L.20

NAME
SCHOOL
TEACHER

Pre-Leaving Certificate Examination, 2018

Mathematics

Paper 2

Higher Level

Time: 2 hours, 30 minutes

300 marks

For
Que

Total

School stamp

Running total

Grade

Section A **Concepts and Skills** **150 marks**

Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) Orla and Liam play a game that consists of tossing an unbiased coin. The first person to get a 'heads' is the winner. If Orla tosses first, find the probability that:

(i) Liam wins the game on his first toss,

5

0 2 4 5

Orla lose & Liam Win

$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(ii) Orla wins the game on her second toss,

5

0 2 4 5

Orla lose & Liam lose & Orla win

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(iii) Orla wins the game.

5

(Orla wins 1^{st} toss) or (Orla wins 2^{nd} toss) or - - - -

$\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \text{———} \quad a = \frac{1}{2} \quad r = \frac{1}{4}$

$S_\infty = \frac{1/2}{1 - 1/4} = \frac{2}{3}$.

(b) A game of chance comprises a player spinning a 'lottery wheel'. There are 100 positions in which the ball has an equal chance of landing but there is only one chance for a player to win the top prize.
Find the minimum number of spins which Carina must attempt in order that the probability she wins the top prize at least once is no less than 25%.

10

0 4 6 8 10

1 − Don't win

win $= \frac{1}{100}$ Don't win $= \frac{99}{100}$ each time

$1 - \left(\frac{99}{100}\right)^n \geq 0.25$

$0.75 \geq (.99)^n$

$\ln .75 \geq n \ln .99$

$\frac{\ln .75}{\ln .99} \geq n$

$28.62 \geq n$

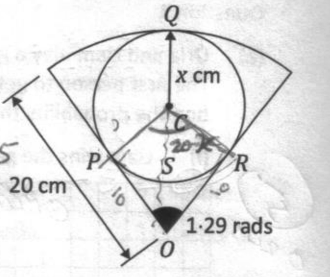
$29 > n$.

page

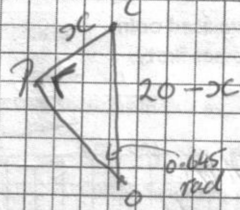
Question 2 **(25 marks)**

The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P, Q and R. $|\angle POR| = 1{\cdot}29$ radians.

(a) By considering the triangle POC, show that x is equal to $7{\cdot}5$ cm, correct to one decimal place.



(10)
0 4 6 8 10



$\theta = 1{\cdot}29 / 2 = 0{\cdot}645$

$\sin(0{\cdot}645) = \dfrac{x}{20 - x}$

$0{\cdot}6011984385(20 - x) = x$

$12{\cdot}02396877 - 0{\cdot}6011984385x = x$

$12{\cdot}02396877 = 1{\cdot}6011984385x$

$7{\cdot}5 = x$

(inside sector outside circle)

(b) Hence, find the sum of the ~~shaded~~ regions in the diagram, correct to three decimal places.

(5)
0 2 4 5

Area Sector − Area circle $\qquad A = \frac{1}{2}r^2\theta$

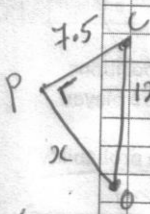
$\left[\frac{1}{2}(20)^2 1{\cdot}29\right] - \pi(7{\cdot}5)^2$

$= 81{\cdot}285 \text{ cm}^2$

(c) Find the perimeter of the region $PORS$ bounded by the arc PSR and the lines OP and OR. Give your answer correct to the nearest cm.

(10)
0 4 6 8 10

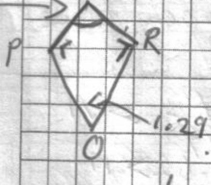
Perimeter = PO + OR + minor arc PR



$(12{\cdot}5)^2 = (7{\cdot}5)^2 + x^2 \qquad x = 10$

$|PO| = 10 \quad |OR| = 10$

1·8516



$90° = \frac{\pi}{2}$ radians. $\qquad \left(\frac{\pi}{2} + \frac{\pi}{2} + 1{\cdot}29\right) = 4{\cdot}4316$ rad

$360° = 2\pi \qquad 2\pi - 4{\cdot}4316 = 1{\cdot}8516$ rad.

