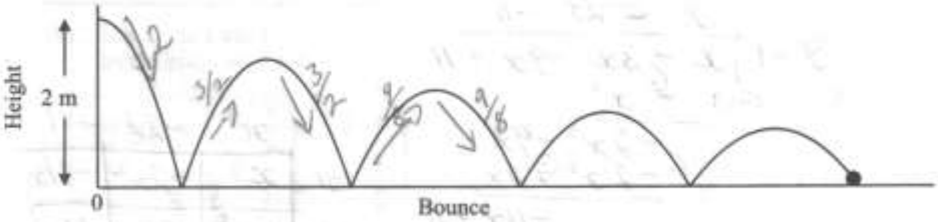
2015 SEC PAPER 1

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

Mary threw a ball onto level ground from a height of 2 m. Each time the ball hit the ground it bounced back up to $\frac{3}{4}$ of the height of the previous bounce, as shown.



(a) Complete the table below to show the maximum height, **in fraction form**, reached by the ball on each of the first four bounces.

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$	$3/2$	$9/8$	$27/32$	$81/128$

10

0 4 8 10

(b) Find, in metres, the total vertical distance (up and down) the ball had travelled when it hit the ground for the 5[th] time. Give your answer in fraction form.

$$2 + 2\left(\frac{3}{2}\right) + 2\left(\frac{9}{8}\right) + 2\left(\frac{27}{32}\right) + 2\left(\frac{81}{128}\right) = \frac{653}{64} \text{ m}$$

5

0 2 4 5

(c) If the ball were to continue to bounce indefinitely, find, in metres, the total vertical distance it would travel.

10

0 4 8 10

$$S_\infty: \quad 2 + 2(3/2) + 2(9/8) + 2(27/32) + \cdots$$

$$\cancel{2\left[1 + \frac{3}{4} + \frac{9}{8} + \frac{27}{32} + \cdots\right]}$$

$$2 + 2\left[\frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \cdots\right] \quad a = 3/2 \quad r = 3/4$$

$$2 + 2\left[\frac{3/2}{1 - 3/4}\right]$$

$$2 + 2(6) = 2 + 12 = 14 \text{ m}$$

Question 2 (25 marks)

Solve the equation $x^3 - 3x^2 - 9x + 11 = 0$.

Write any irrational solution in the form $a + b\sqrt{c}$, where $a, b, c \in \mathbb{Z}$.

$f(1)$ $(1)^3 - 3(1)^2 - 9(1) + 11 = 0$ ✓ $\Rightarrow (x-1)$ a factor

$$\begin{array}{r} x^2 - 2x - 11 \\ x - 1 \overline{\smash{)}\, x^3 - 3x^2 - 9x + 11} \\ -x^3 + x^2 \\ \hline -2x^2 - 9x \\ -2x^2 + 2x \\ \hline -11x + 11 \\ -11x + 11 \end{array}$$

	x^2	$-2x$	-11
x	x^3	$-2x^2$	$-11x$
-1	$-x^2$	$2x$	11

$x^2 - 2x - 11 = 0$

25

0, 5, 10, 15, 20, 25

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$$

$1 + 2\sqrt{3}$ $\quad$ $1 - 2\sqrt{3}$

~~Factors are~~ Roots are $x = 1$ $\quad x = 1 + 2\sqrt{3}$ $\quad x = 1 - 2\sqrt{3}$

mwujvc

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Question 5 **(25 marks)**

(a) Prove that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$

15D

0471115

$[\div \text{ all by } \cos A\cos B]$

$$= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$$

(b) Find all the values of x for which $\sin(3x) = \frac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

120 S | A 60
T | C

$\sin 3x = \sqrt{3}/2$ $\qquad 3x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60$

