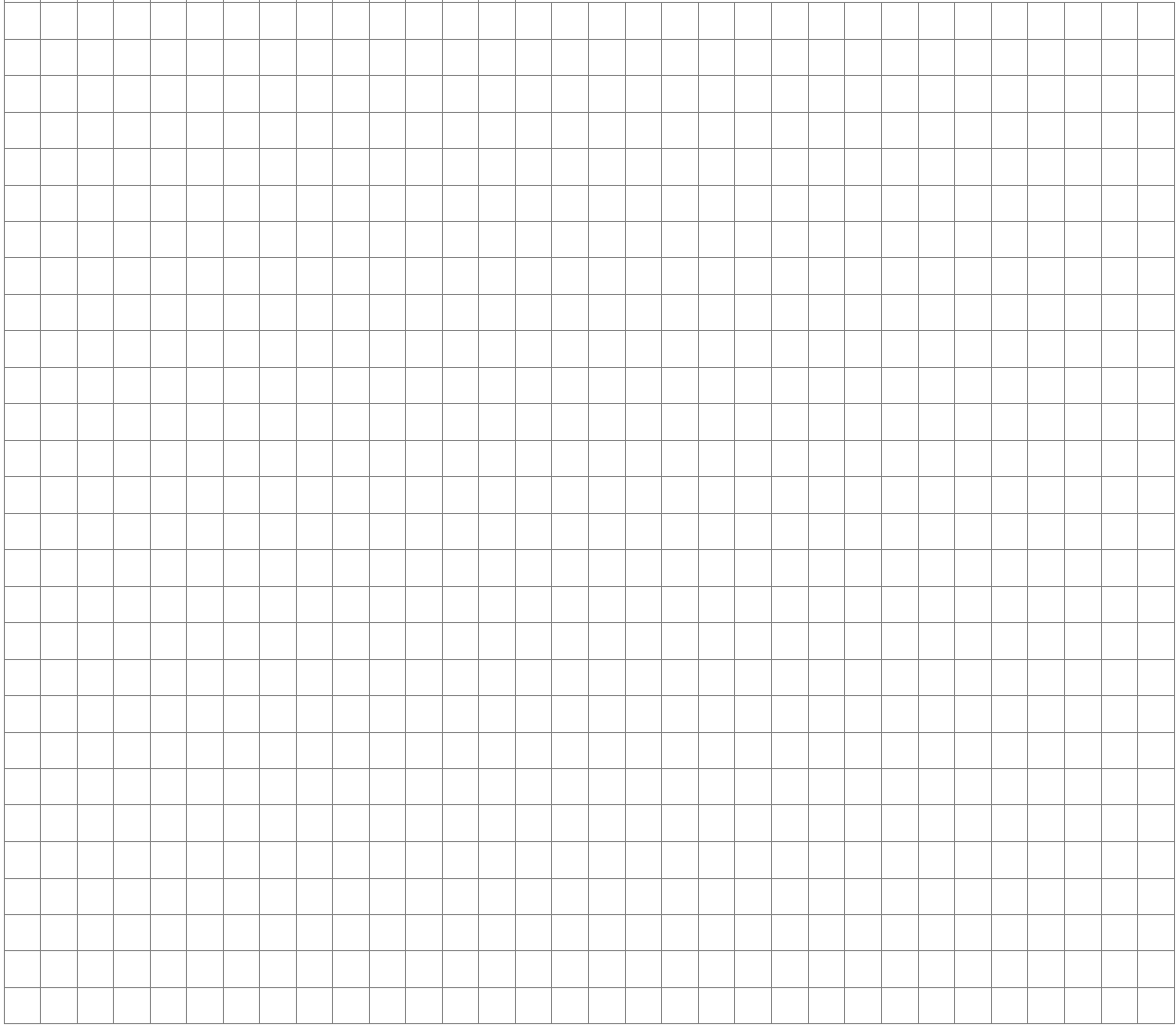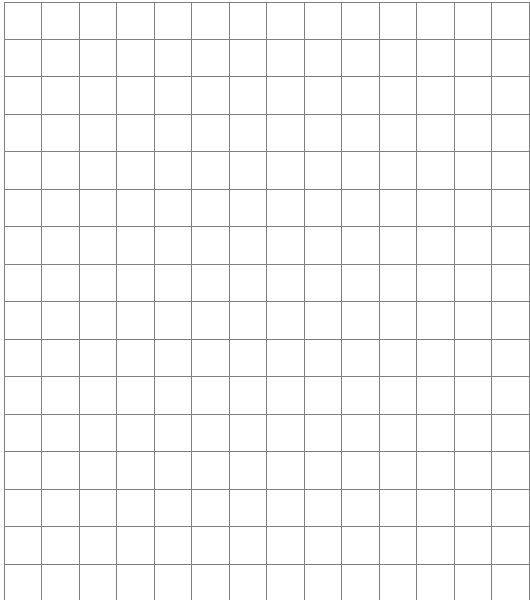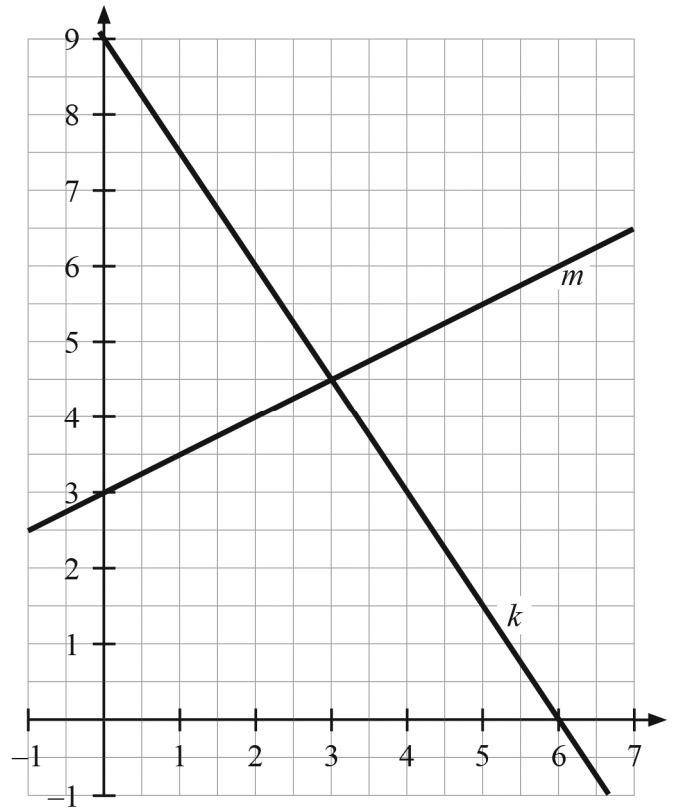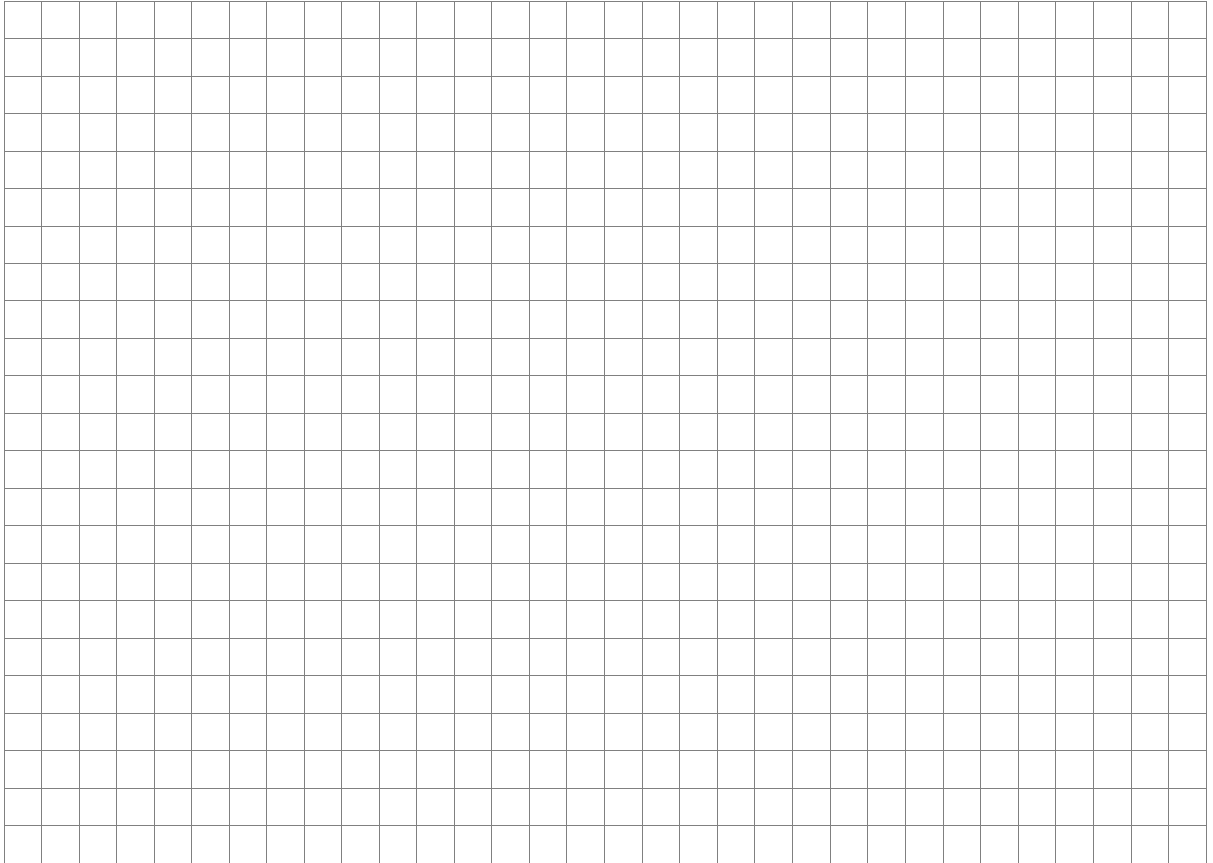
Question 4   (25 marks)