$L = r\theta = 7{\cdot}5(1{\cdot}8516) = 13{\cdot}8869$

$13{\cdot}8869 + 10 + 10 = 33{\cdot}886 = 34$ cm

Question 3 **(25 marks)**

Two circles, k_1 and k_2, touch externally.

(a) The equation of the circle k_1 is $x^2 + y^2 - 6x + 2y - 15 = 0$.
Find the centre and radius of k_1.

5
0 2 4 5

$(3, -1)$ $\quad r = \sqrt{3^2 + (-1)^2 + 15} = 5$

(b) The centres of the two circles lie on the line $4x + 3y - 9 = 0$. The radius of circle k_2 is 10 units.
If the co-ordinates of the centre of circle k_2 are expressed in the form $(-g, -f)$, show that $(3+g)^2 + (f-1)^2 = 225$.

5
0 2 4 5

centre $(-g, -f)$ $\quad r = 10$. $\quad (-g, -f)$ on $4x + 3y - 9 = 0$

$-4g - 3f = 9$

$r_1 + r_2 = $ dis $(3, -1)$ to $(-g, -f)$

$(5+10)^2 = \left(\sqrt{(3+g)^2 + (-1+f)^2}\right)^2$

$225 = (3+g)^2 + (f-1)^2$

15
0 5 9 12 15

(c) Hence, or otherwise, find the possible equations of k_2.

(A) $(3+g)^2 + (f-1)^2 = 225$

(B) $-4g - 3f = 9$

$f = \dfrac{-4g - 9}{3}$

Sub into (A) $\quad (3+g)^2 + \left(\dfrac{-4g-9}{3} - 1\right)^2 = 225$

$9 + 6g + g^2 + \dfrac{16g^2 + 72g + 81}{9} - 2\left(\dfrac{-4g-9}{3}\right) + 1 = 225$ $\quad (\times 9)$

$81 + 54g + 9g^2 + 16g^2 + 72g + 81 + 24g + 54 + 9 = 2025$

$25g^2 + 150g - 1800 = 0$

$g^2 + 6g - 72 = 0$

$(g + 12)(g - 6) = 0$

$f = \dfrac{-4g-9}{3}$ $\quad g = -12 \quad g = 6$

$f = 13 \quad f = -11$

Centre $(12, -13)$ $\quad r = 10$

$(x - 12)^2 + (y + 13)^2 = 100$

Centre $(-6, 11)$ $\quad r = 10$

$(x + 6)^2 + (y - 11)^2 = 100$

previous	page	running

Question 4 (25 marks)

(a) Find the equation of the line l through the point $(-3, 2)$, which divides the line segment $(-6, 2)$ to $(-3, -4)$ internally in the ratio 1:2.

(10)
0 4 6 8 10

$x_1\ y_1 \quad x_2\ y_2 \quad a\ b$

$$\left(\frac{bx_1 + ax_2}{b+a}, \frac{by_1 + ay_2}{b+a}\right)$$

$$\left(\frac{2(-6)+1(-3)}{2+1}, \frac{2(2)+1(-4)}{2+1}\right)$$

$(-5, 0)$

$(-3, 2)\ (-5, 0)$

$m = \frac{0-2}{-5+3} = 1$

$y - 2 = 1(x+3)$

$y - x - 5 = 0$

(b) Find the co-ordinates of the points where l cuts the x-axis and the y-axis and hence, find the area of the triangle formed by l and the two axes.

(5)
0 2 4 5

cuts $x \Rightarrow y = 0$
$\therefore x = -5$
$(-5, 0)$

cuts $y \Rightarrow x = 0$
$y = 5$
$(0, 5)$

Area $= \frac{1}{2}\left|(-5)(5) - (0)(0)\right| = 25/2$ or $12 \cdot 5$ units^2

(c) A second line, $y = mx + c$, where m and c are positive constants, passes through $(-3, 2)$ and forms a triangle with the axes of equal area to that in part **(b)** above.
Find the equation of this line.