10D

025816

① $3x = 60 + 360n$ $\qquad$ ② $3x = 120 + 360n$

$x = 20 + 120n$ $\qquad$ $x = 40 + 120n$

$n=0$ $\quad x = 20$ $\qquad x = 40$

$n=1$ $\quad x = 20 + 120 = 140$ $\qquad x = 40 + 120 = 160$

$n=2$ $\quad x = 20 + 240 = 260$ $\qquad x = 40 + 240 = 280$

$n=3$ $\quad x = 20 + 360 = \cancel{380}$

$x = 20, 40, 140, 160, 260, 280.$

hnexrv

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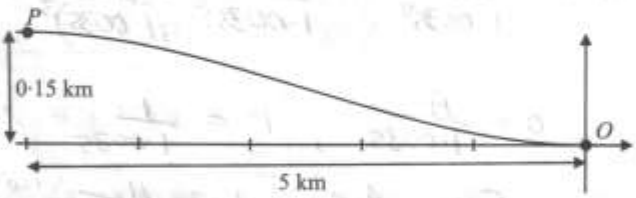
2015 SEC PAPER 2

Section B	Contexts and Applications	150 marks

Answer **all three** questions from this section.

Question 7 **(50 marks)**

A plane is flying horizontally at P at a height of 150 m above level ground when it begins its descent. P is 5 km, horizontally, from the point of touchdown O. The plane lands horizontally at O.



Taking O as the origin, $(x,\ f(x))$ approximately describes the path of the plane's descent where $f(x) = 0{\cdot}0024x^3 + 0{\cdot}018x^2 + cx + d,\ -5 \leq x \leq 0$, and both x and $f(x)$ are measured in km.

(a) (i) Show that $d = 0$.

5

0, 2, 5

pt $(0,0)$ is on graph.

$0 = 0{\cdot}0024(0)^3 + 0{\cdot}018(0)^2 + c(0) + d$

$0 = 0 + d$

$0 = d.$

(ii) Using the fact that P is the point $(-5,\ 0{\cdot}15)$, or otherwise, show that $c = 0$.

5

0, 2, 5

$0{\cdot}15 = 0{\cdot}0024(-5)^3 + 0{\cdot}018(-5)^2 + c(-5) + 0$

$0{\cdot}15 = 0{\cdot}15 - 5c$

$5c = 0$

$c = 0.$

(b) (i) Find the value of $f'(x)$, the derivative of $f(x)$, when $x = -4$.

10

0, 4, 8, 10

$f(x) = 0{\cdot}0024x^3 + 0{\cdot}018x^2$

$f'(x) = 0{\cdot}0072x^2 + 0{\cdot}036x$

@ $x = -4$ $f'(-4) = 0{\cdot}0072(-4)^2 + 0{\cdot}036(-4)$

$= -0{\cdot}0288$

(ii) Use your answer to part **(b) (i)** above to find the angle at which the plane is descending when it is 4 km from touchdown. Give your answer correct to the nearest degree.

5

0,2,5

$f(-4) = -0 \cdot 0288 \quad \Rightarrow Slope = Tan\,\theta$

$Tan\,\theta = -0 \cdot 0288$

$\theta = Tan^{-1}(-0 \cdot 0288)$

$\theta = -1 \cdot 64966$

$\theta = 2^{\circ}$

(c) Show that $(-2 \cdot 5,\ 0 \cdot 075)$ is the point of inflection of the curve $y = f(x)$.

10

0,2,5,8,10

$f''(x) = 0$ at pt inflection

$f''(x) = 0 \cdot 0144x + 0 \cdot 036$

$0 \cdot 0144x + 0 \cdot 036 = 0$

$0 \cdot 0144x = -0 \cdot 036$

$x = \frac{-0 \cdot 036}{0 \cdot 0144} = -2 \cdot 5$

$y = 0 \cdot 0024(-2 \cdot 5)^3 + 0 \cdot 018(-2 \cdot 5)^2 = 0 \cdot 075$

$(-2 \cdot 5,\ 0 \cdot 075)$

(d) **(i)** If (x, y) is a point on the curve $y = f(x)$, verify that $(-x-5, -y+0 \cdot 15)$ is also a point on $y = f(x)$.