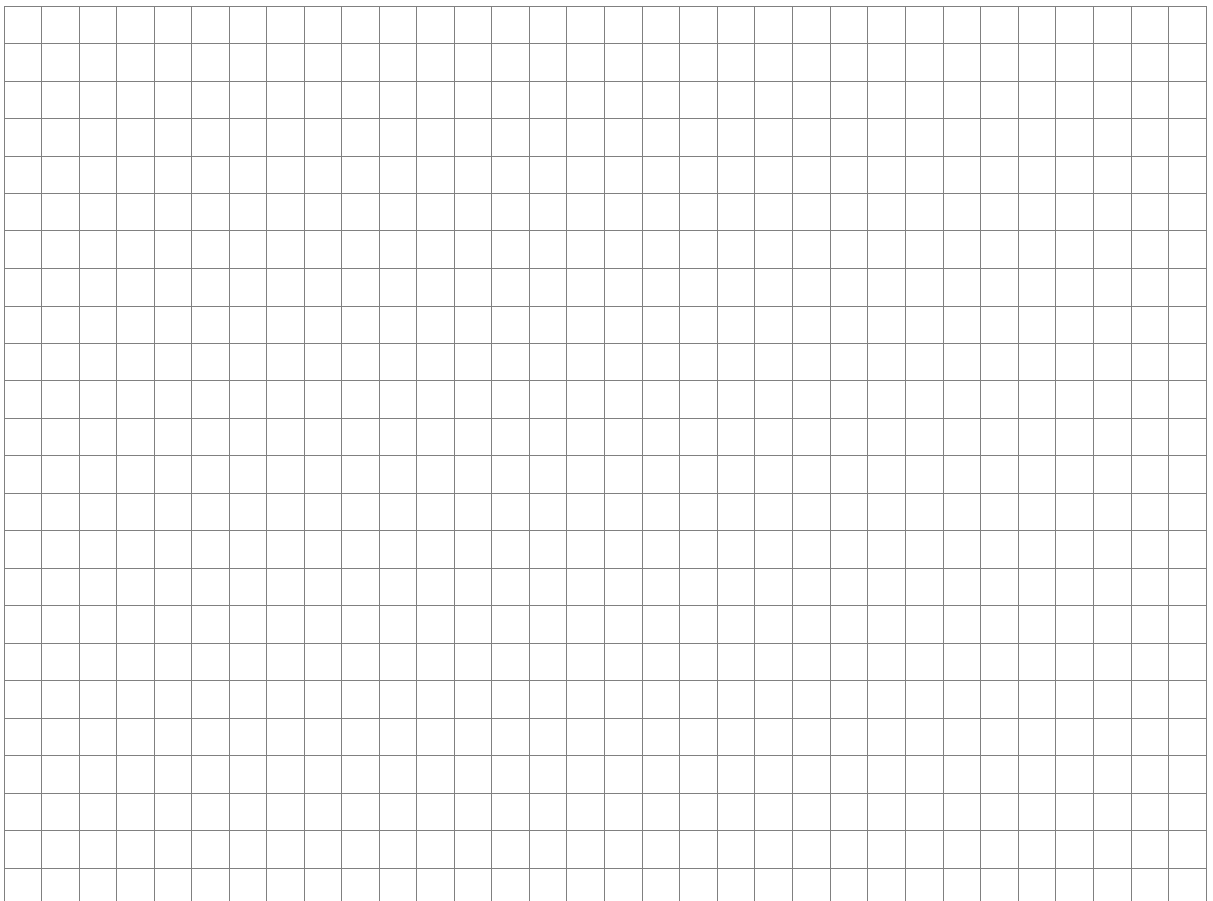The lines *m* and *k* are shown in the co-ordinate diagram.

(a)   **(i)**   Find the equations of the lines *m* and *k*.

(ii)   Find the area of the region bounded by the lines *m* and *k*, the *x*-axis and the *y*-axis.