(10)
0 4 6 8 10

$y - 2 = m(x+3)$

$y - mx - 2 - 3m = 0$

cuts x at $y = 0$

$mx = -2 - 3m$

$x = \frac{-2-3m}{m}$

$\left(\frac{-2-3m}{m}, 0\right)$

cuts y at $x = 0$.

$y = 2 + 3m$

$(0, 2+3m)$

$\frac{25}{2} = \frac{1}{2}\left|\left(\frac{-2-3m}{m}\right)(2+3m) - (0)(0)\right|$

$25 = \left|\frac{-4+12m-9m^2}{m}\right|$

$\pm 25m = -4 - 12m - 9m^2$

$9m^2 + 37m + 4 = 0$

$(9m+1)(m+4) = 0$

$m = -1/9 \quad m = -4$

invalid as negative

$9m^2 - 13m + 4$
$(9m-4)(m-$
$m = 4/9 \quad m =$

$y - 4/9x - 2 - 3(4/9) = 0$

$9y - 4x - 30 = 0$

Question 5 **(25 marks)**

(a) A jury of 12 people is to be selected from a panel of 8 men and 8 women. 16

(i) In how many ways can the jury be selected?

8

0 2 4 5

$\binom{16}{12} = 1820$

(ii) Find the probability that the jury selected has more women than men.

10

0 4 6 8 10

7w & 5m
or
8w & 4m

$\rightarrow \binom{8}{7} \times \binom{8}{5} + \binom{8}{8} \times \binom{8}{4}$

$= 518$

$\text{Prob} = \frac{518}{1820} = \frac{37}{130}$

(b) A Maths teacher tells her class of 23 students that "There is a greater than 50% chance of two of more of them having the same birthday."
Do you agree with the teacher? Justify your answer by calculation. — 365 days in Yr.

10

0 4 6 8 10

?

$1 - \text{Prob (none with same b'day)}$

$1 - \left[\left(\frac{365}{365}\right)\left(\frac{364}{365}\right)\left(\frac{363}{365}\right) \cdots \left(\frac{343}{365}\right)\right]$

$1 - \left[\frac{(365)(364)(363)(362) \cdots (343)}{365^{23}}\right]$

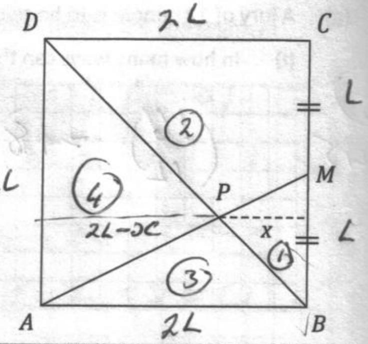
$1 - 0.492702\ldots$

$= 0.507279 = 50.73\%$ Teacher is correct.

previous	page	running

Question 6 **(25 marks)**

(a) The diagram shows a square $ABCD$ and the point M, the midpoint of $[BC]$. $[AM]$ and $[BD]$ intersect at the point P and divide the square into four regions. x is the perpendicular height of the triangle BMP.



(i) Let $|AB| = 2l$.
Find an equation for the sum of the areas of the four regions, in terms of x and l, and hence, show that $x = \dfrac{2l}{3}$.

10
0 4 6 8 10

Area $\Delta = \frac{1}{2}$ base $\times$ height.

Area 1 $= \frac{1}{2} L x = \frac{Lx}{2}$

Area 4 $= \frac{1}{2}(2L)(2l-x) = 2l^2 - Lx$

Area 2 $=$ Area $\Delta DCB - \Delta PMB =$
$= \frac{1}{2}(2L)(2L) - \frac{Lx}{2} = 2L^2 - \frac{Lx}{2}$

Area 3 $=$ Area $\Delta AMB - \Delta PMB$
$= \frac{1}{2}(2L)(L) - \frac{Lx}{2} = L^2 - \frac{Lx}{2}$

Total Area $= \frac{Lx}{2} + 2l^2 - Lx + 2l^2 - \frac{Lx}{2} + l^2 - \frac{Lx}{2}$
$= 5l^2 - \frac{3Lx}{2}$

$5l^2 - \frac{3Lx}{2} = 4l^2$

$10l^2 - 3lx = 8l^2$

$-3lx = -2l^2$

$x = \frac{2l^2}{3l}$

$x = \frac{2l}{3}$

5
0 2 4 5

(ii) Hence, find the ratio of the areas of the four regions.

