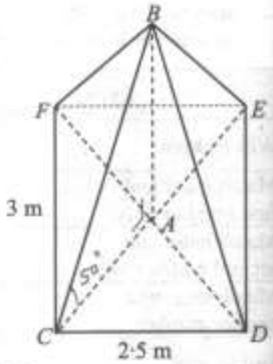
Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

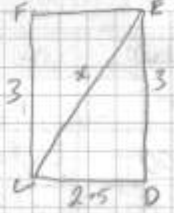
Question 7 (55 marks)

A glass Roof Lantern in the shape of a pyramid has a rectangular base *CDEF* and its apex is at *B* as shown. The vertical height of the pyramid is $|AB|$, where *A* is the point of intersection of the diagonals of the base as shown in the diagram.
Also $|CD| = 2\cdot5$ m and $|CF| = 3$ m.



10 0 3 7 10

(a) (i) Show that $|AC| = 1\cdot95$ m, correct to two decimal places.



$x^2 = 3^2 + 2\cdot5^2$

$x^2 = 15\cdot25$

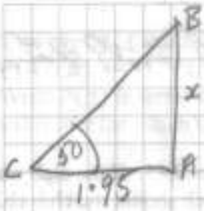
$x = \frac{\sqrt{61}}{2}$

$|AC| = \frac{\sqrt{61}}{2} \div 2$

$|AC| = 1\cdot95$ to 2d.p

10 0 5 10

(ii) The angle of elevation of *B* from *C* is 50° (i.e. $|\angle BCA| = 50°$).
Show that $|AB| = 2\cdot3$ m, correct to one decimal place.



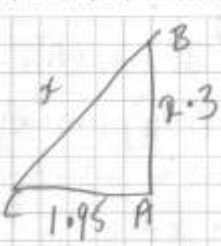
$\text{Tan } 50 = \frac{x}{1\cdot95}$

$1\cdot95 \text{ Tan}(50) = x$

$2\cdot3 = x$ to 1d p

10 0 3 7 10

(iii) Find $|BC|$, correct to the nearest metre.



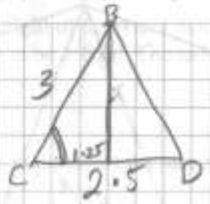
$x^2 = 2\cdot3^2 + 1\cdot95^2$

$x = \sqrt{(2\cdot3)^2 + (1\cdot95)^2}$

$x = 3$m to nearest m

(10) 0 3 7 10

(iv) Find $|\angle BCD|$, correct to the nearest degree.



$\cos C = \frac{1.25}{3}$

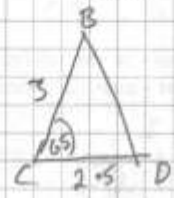
$C = \cos^{-1}\left(\frac{1.25}{3}\right)$

$C = 65°$ to nearest degree

(v) Find the area of glass required to glaze all four triangular sides of the pyramid. Give your answer correct to the nearest m^2.

(10) 0 3 5 8 10

$\text{Area} = \frac{1}{2}ab\sin C$

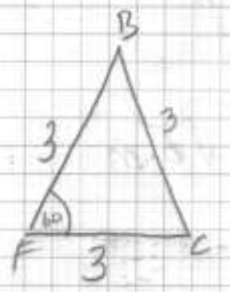


$\text{Area} = \frac{1}{2}(3)(2.5)\sin(65)$

$= 3.39865\ldots$

$\times\ 2$

$6.797\ m^2$



equilateral $\Delta \Rightarrow \angle BFC = 60°$

$\text{Area} = \frac{1}{2}(3)(3)\sin(60)$

$= \frac{9\sqrt{3}}{4}$

$\times\ 2$

$7.794\ldots m^2$

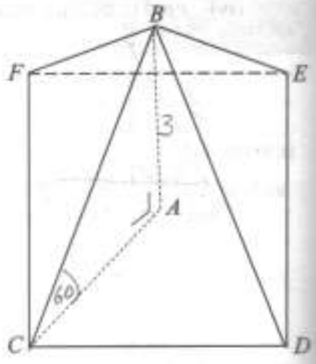
Total Area $= 6.797\ldots + 7.794\ldots$

$= 14.59$

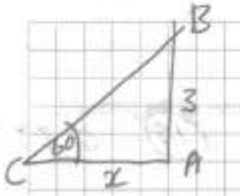
$= 15\ m^2$ to nearest m^2

Previous	page	running

(b) Another Roof Lantern, in the shape of a pyramid, has a square base $CDEF$. The vertical height $|AB| = 3$ m, where A is the point of intersection of the diagonals of the base as shown.
The angle of elevation of B from C is $60°$
(i.e. $|\angle BCA| = 60°$).
Find the length of the side of the square base of the lantern.
Give your answer in the form $\sqrt{a}$ m, where $a \in \mathbb{N}$.



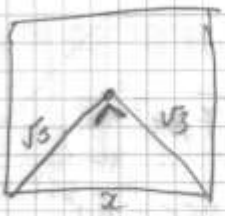
5 0, 2, 4, 5



$\tan(60) = \frac{3}{x}$

$x = \frac{3}{\tan(60)}$

$x = \sqrt{3}$

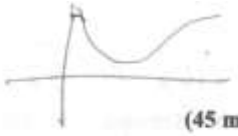


$x^2 = \sqrt{3}^2 + \sqrt{3}^2$

$x^2 = 6$

$x = \sqrt{6}$

side of square $= \sqrt{6}$ m.



Cos 7/4 θ

Question 8 **(45 marks)**

The height of the water in a port was measured over a period of time. The average height was found to be 1·6 m. The height measured in metres, $h(t)$, was modelled using the function

$$h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$$

where t represents the number of hours since the last recorded high tide and $\left(\frac{\pi}{6}t\right)$ is expressed in radians.