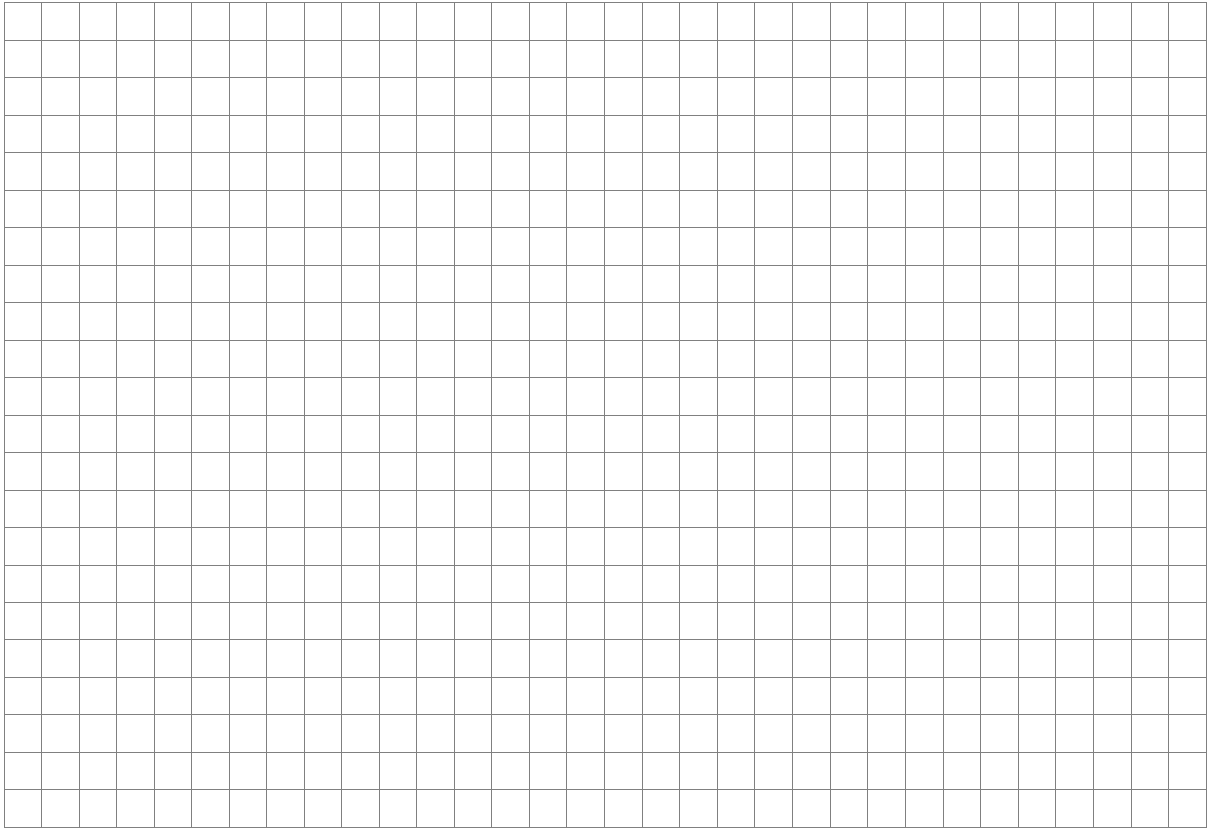**(b)**   Find the measure of the acute angle between the two lines, correct to the nearest degree.

Question 5 (25 marks)

(a) Find the values of θ given that $\tan 3\theta = 1$ for $0° \le \theta \le 360°$.

(b) β is an acute angle such that $\cos\beta = \dfrac{15}{17}$. Without evaluating β, find the value of $\sin 2\beta$.
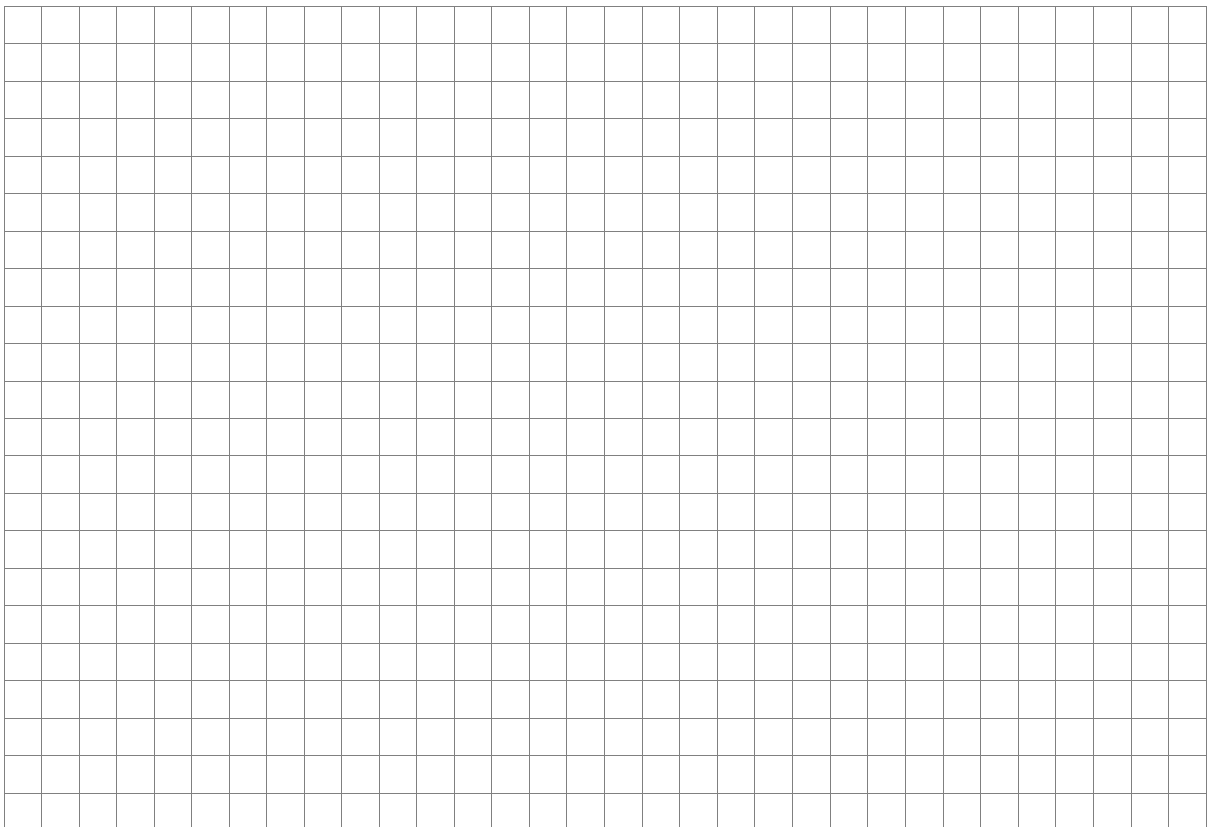
(c) **(i)** Prove that $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$.
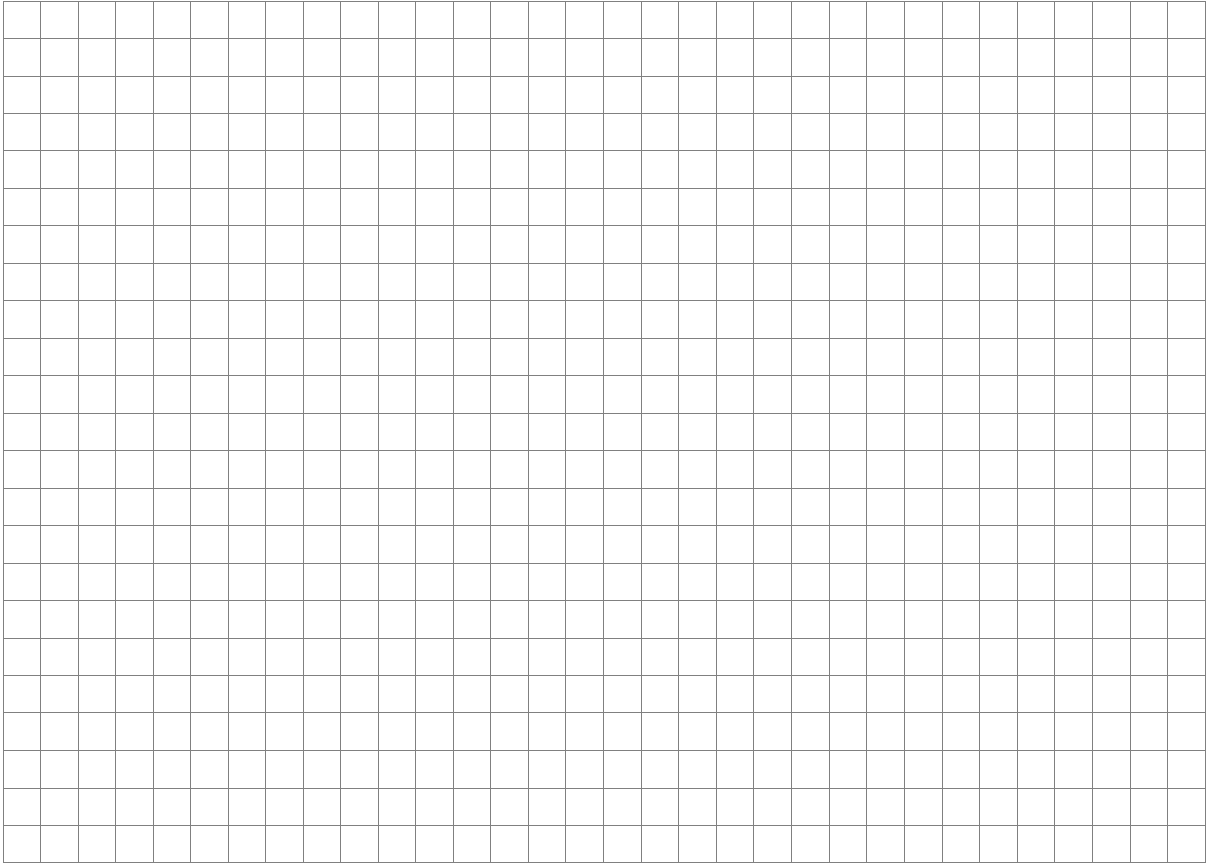
(ii) Hence, or otherwise, show that $\cos 2\theta = 2\cos^2 \theta - 1$.