$\frac{Lx}{2} : 2l^2 - Lx : 2l^2 - \frac{Lx}{2} : l^2 - \frac{Lx}{2}$

$\frac{l(2l/3)}{2} : 2l^2 - l\left(\frac{2l}{3}\right) : 2l^2 - \frac{l(2l/3)}{2} : l^2 - \frac{l(2l/3)}{2}$

$\frac{2l^2}{3} \times \frac{1}{2} : 2l^2 - \frac{2l^2}{3} : \frac{12l^2 - 2l^2}{6} : \frac{6l^2 - 2l^2}{6}$

$\frac{2l^2}{6} : \frac{6l^2 - 2l^2}{3} : \frac{10l^2}{6} : \frac{4l^2}{6}$

$\frac{1}{3}l^2 : \frac{4l^2}{3} : \frac{5l^2}{3} : \frac{2l^2}{3}$

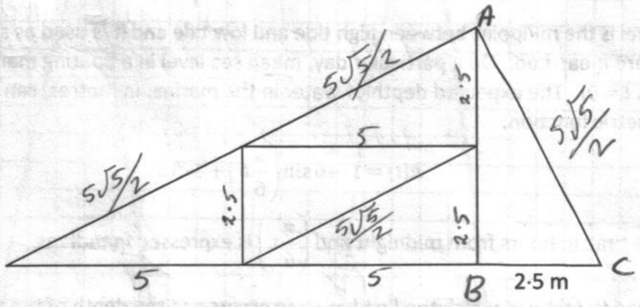
$\Rightarrow 1 : 4 : 5 : 2$

(10)

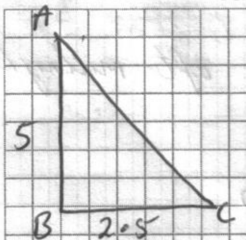
0 4 6 8 10

(b) The diagram shows the support framework for the roof configuration of a new house. The design is based on a larger triangle which is subdivided into five identical triangles that are similar to the larger triangle.

Each of the frameworks, or *roof trusses*, is constructed using timber. The quantity surveyor for the project needs to determine the total length of timber required for each truss.



Given that the shortest side of each of the smaller triangles in the design is 2·5 m, find the total length of timber required to make each truss.
Give your answer in the form $a + b\sqrt{c}$, where a, b and $c \in \mathbb{N}$.



$$x^2 = (5)^2 + (2·5)^2$$

$$x = \sqrt{5^2 + (2·5)^2} = \frac{5\sqrt{5}}{2}$$

Total timber

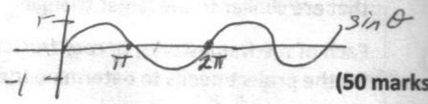
$$5 + 5 + 2·5 + \frac{5\sqrt{5}}{2} + \frac{5\sqrt{5}}{2} + \frac{5\sqrt{5}}{2} + 2·5 + 5 + \frac{5\sqrt{5}}{2} + 2·5 + 2·5$$

$$= 25 + 4\left(\frac{5\sqrt{5}}{2}\right) = 25 + 10\sqrt{5}$$

previous	page	running

Section B Contexts and Applications 150 marks

Answer **all three** questions from this section.



Question 7 **(50 marks)**

Mean sea level is the midpoint between high tide and low tide and it is used as a datum from which all altitudes are measured. On a particular day, mean sea level in a boating marina first occurs at midnight (*i.e.* $t = 0$). The expected depth of water in the marina, in metres, can be modelled using the trigonometric function:

$$h(t) = 1{\cdot}46\sin\left(\frac{\pi}{6}t\right) + 1{\cdot}56$$

where t is the time in hours from midnight and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

(i) Find the time at which the first high tide occurs and the depth of the water in the marina at that time.

10

0 4 7 10

Time:

$\Rightarrow$ 1 full period = 12hrs.

$\Rightarrow$ first high tide = 3hrs after midnight = 3am

Depth:

$1{\cdot}46 + 1{\cdot}56 = 3{\cdot}02$m

(ii) Find, by calculation, the period of $h(t)$.