5 0, 2, 4, 5

(a) Find the period and range of $h(t)$.

Period: $\frac{2\pi}{\pi/6} = 12$ hrs

Range: $[0{\cdot}1, 3{\cdot}1]$ $[1{\cdot}6 - 1{\cdot}5,\ 1{\cdot}6 + 1{\cdot}5]$

(b) Find the maximum height of the water in the port. max $\Rightarrow dy/dx = 0$ 0, 2, 5 5

3·1 m

(c) Find the rate at which the height of the water is changing when $t = 2$, correct to two decimal places. Explain your answer in the context of the question.

0 2 4 5

5

Rate: $\left(\frac{dy}{dx}\right)$ $h(t) = 1{\cdot}6 + 1{\cdot}5\cos\left(\frac{\pi}{6}t\right)$

$h'(t) = 1{\cdot}5\left(-\sin\frac{\pi}{6}t\right)\left(\frac{\pi}{6}\right)$

at $t=2$ $-1{\cdot}5\sin\frac{2\pi}{6}\left(\frac{\pi}{6}\right)$

$= -0{\cdot}680 17\ldots$

$= 0{\cdot}68$ 2d.p

Explanation: tide is falling at a rate of 0·68 m per hour when $t = 2$

Previous	page	running

2016 SEC PAPER 2

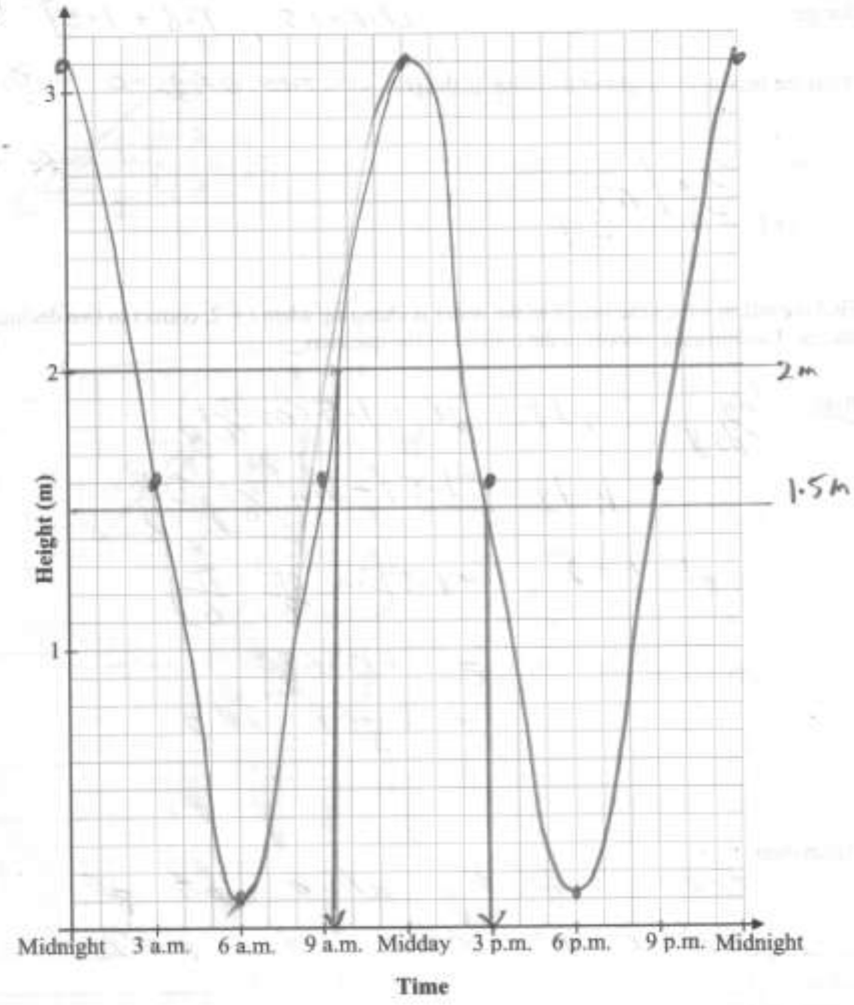
(10) 0 3710

(d) (i) On a particular day the high tide occurred at midnight (i.e. $t = 0$). Use the function to complete the table and show the height, $h(t)$, of the water between midnight and the following midnight.

$h(t) = 1\cdot6 + 1\cdot5\cos\left(\frac{\pi}{6}t\right)$									
Time	Midnight	3 a.m.	6 a.m.	9 a.m.	12 noon	3 p.m.	6 p.m.	9 p.m.	Midnight
t (hours)	0	3	6	9	12	15	18	21	24
$h(t)$ (m)	3·1	1·6	0·1	1·6	3·1	1·6	0·1	1·6	3·1

(10) 03710

(ii) Sketch the graph of $h(t)$ between midnight and the following midnight.



(5) 0 25

(e) Find, from your sketch, the difference in water height between low tide and high tide.

3m

$3 \cdot 1 - 0 \cdot 1 = 3m$

(f) A fully loaded barge enters the port, unloads its cargo and departs some time later.
The fully loaded barge requires a minimum water level of 2 m.
When the barge is unloaded it only requires 1·5 m.
Use your graph to estimate the **maximum** amount of time that the barge can spend in port, without resting on the sea-bed.

(5) 0, 2, 5

2016 SEC PAPER 2

9:30am → 3pm

$5\frac{1}{2}$ hours

$5\frac{1}{2}$ hrs

Previous	page	running