Question 6 (25 marks)

The cross section of a drinking trough of is shown.

The trough has sides of fixed length l.

(a) Show that the vertical height of the trough can be expressed as $h = l\sin\theta$.

(b) (i) Show that the area of the trough can be expressed as $l^2 \sin\theta(1 + \cos\theta)$.
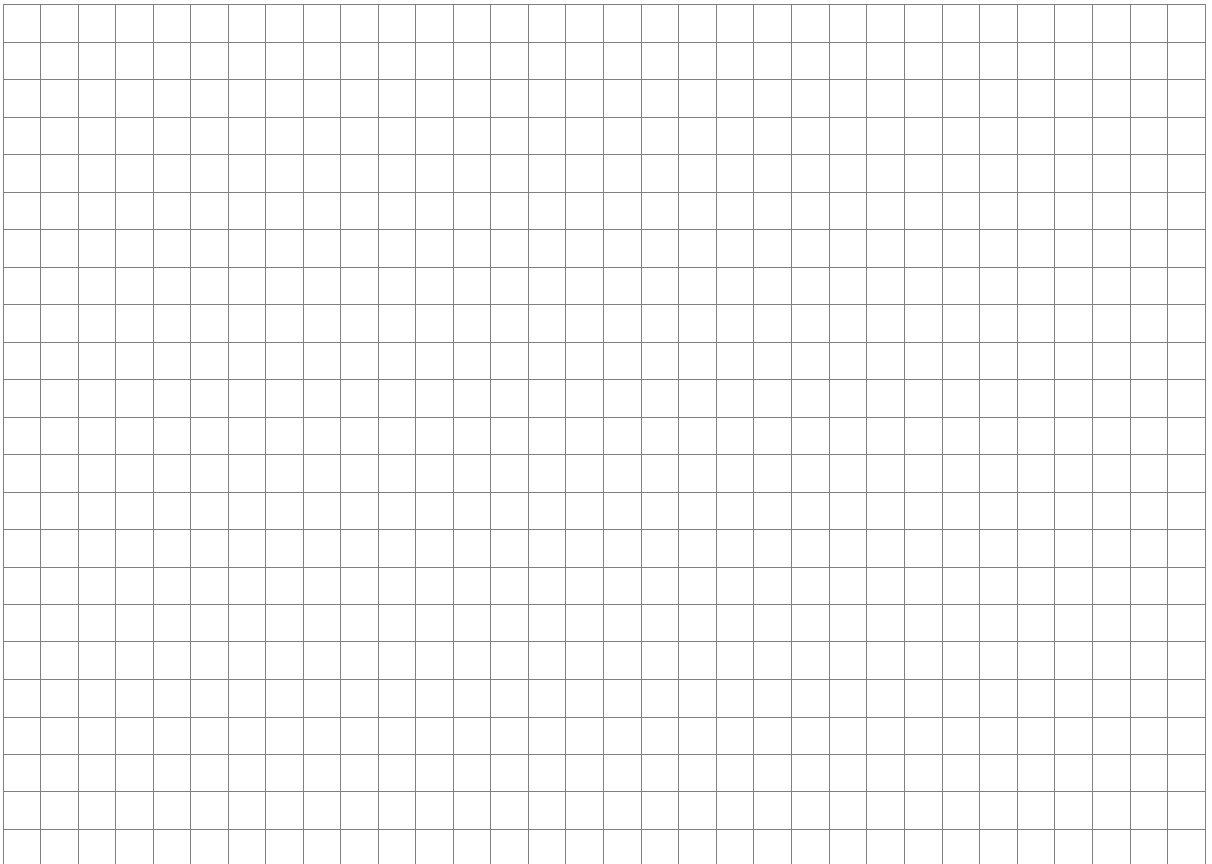
(ii) Hence, show that the area of the trough is maximised when $\theta = \dfrac{\pi}{3}$.

Answer **all three** questions from this section.

Question 7   **(60 marks)**

Oil is stored in a cylindrical shaped tank as shown, of radius R.

John wants to be able to calculate the volume of oil in his tank by dipping a stick vertically into the tank and getting a reading of the depth of the oil.
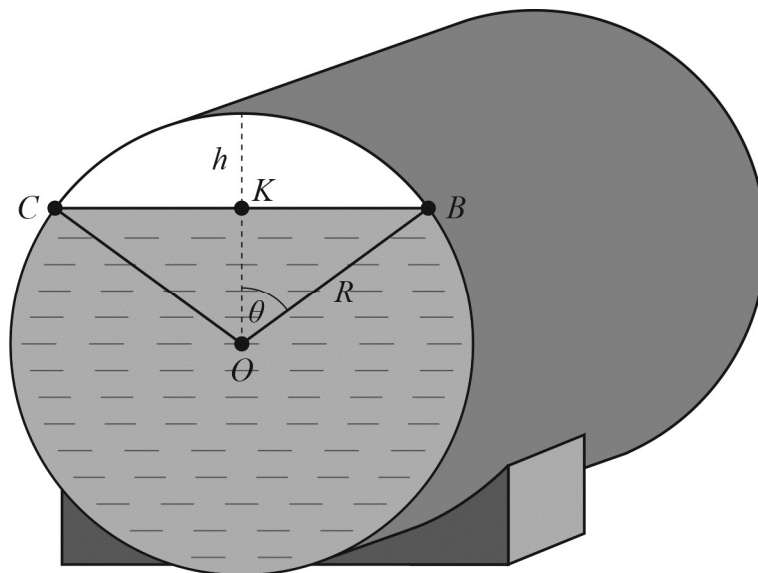
(a)   Show that $|OK| = (R - h)$.