5

0 2 5

$\dfrac{2\pi}{\pi/6} = 12$ hours.

(b) **(i)** Use the depth function, $h(t)$, to show the expected depth of water in the marina between midnight and the following midnight.

10

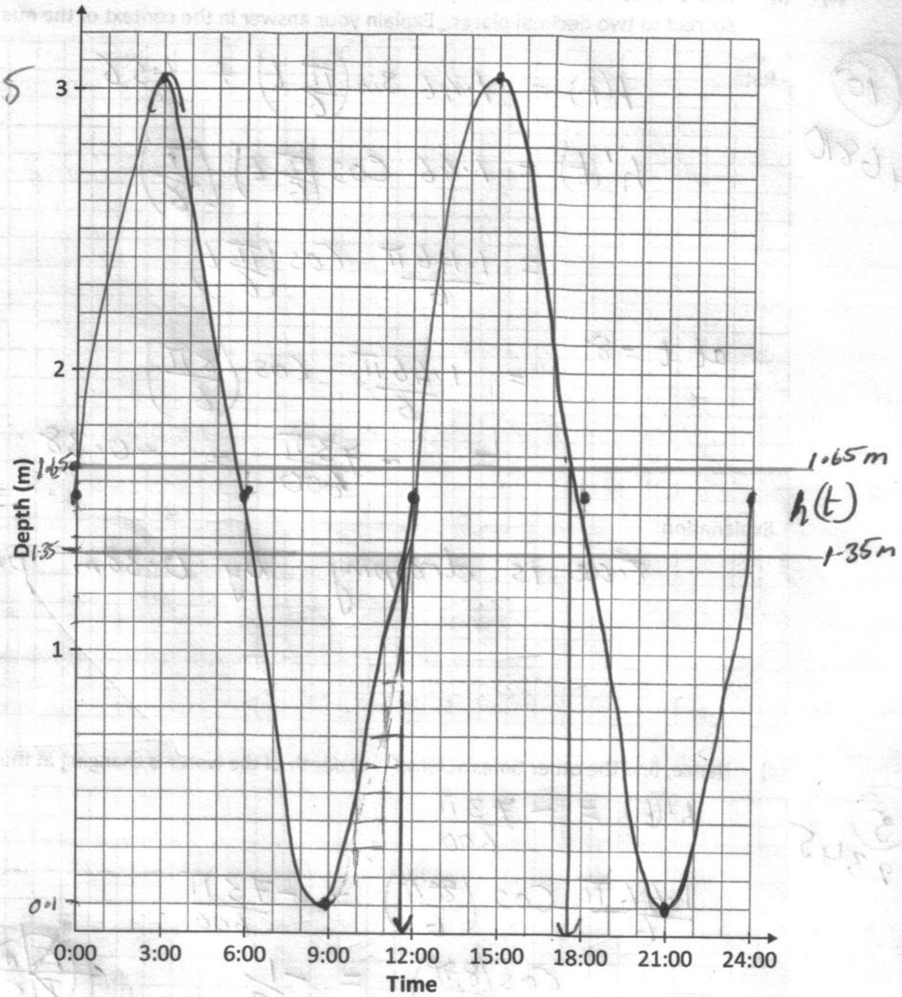
0 4 6 8 10

$$h(t) = 1{\cdot}46\sin\left(\frac{\pi}{6}t\right) + 1{\cdot}56$$

Time	0:00	3:00	6:00	9:00	12:00	15:00	18:00	21:00	00:00
t (hours)	0	3	6	9	12	15	18	21	24
$h(t)$ (m)	1·56	3·02	1·56	0·1	1·56	3·02	1·56	0·1	1·56

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.

(5)
0 2 4 5



(iii) A large cruiser wishes to enter the marina to refuel. The boat requires a minimum water level of 1·35 m. When it is fully fuelled, the boat requires at least 1·65 m. Use your graph to estimate the largest time interval for which the cruiser can enter the marina in order that it is not grounded on the sea-bed if refuelling takes 4·5 hours.

(5)
0 2 4 5

from 11:45 till 17:30.