5

0,2,4,5

$y = 0 \cdot 0024x^3 + 0 \cdot 018x^2$

@ $x = (-x-5)$ $\quad y = 0 \cdot 0024(-x-5)^3 + 0 \cdot 018(x-5)^2$

$y = 0 \cdot 0024[(-x-5)(x^2+10x+25)] + 0 \cdot 018(x^2+10x+25)$

$y = 0 \cdot 0024(-x^3 - 10x^2 - 25x - 5x^2 - 50x - 125) + 0 \cdot 018x^2 + 0 \cdot 18x + 0 \cdot 45$

$y = -0 \cdot 0024x^3 - 0 \cdot 036x^2 + \cancel{0 \cdot 18x} - 0 \cdot 3 + 0 \cdot 018x^2 - \cancel{0 \cdot 18x} + 0 \cdot 45$

$y = \underline{-0 \cdot 0024x^3 - 0 \cdot 018x^2} + 0 \cdot 15$

$y = -y + 0 \cdot 15 \quad \Rightarrow (-x-5, -y+0 \cdot 15)$ is on $f(x)$

(ii) Find the image of $(-x-5, -y+0 \cdot 15)$ under symmetry in the point of inflection.

10

0,4,8,10

$(-x-5, -y+0 \cdot 15) \rightarrow (-2 \cdot 5, 0 \cdot 075) \rightarrow (x, y)$

$+x+2 \cdot 5 \qquad +x+2 \cdot 5$

$+y-0 \cdot 075 \qquad +y-0 \cdot 075$

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aoesck

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Question 8 **(50 marks)**

An oil-spill occurs off-shore in an area of calm water with no currents. The oil is spilling at a rate of 4×10^6 cm^3 per minute. The oil floats on top of the water.

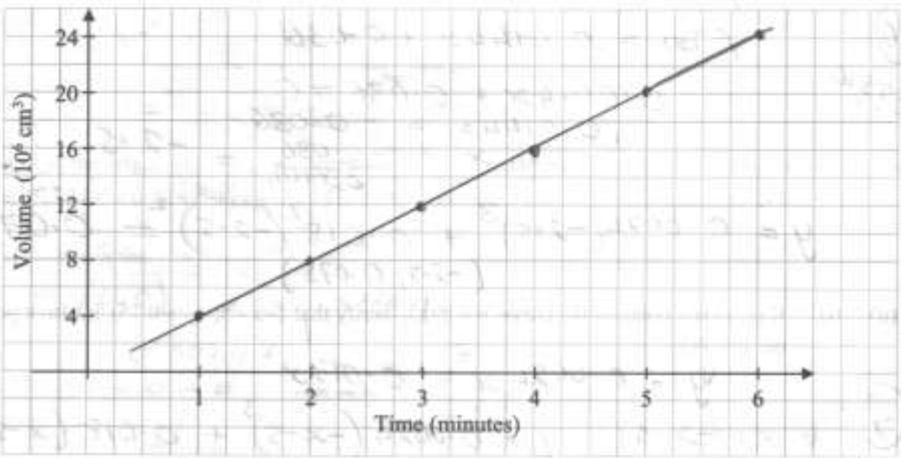
(a) **(i)** Complete the table below to show the total volume of oil on the water after each of the first 6 minutes of the oil-spill.

5
0,25

Time (minutes)	1	2	3	4	5	6
Volume (10^6 cm^3)	4	8	12	16	20	24

(ii) Draw a graph to show the total volume of oil on the water over the first 6 minutes.

5
0,25



(iii) Write an equation for $V(t)$, the volume of oil on the water, in cm^3, after t minutes.

5
0,25

$V = (4\times10^6)t \text{ cm}^3$

(b) The spilled oil forms a circular oil slick **1 millimetre** thick.

(i) Write an equation for the volume of oil in the slick, in cm^3, when the radius is r cm.

5
0,25

$V = \pi r^2 h$ $\quad$ $1\text{mm} = 0{\cdot}1\text{cm}$

$V = 0{\cdot}1\pi r^2$

(ii) Find the rate, in cm per minute, at which the radius of the oil slick is increasing when the radius is 50 m.

10

0,2,5,8,10

Given $\frac{dV}{dt} = 4 \times 10^6$ want $\frac{dr}{dt}$ need $\frac{dr}{dt} = \frac{dV}{dt} \cdot \frac{dr}{dV}$

$V = 0.1\pi r^2$

$\frac{dV}{dr} = 0.2\pi r \Rightarrow \frac{dr}{dV} = \frac{1}{0.2\pi r}$

$\frac{dr}{dt} = (4 \times 10^6) \cdot \left(\frac{1}{0.2\pi r}\right)$ at $r = 50m \Rightarrow 5000cm$.