Explain your answer.
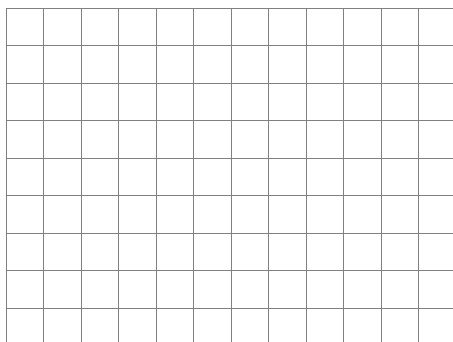
(b)   Show that the area of the triangle OBK can be written as Area $= \dfrac{1}{2}(R-h)\sqrt{2Rh - h^2}$.
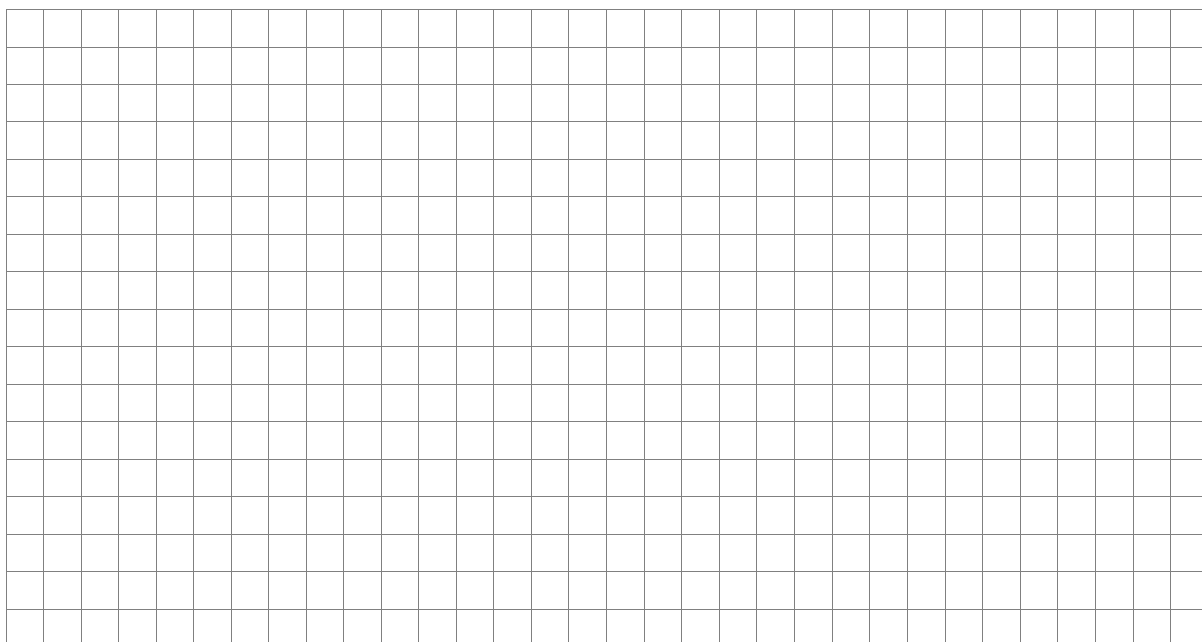
(c) Show that the area of the sector OCB can be expressed as $\theta = R^2 \cos^{-1}\left(\dfrac{R-h}{R}\right)$ where the angle θ is in radians.
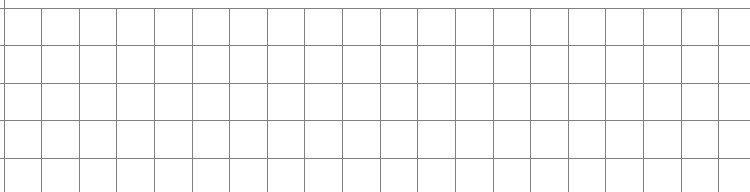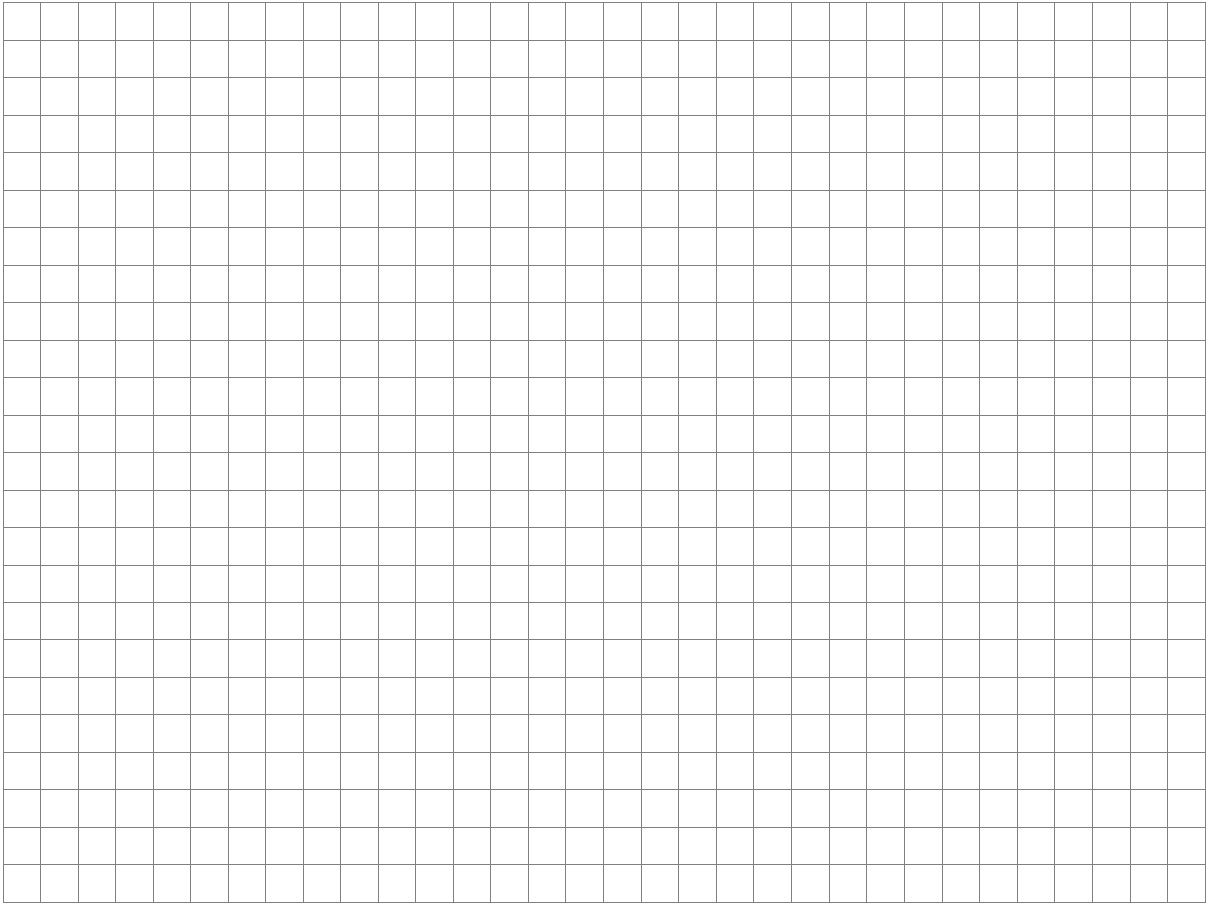
(d) Hence, show the volume of liquid in the tank of length l, is given by