⇒ 5hrs 45 mins.

previous	page	running

(c) (i) **Find the rate at which the depth of the water in the marina is changing at 8:00 a.m., correct to two decimal places. Explain your answer in the context of the question.**

10

0 4 6 8 10

Rate:

$h(t) = 1.46 \sin\left(\frac{\pi}{6}t\right) + 1.56$

$h'(t) = 1.46 \cos\left(\frac{\pi}{6}t\right)\left(\frac{\pi}{6}\right)$

$= \frac{1.46\pi}{6} \cos\left(\frac{\pi}{6}t\right)$

at $t = 8$ $= \frac{1.46\pi}{6} \cos\left(\frac{8\pi}{6}\right)$

$= -\frac{73\pi}{600} = -0.38$

Explanation:

tide is dropping by 0.38m per hr.

(ii) **Hence, find the other times at which the depth of the water is changing at the same rate.**

5

0 2 3 4 5

$h'(t) = -\frac{73\pi}{600}$

$\frac{1.46\pi}{6} \cos\left(\frac{\pi t}{6}\right) = -\frac{73\pi}{600}$

$\cos\left(\frac{\pi t}{6}\right) = -\frac{1}{2}$

ref angle $= \frac{\pi}{3}$

① $\frac{2\pi}{3}$ ② $\frac{4\pi}{3}$

$\frac{\pi t}{6} = \frac{2\pi}{3} + 2n\pi$ | $\frac{\pi t}{6} = \frac{4\pi}{3} + 2n\pi$

$t = 4 + 12n$ | $t = 8 + 12n$

$n=0$ $t = 4 = 4am$ | $t = 8 = 8\,am$

$n=1$ $t = 16 = 4\,pm$ | $t = 20 = 8\,pm$

$n=2$ $\cancel{t = 28}$ | $\cancel{t = 32}$

times are: 4 am, 8 am, 4pm & 8pm.

Question 8 **(50 marks)**

(a) A recent flood damaged a warehouse that contained 3000 computers. An insurance assessor, trying to estimate the damage, takes a random sample of 140 computers and finds that 34 of them are damaged.

10
0 4 6 8 10

(i) Create a 95% confidence interval for the proportion of computers that are damaged.

$\frac{34}{140} = 0.2428571 = \frac{17}{70}$

$\frac{17}{70} - 1.96\sqrt{\frac{\frac{17}{70}(1-\frac{17}{70})}{140}} < p < \frac{17}{70} + 1.96\sqrt{\frac{\frac{17}{70}(1-\frac{17}{70})}{140}}$

$0.1718 < p < 0.3138$

5
0 2 4 5

(ii) Find the 95% confidence interval for the number of computers that are damaged.

$0.1718(3000) = 515.4 \qquad 0.3138(3000) = 941.4$

$\Rightarrow$ between 516 ~~and~~ 942 computers damaged.

5
0 2 4 5

(iii) The assessor wishes to halve the margin of error in **part (i)** above. Assuming that the proportion of computers that are damaged remains unchanged, how many computers should he include in the random sample?

$E = \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{n}} = \frac{1}{2}\left(\frac{1}{\sqrt{140}}\right)$

$\frac{1}{\sqrt{n}} = 0.0422577127 4$

$\frac{1}{Ans} = \sqrt{n}$

$\left(\frac{1}{Ans}\right)^2 = n$

$560. = n$

previous	page	running

$$\text{Error} = 1.96\sqrt{\frac{17/40\,(1-17/40)}{140}}$$

$$= 0.08188803331$$

$$\tfrac{1}{2}(\text{Ans}) = 0.04094401666$$

$$1.96\sqrt{\frac{17/40\,(1-17/40)}{n}} = 0.04094401666$$

$$\sqrt{\frac{391/1600}{n}} = 0.02088980442$$

$$\frac{391/1600}{n} = \frac{391}{896000}$$

$$\frac{391}{1600} \times \frac{896000}{391} = n$$

$$560 = n$$

(b) Lactate dehydrogenase (LDH) is an enzyme found in nearly all living cells. Measuring LDH levels can be helpful in monitoring the effectiveness of certain medical treatments. For a particular group of patients in a research study, the distribution of LDH levels was normal with a mean of 210 and a standard deviation of 15.