$(4 \times 10^6) \frac{1}{0.2\pi(5000)} = 1273.239$

$\simeq 1273$ cm/min

(c) Show that the area of water covered by the oil slick is increasing at a constant rate of 4×10^7 cm^2 per minute.

10

0,4,8,10

want $\frac{dA}{dt}$. given $\frac{dr}{dt} = \frac{4 \times 10^6}{0.2\pi r}$.

$\frac{dA}{dt} = \frac{dr}{dt} \cdot \frac{dA}{dr}$

$A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{4 \times 10^6}{0.2\pi r} \times 2\pi r$

$= 4 \times 10^7$ cm^2/min

(d) The nearest land is 1 km from the point at which the oil-spill began. Find how long it will take for the oil slick to reach land. Give your answer correct to the nearest hour.

10

0,4,8,10

$A = \pi r^2$ when $r = 1km = 1000m = 100\,000cm$.

$A = \pi(100000)^2 = 10^{10}\pi$

$\frac{dA}{dt} = 4 \times 10^7$

$T = \frac{D}{S} = \frac{10^{10}\pi}{4 \times 10^7} = 250\pi$ minutes

$250\pi \div 60 = 13.08996939$

$\simeq 13$ hrs.

movxph

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Question 3 $x\ y_1 \quad y_2\ y_2$ **(25 marks)**

(a) The co-ordinates of two points are $A(4, -1)$ and $B(7, t)$.

The line $l_1: 3x - 4y - 12 = 0$ is perpendicular to AB. Find the value of t.

$m_{l_1} = \frac{-3}{-4} = \frac{3}{4} \Rightarrow m_{AB} = -\frac{4}{3}$

$m_{AB} = \frac{t+1}{7-4} = -\frac{4}{3}$

$3t + 3 = -28 + 16$

$3t = -15$

$t = -5$.

10 D

0 2 5 8 10

(b) Find, in terms of k, the distance between the point $P(10, k)$ and l_1.

$d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}} \qquad 3x - 4y - 12 \qquad (10, k)$

$d = \frac{|3(10) - 4(k) - 12|}{\sqrt{3^2 + (-4)^2}} = \ = \frac{|18-4k|}{5}.$

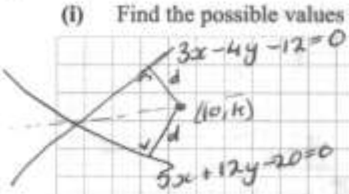
10 C

0 4 8 10

2015 SEC PAPER 2

(c) $P(10, k)$ is on a bisector of the angles between the lines l_1 and $l_2: 5x + 12y - 20 = 0$.

(i) Find the possible values of k.



⊥ Dis from (10,k) to each line is $\frac{|18-4k|}{5}$.

5 D

0 2 3 4 5

$5x + 12y - 20 \quad (10, k)$

$\frac{|5(10) + 12(k) - 20|}{\sqrt{5^2+12^2}} = \frac{|18-4k|}{5}$

$\frac{|30+12k|}{13} = \frac{18-4k}{5}$

$(5|30+12k|)^2 = (13|18-4k|)^2$

$25(900 + 720k + 144k^2) = 169(324 - 144k + 16k^2)$

$22500 + 18000k + 3600k^2 = 54756 - 24336k + 2704k^2$

$0 = 896k^2 + 42336k - 32256$

$k = \frac{-42336 \pm \sqrt{42336^2 - 4(896)(-32256)}}{2(896)}$

$k = 3/4 \qquad k = -48$

(ii) If $k > 0$, find the distance from P to l_1.

$k = 3/4.$

$\frac{|18 - 4(3/4)|}{5} = \frac{|15|}{5} = 3.$

Question 5 **(25 marks)**

(a) Prove that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A\cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

[÷ all by $\cos A \cos B$]

$$= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}}$$

$$= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{1 - \frac{\sin A}{\cos A}\cdot\frac{\sin B}{\cos B}}$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

15 D

047 11/15

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(b) Find all the values of x for which $\sin(3x) = \dfrac{\sqrt{3}}{2}$, $0 \le x \le 360$, x in degrees.