$$\text{Volume} = l\left(\pi R^2 + (R-h)\sqrt{2Rh - h^2} - R^2 \cos^{-1}\left(\dfrac{R-h}{R}\right)\right).$$
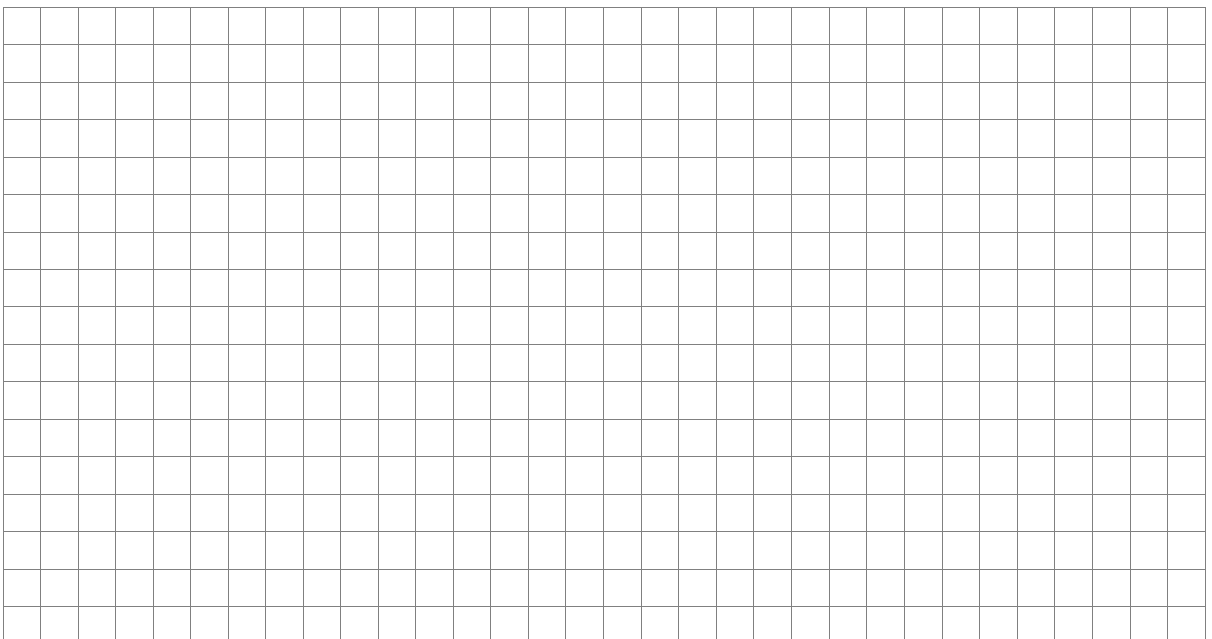
(e)   A tank has radius 0.75 m and length 2.5 m.
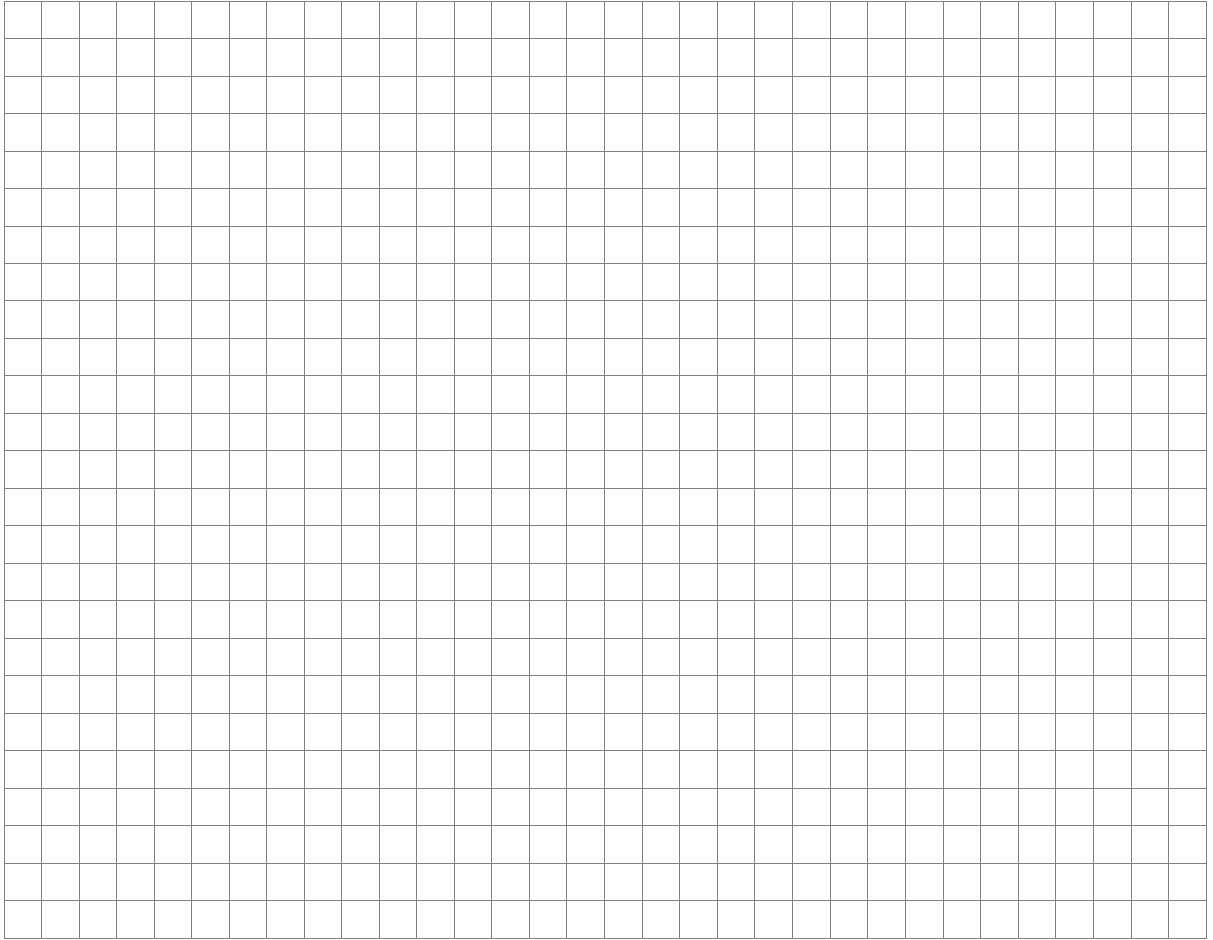
(i)   Calculate the volume of oil in the tank when full in terms of π.