10
0 4 6 8 10

(i) Find the proportion of patients in the study with LDH levels of between 200 and 240.

$\mu = 210$
$\sigma = 15$

$z = \dfrac{x - \mu}{\sigma}$ $\quad \dfrac{200 - 210}{15} = -0.67$

$\dfrac{240 - 210}{15} = 2$

$P(-.67 < z < 2)$ $\quad (1 - .7486) \leftarrow .2514$

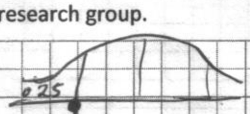
$0.9772 - .2514$

$= 0.7258.$

5
0 2 4 5

(ii) Reduced levels of LDH usually indicate that the medical treatments are working. Find the lower quartile of LDH levels for patients in this research group.

0.25



Tables $.75 \Rightarrow z = -0.67$

$-0.67 = \dfrac{x - 210}{15}$

$\underline{199.95} = x$

5
0 2 4 5

$\mu = 208$
$\sigma = 15$
$n = 100$

(iii) One month later, 100 of these patients were randomly selected and underwent further testing. It was found that their LDH levels were normally distributed with a mean of 208 and the same standard deviation.
Using the sample mean, find a 95% confidence interval for the mean LDH level in this group of patients.

$208 - 1.96\left(\dfrac{15}{\sqrt{100}}\right) < \mu < 208 + 1.96\left(\dfrac{15}{\sqrt{100}}\right)$

$205.06 < \mu < 210.94.$

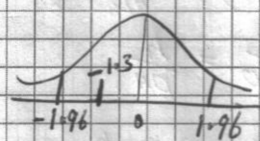
(iv) **Test the hypothesis, at the 5% level of significance, that the mean LDH level has not changed in this period of time.**
State clearly the null hypothesis and the alternative hypothesis.
Give your conclusion in the context of the question.

8
0 2 4 5

$H_0 : \mu = 210$

$H_1 : \mu \neq 210$

$z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \dfrac{208 - 210}{\frac{15}{\sqrt{100}}} = -\frac{4}{3} = -1.\dot{3}$



$\Rightarrow$ fail to reject H_0 and conclude that mean LDH level has not changed.

(v) **Find the *p*-value of the test you performed in part (iv) above and explain what this value represents in the context of the question.**

5
0 2 3 4 5

p-value:

$P(z < -1.33) = 1 - 0.9082 = 0.0918$

$2 \times (0.0918)$

$= 0.1836$

Since $0.1836 > 0.05$ fail to reject H_0.

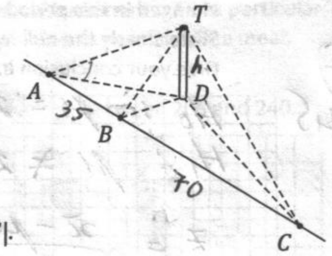
Explanation:

P value is the prob the test stastic could occur if ~~Null~~ H_0 is true. In this case P value is 18.36% which is more than a 5% chance $\therefore$ fail to reject H_0

previous	page	running

Question 9 **(50 marks)**

Three points, A, B and C, are on a horizontal roadway such that $|AB| = 35$ m and $|BC| = 70$ m. A vertical mobile phone mast $[DT]$ has its base, D, at the same level as the roadway. The angles of elevation from A, B and C to the top of the tower, T, are such that $\tan|\angle TAD| = \frac{3}{20}$, $\tan|\angle TBD| = \frac{1}{5}$ and $\tan|\angle TCD| = \frac{3}{13}$.



(10)
0 4 6 8 10

(a) **(i)** Let h be the height of the mobile phone mast, $|DT|$. Express $|DA|$, $|DB|$ and $|DC|$, in terms of h.

$|DA| = \frac{20h}{3}$

$\tan(TAD) = \frac{|TD|}{|AD|} = \frac{3}{20}$

$\frac{3}{20} = \frac{h}{DA}$

$|DB| = 5h$

$\tan(DBT) = \frac{h}{BD}$

$\frac{1}{5} = \frac{h}{BD}$

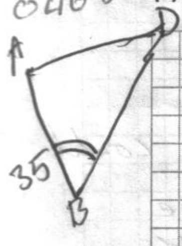
$|DC| = \frac{13h}{3}$

$\tan(DCT) = \frac{h}{DC}$

$\frac{3}{13} = \frac{h}{DC}$

(10)
0 4 6 8 10

(ii) Use the cosine rule to find $\cos|\angle ABD|$ in the form $\frac{a - h^2}{bh}$, where $a, b \in \mathbb{N}$.