120 60 S A T C

$\sin 3x = \sqrt{3}/2$ $\quad 3x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60$

10 D

0258 16

1 $3x = 60 + 360n$ $\quad$ 2 $3x = 120 + 360n$

$x = 20 + 120n$ $\quad x = 40 + 120n$

$n=0$ $\quad x = 20$ $\quad x = 40$

$n=1$ $\quad x = 20 + 120 = 140$ $\quad x = 40 + 120 = 160$

$n=2$ $\quad x = 20 + 240 = 260$ $\quad x = 40 + 240 = 280$

$n=3$ $\quad x = 20 + 360 = \cancel{380}$

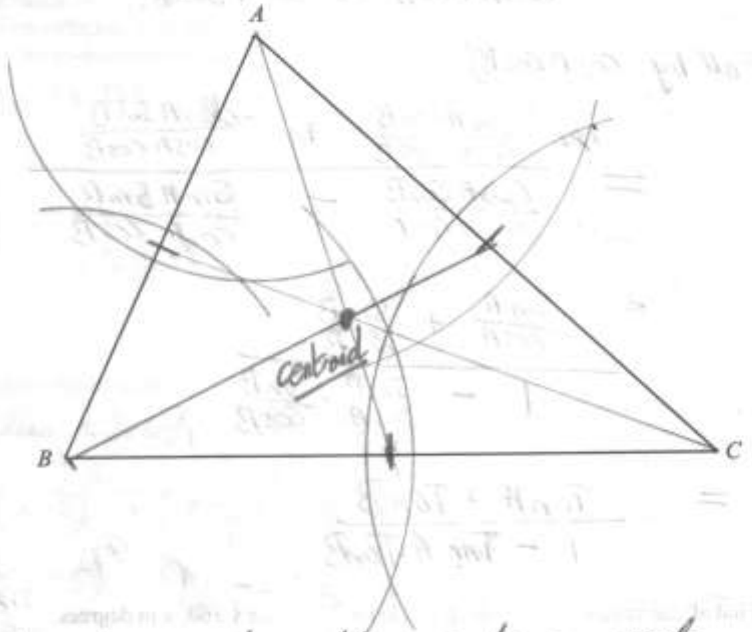
$x = 20, 40, 140, 160, 260, 280.$

Question 6 **(25 marks)**

(a) Construct the centroid of the triangle ABC below. Show all construction lines.
(Where measurement is used, show all relevant measurements and calculations clearly.)

54

0 2 3 4 5

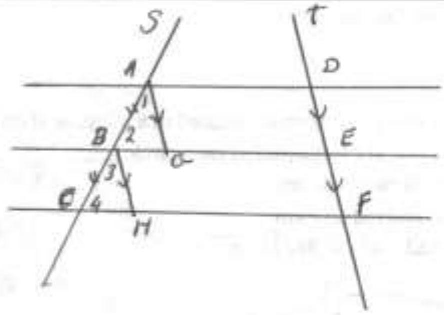


Centroid is where the medians meet.
Mid pt of a side Joined to opposite Vertex.

1. Construct ⊥ bisectors
2. Join midpts to opp vertex = Medians
3. Where medians intersect = centroid

(b) Prove that, if three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal line.

Diagram:



5B

025

Given: S and t are transversals of 3 parallel lines crossing at ABC and DEF.
$|AB| = |BC|$

To Prove: $|DE| = |EF|$

Construction: Draw $[AG]$ and $[BH]$ both parallel to t meeting parallel lines at G and H.
mark angles 1, 2, 3, 4.

5B

025

Proof:

In Δ's ABG and BCH

$|\angle 1| = |\angle 3|$ corresponding

$|AB| = |BC|$ given

$|\angle 2| = |\angle 4|$ corresponding

$\therefore \Delta ABG \equiv \Delta BCH$ ASA

$\Rightarrow |AG| = |BH|$ corresponding sides

AGED and BHFE are parallelograms as AG and BH are parallel to t

$\Rightarrow |AG| = |DE|$ and $|BH| = |EF|$

But $|AG| = |BH| \Rightarrow |DE| = |EF|$

10C

04810

urwmcx
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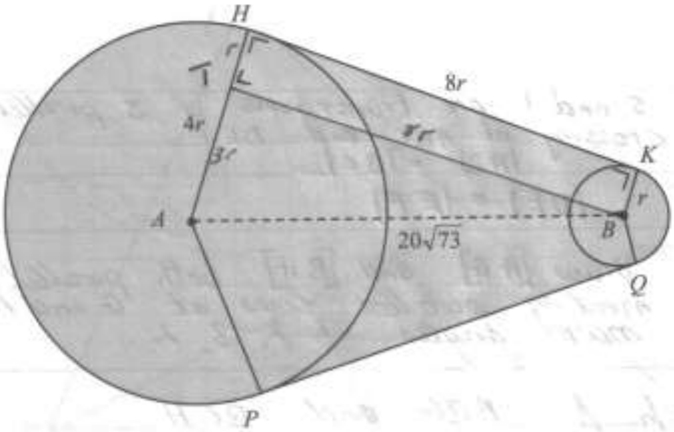
2015 SEC PAPER 2

Section B	**Contexts and Applications**	**150 marks**

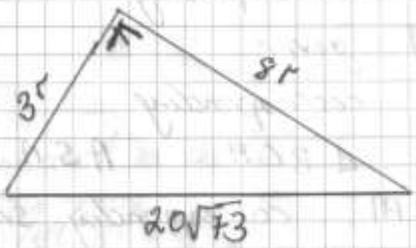
Answer **all three** questions from this section.

Question 7 **(40 marks)**

A flat machine part consists of two circular ends attached to a plate, as shown (diagram not to scale).
The sides of the plate, HK and PQ, are tangential to each circle.
The larger circle has centre A and radius $4r$ cm.
The smaller circle has centre B and radius r cm.
The length of $[HK]$ is $8r$ cm and $|AB| = 20\sqrt{73}$ cm.



(a) Find r, the radius of the smaller circle. (Hint: Draw $BT \parallel KH$, $T \in AH$.)



$(20\sqrt{73})^2 = (3r)^2 + (8r)^2$

$29200 = 9r^2 + 64r^2$

$29200 = 73r^2$

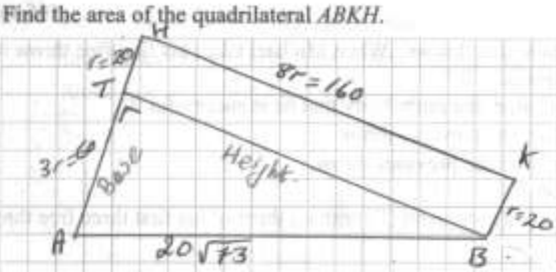
$\sqrt{\frac{29200}{73}} = r$

$20 = r$

15c

05 12 15

(b) Find the area of the quadrilateral *ABKH*.



15C

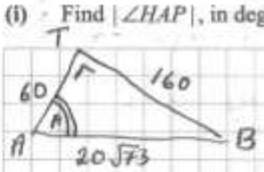
0512 15

Area $THKB = L \times W = 160 \times 20 = 3200$

Area $\Delta ATB = \frac{1}{2}(60) \times 160 = 4800$

Total Area $= 3200 + 4800 = 8000\,cm^2$

(c) **(i)** Find $|\angle HAP|$, in degrees, correct to one decimal place.



$\sin A = \frac{160}{20\sqrt{73}}$

$A = \sin^{-1}\left(\frac{160}{20\sqrt{73}}\right) = 69.44395...$

5C

0245

$|\angle HAP| = 2 \times 69.44 = 138.9^\circ$ (to 1dp)

(ii) Find the area of the machine part, correct to the nearest cm^2.

Area of Sector HAP

$$\begin{array}{r} 360 \\ -\ 138.9 \\ \hline 221.1 \end{array}$$

Area $= \frac{221.1}{360}(\pi.80^2)$

$= \boxed{12348.55\,cm^2}$

5D

02345

Area $HKQP = 2 \times 8000 = \boxed{16000\,cm^2}$

Area Sector KBQ

Area $= \frac{138.9}{360}(\pi.20^2)$

$= \boxed{484.85\,cm^2}$

Total Area: $12348.55 + 16000 + 484.85$

$= 28833\,cm^2$ to nearest cm^2