(ii)   On a certain day John dips the tank. The depth of oil in the tank is 0.9 m from the bottom of the tank. Calculate the volume of oil in the tank, correct to two decimal places.
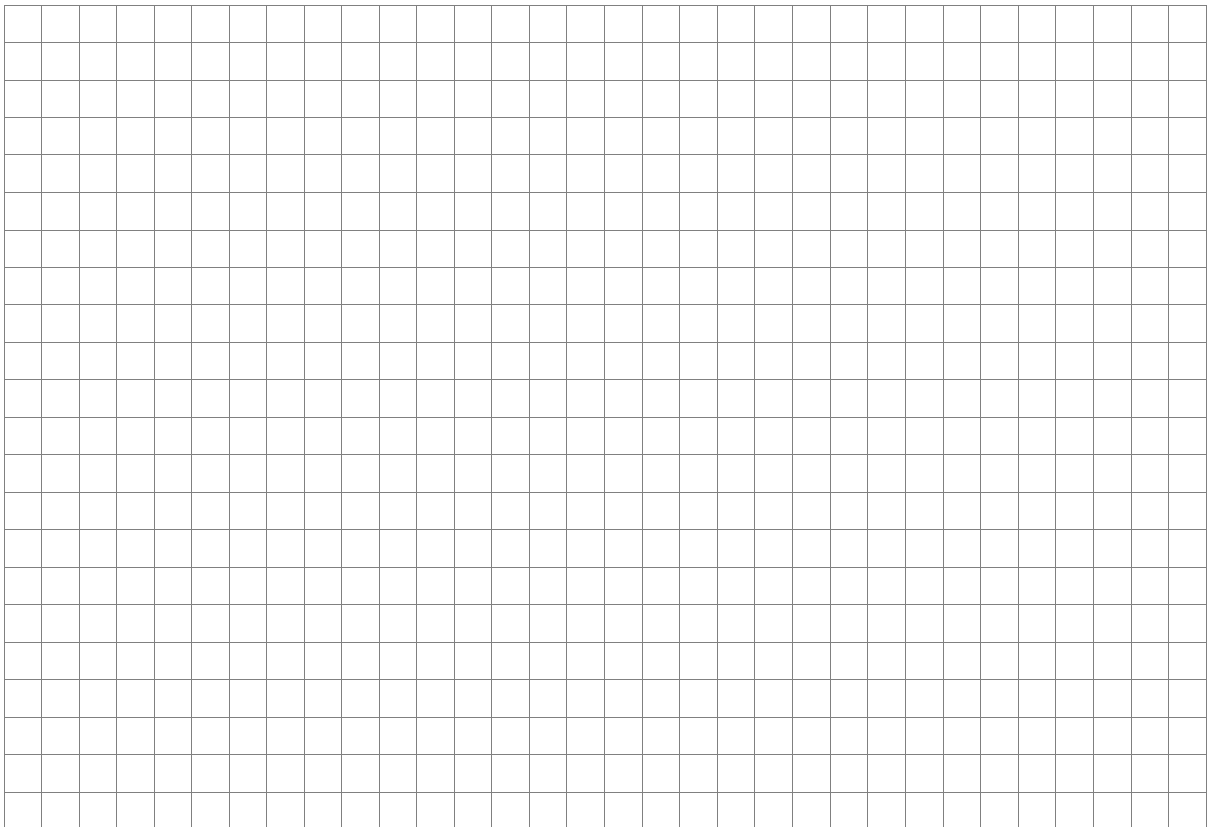
(f) Another cylindrical tank has length 8 m. On a certain day the volume of oil in this tank is 99.5 m³. The oil is $\dfrac{R}{2}$ m from the top of the tank. Calculate the radius of the tank, correct to one decimal place.

Question 8 (55 marks)

The following table shows the average rent for a two bedroom apartment in areas of Dublin in 2015.

Area	€	Area	€
Balbriggan	821.01	I.F.S.C.	1533.78
Blackrock	1420.73	Parnell Street	1317.00
Booterstown	1468.77	Spencer Dock	1629.02
Citywest	1077.95	Smithfield	1343.24
Dalkey	1521.48	Grand Canal Dock	2002.70
Donabate	973.77	Grand Canal Square	2098.65
Dun Laoghaire	1423.06	Hanover Quay	2091.94
Howth	1382.18	Pearse Street	1449.12
Killiney	1704.34	Temple Bar	1416.25
Lucan	1047.33	Townsend Street	1578.70
Malahide	1333.39	Christchurch	1231.25
Monkstown	1230.47	Clontarf	1284.93
Portmarnock	1201.19	East Wall	1214.18
Rathcoole	951.06	Fairview	1192.35
Rush	921.54	Rathmines	1312.74
Saggart	1049.23	Ballsbridge	1723.31

(Source: Central Statistic Office www.cso.ie)

(a) Daft.ie wish to carry out a similar study in Cork city. Suggest what sampling method they may use to collect their data. Explain your answer.

(b) **(i)** Calculate the mean for the rents shown for 2015.

(ii) Calculate the standard deviation for the rents in 2015.

(c) Previous years' statistics have shown that the rents across Dublin are normally distributed. From your calculations in parts **(b)(i)** and **(b)(ii)**, what is the range of rents that you would expect to find 95% of the 2 bedroom apartments in these areas to lie between?

The rents for the same areas in Dublin for 2014 are shown below.

Area	€	Area	€
Balbriggan	754.23	I.F.S.C.	1380.75
Blackrock	1332.63	Parnell Street	1211.54
Booterstown	1339.26	Spencer Dock	1566.53
Citywest	984.69	Smithfield	1228.95
Dalkey	1419.21	Grand Canal Dock	1835.69
Donabate	909.64	Grand Canal Square	2008.44
Dun Laoghaire	1290.62	Hanover Quay	1955.71
Howth	1270.19	Pearse Street	1278.59
Killiney	1531.06	Temple Bar	1307.45
Lucan	962.45	Townsend Street	1498.42
Malahide	1235.20	Christchurch	1155.00
Monkstown	1149.51	Clontarf	1190.27
Portmarnock	1090.34	East Wall	1101.94
Rathcoole	873.75	Fairview	1120.98
Rush	844.12	Rathmines	1235.38
Saggart	961.41	Ballsbridge	1586.50

(Source: Central Statistic Office www.cso.ie)

(d) Choose 6 areas from the tables and display the 2014 and 2015 rent. What observation can you make about the rents between the years? Comment on any trends you notice in the price of the rents.
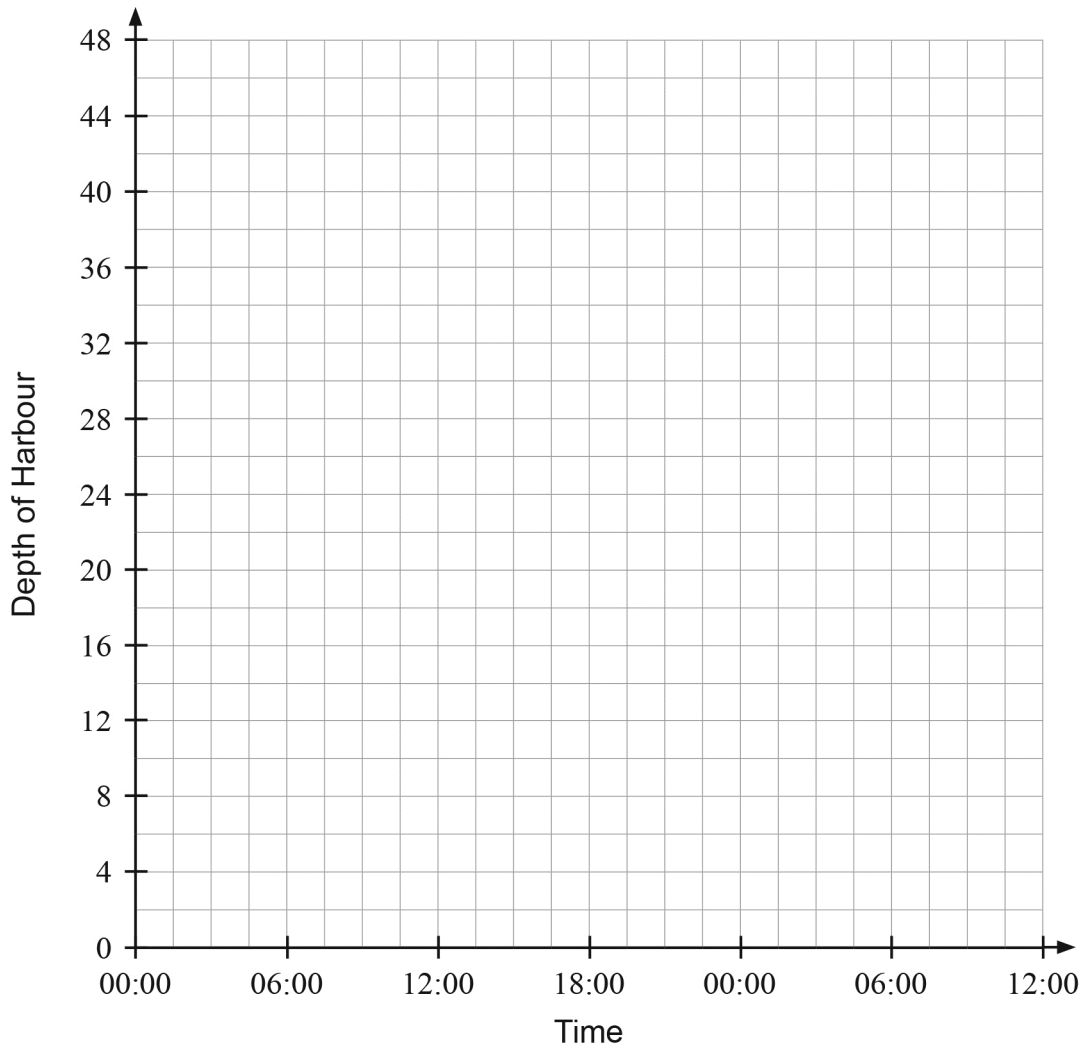
The rents for all areas for 2014 and 2015 are compared in the following graph.