$a^2 = b^2 + c^2 - 2bc\cos A$

$|AD|^2 = |AB|^2 + |BD|^2 - 2|AB||BD|\cos B$

$\left(\frac{20h}{3}\right)^2 = (35)^2 + (5h)^2 - 2(35)(5h)\cos B$

$\frac{400h^2}{9} = 1225 + 25h^2 - 350h\cos B \quad (\times 9)$

$400h^2 = 11025 + 225h^2 - 3150h\cos B$

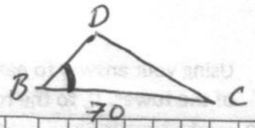
$175h^2 - 11025 = -3150h\cos B$

$(\div 175)$ $\frac{11025 - 175h^2}{3150h} = \cos B$

$\frac{63 - h^2}{18h} = \cos B$

5
0 2 3 4 5

(iii) **Similarly, show that $\cos|\angle DBC| = \dfrac{1575 + 2h^2}{225h}$.**



$|DC|^2 = |DB|^2 + |BC|^2 - 2|DB||BC| \cos B.$

$\left(\frac{13h}{3}\right)^2 = (5h)^2 + (70)^2 - 2(5h)(70)\cos B$

$(\times 9)$ $\frac{169h^2}{9} = 25h^2 + 4900 - 700h \cos B$

$169h^2 = 225h^2 + 44100 - 6300h \cos B$

$-56h^2 - 44100 = -6300h \cos B$

$(\div 28)$ $\frac{56h^2 + 44100}{6300h} = \cos B$

$\frac{2h^2 + 1575}{225h} = \cos B.$

5
0 2 4 5

(iv) **Show that $\cos(180 - \theta) = -\cos\theta$.**

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\cos(180 - \theta) = \cos 180 \cos\theta + \sin 180 \sin B$

$= (-1)\cos\theta + (0)\sin B$

$= -\cos\theta$

10
0 4 6 8 10

(v) **Hence, or otherwise, find the value of h.**

$|\angle ABD| + |\angle DBC| = 180^\circ$

$|\angle ABD| = 180 - |\angle DBC|$

$\cos(ABD) = \cos(180 - DBC)$

$\cos(ABD) = -\cos DBC$

$\frac{63 - h^2}{18h} = -\left(\frac{2h^2 + 1575}{225h}\right)$

$(63 - h^2)(225h) = -18h(2h^2 + 1575)$

$14175\cancel{h} - 225h^{\cancel{3}2} = -36h^{\cancel{3}2} - 28350\cancel{h}$ $(\div h)$

$42525 = 189h^2$

$225 = h^2$

$15\text{m} = h$

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10 **(b)** Using your answer to **part (a)(v)**, or otherwise, find the shortest distance from the foot of the tower, *D*, to the roadway *ABC*.

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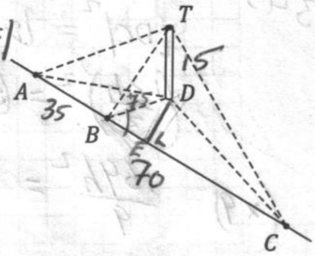
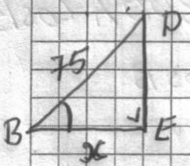


Shortest is the $\perp$ distance $|DE|$

$|BD| = 5h = 5(15) = 75$

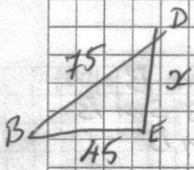
$\cos(DBC) = \dfrac{1575 + 2(15)^2}{225(15)} = \dfrac{3}{5}$



$\cos B = \dfrac{A}{H}$

$\dfrac{3}{5} = \dfrac{x}{75}$

$45 = x$



$75^2 = 45^2 + x^2$

$\sqrt{75^2 - 45^2} = x$

$60 = x.$