(e) Does this graph support your claim in part **(d)**. Explain your answer.

It is claimed that the average rent for a two bedroom apartment in 2015 in Dublin was €994 per month. An estate agent conducted a survey of 50 such properties in the Dublin area. The sample had a mean of rent of €1050 and a standard deviation of €390.
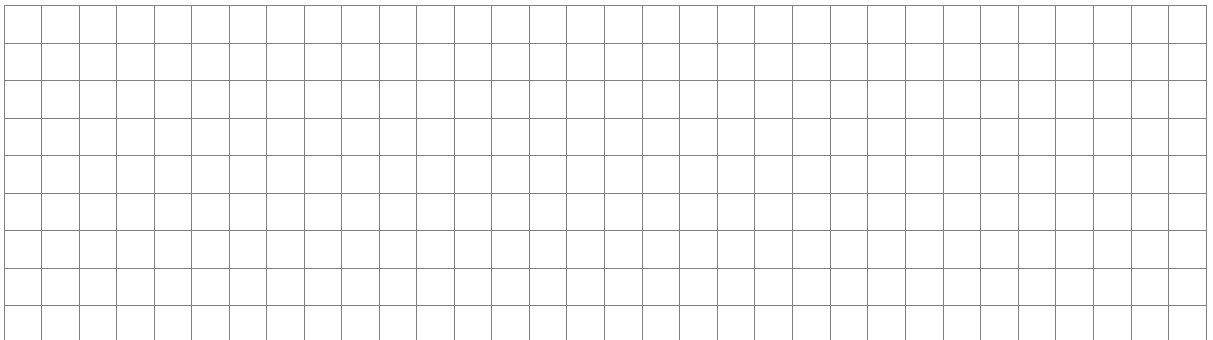
(f) Using the estate agent's sample, conduct a 95% confidence interval for the mean monthly rent of a two bedroom apartment in Dublin.

Question 9 **(35 marks)**

A certain model of a buoy is used to monitor the depth of water in a harbour.

At a certain time of year high tide occurs at midnight. Low tide occurs 12 hours later. During thecourse of a 24 hour period the buoy measures a range of depths between 21 m and 39 m. The tide can be modelled using a sinusoidal function.

(a) **(i)** Write down the equation for the central line of the function.

(ii) Write down the period of the tide.

(b) Explain why the tide can be modelled using the equation $f(t) = 30 + 9\cos\left(\dfrac{\pi}{12}t\right)$.

(c) Draw a graph of the function $f(t)$ over a 24 hour period.

Depth of Harbour

48

44

40

36

32

28

24

20

16

12

8

4

0

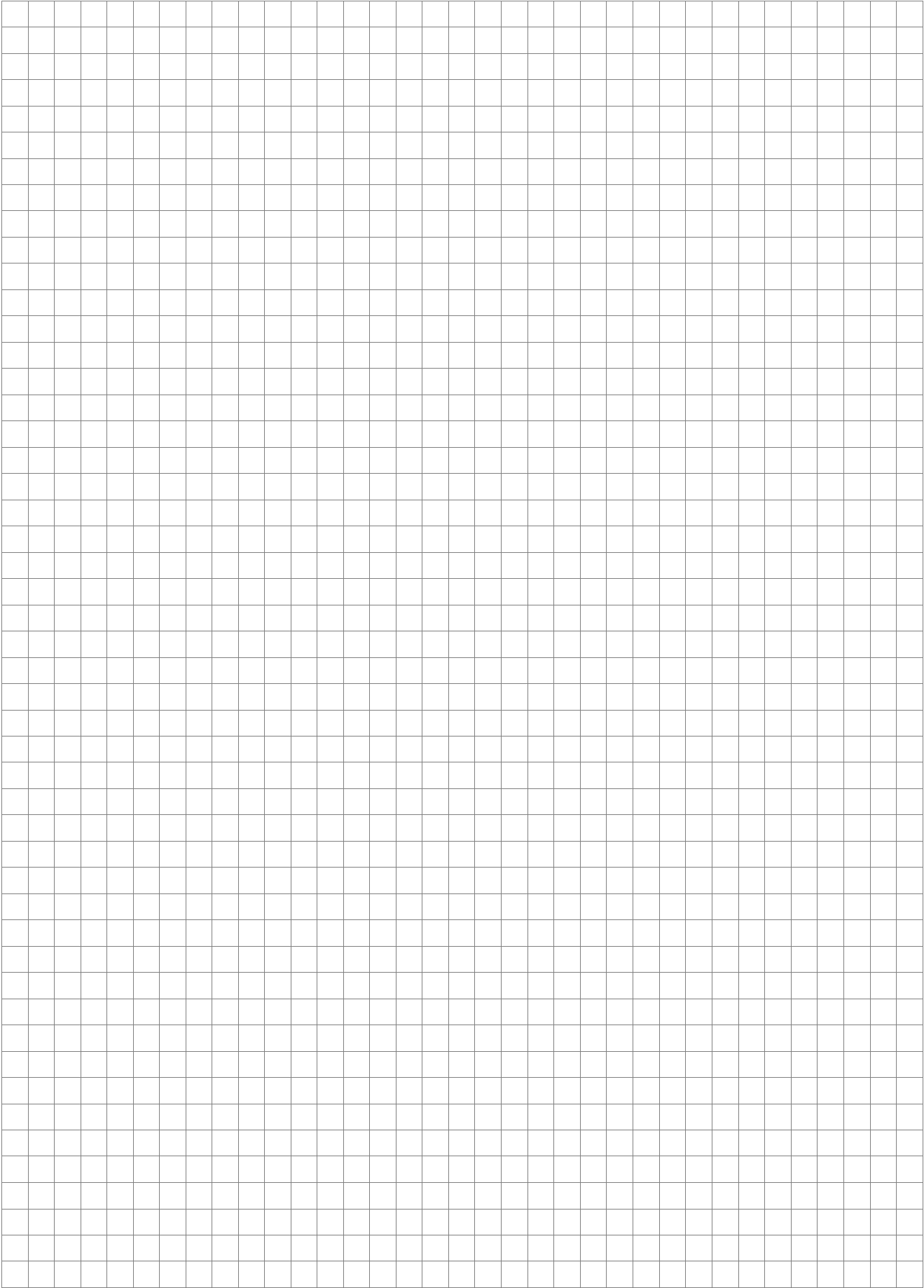00:00   06:00   12:00   18:00   00:00   06:00   12:00

Time

(d) At a different time of year low tide occurs at midnight. What affect will this have on the function $f(t)$?

You may use this page for extra work.

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