Section A	Concepts and Skills	150 marks

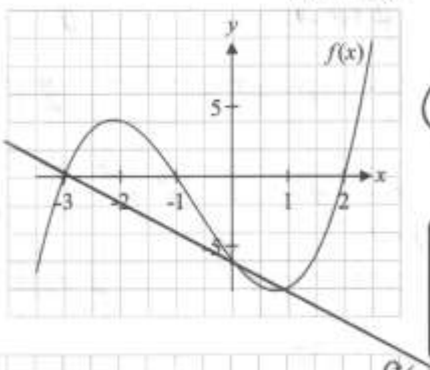
Answer **all six** questions from this section.

Question 1 **(25 marks)**

(a) The graph of a cubic function $f(x)$ cuts the x-axis at $x=-3$, $x=-1$ and $x=2$, and the y-axis at $(0,-6)$, as shown.

Verify that $f(x)$ can be written as

$$f(x)=x^3+2x^2-5x-6.$$



g(x)

15

roots: $x=-3$, $x=-1$, $x=2$.

factors: $(x+3)(x+1)(x-2)$

eqn: $(x^2+4x+3)(x-2)$

$x^3-2x^2+4x^2-8x+3x-6$

x^3+2x^2-5x-6

(b) **(i)** The graph of the function $g(x)=-2x-6$ intersects the graph of the function $f(x)$ above. Let $f(x)=g(x)$ and solve the resulting equation to find the co-ordinates of the points where the graphs of $f(x)$ and $g(x)$ intersect.

5

$x^3+2x^2-5x-6=-2x-6.$

$x^3+2x^2-3x=0.$

$f(1)$ $(1)^3+2(1)^2-3(1)=0$ ✓ $\Rightarrow x=1$ a root $\therefore (x-1)$ a factor

	x^2	$+3x$	
x	x^3	$+3x^2$	$+0x$
-1	$-x^2$	$-3x$	0

$x^2+3x=0$

$x(x+3)=0$

$x=0$ $x=-3.$

y co-ords

$x=1$ $y=-2(1)-6=-8$ $(1,-8)$

$x=0$ $y=-2(0)-6=-6$ $(0,-6)$

$x=-3$ $y=-2(-3)-6=0$ $(-3,0)$

(ii) Draw the graph of the function $g(x)=-2x-6$ on the diagram above.

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Question 4 **(25 marks)**

(a) Differentiate the function $2x^2-3x-6$ with respect to x from first principles.

$f(x) = 2x^2-3x-6$

$f(x+h) = 2(x+h)^2 - 3(x+h) - 6$

$2x^2 + 4xh + 2h^2 - 3x - 3h - 6$

$f(x+h) - f(x) = 4xh + 2h^2 - 3h$

15

0, 5, 9, 12, 15

$\frac{f(x+h)-f(x)}{h} = 4x + 2h - 3$

$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = 4x - 3.$

$f'(x) = 4x - 3$

(b) Let $f(x) = \frac{2x}{x+2}$, $x \neq -2$, $x \in \mathbb{R}$. Find the co-ordinates of the points at which the slope of the tangent to the curve $y = f(x)$ is $\frac{1}{4}$.

Slope $= \frac{1}{4} \Rightarrow dy/dx = 1/4$

10

0, 3, 7, 8, 10

$f(x) = \frac{2x}{x+2}$

$f'(x) = \frac{(x+2)(2) - (2x)(1)}{(x+2)^2} = \frac{2x+4-2x}{x^2+4x+4}$

$\frac{4}{x^2+4x+4} = \frac{1}{4}$

$16 = x^2 + 4x + 4$

$0 = x^2 + 4x - 12$

$0 = (x+6)(x-2)$

$x = -6 \quad x = 2$

y co-ords

$x = 2 \quad y = \frac{2(2)}{2+2} = 1$

$(2, 1)$

$x = -6$

$y = \frac{2(-6)}{-6+2} = \frac{-12}{-4} = 3$

$(-6, 3)$

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Question 6 **(25 marks)**

The n^{th} term of a sequence is $T_n = \ln a^n$, where $a > 0$ and a is a constant.

(a) **(i)** Show that T_1, T_2, and T_3 are in arithmetic sequence.

$T_1 = \ln a^1 \quad T_2 = \ln a^2 \quad T_3 = \ln a^3$

If Arith Then $T_3 - T_2 = T_2 - T_1$

$\ln a^3 - \ln a^2 = \ln a^2 - \ln a^1$

$3\ln a - 2\ln a = 2\ln a - \ln a$

$\ln a = \ln a.$ True ∴ It is an Arith Seq.

10

(ii) Prove that the sequence is arithmetic and find the common difference.

Proof : $T_{n+1} - T_n = \text{constant}$

$\ln a^{n+1} - \ln a^n$

$(n+1)\ln a - n\ln a$ $\quad d = \ln a.$

$\ln a(n+1-n)$

$\ln a(1)$

$\ln a =$ a constant.

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(b) Find the value of a for which $T_1 + T_2 + T_3 + \cdots + T_{98} + T_{99} + T_{100} = 10\,100$.

$S_n = \frac{n}{2}[2a + (n-1)d] \quad a = \ln a \quad d = \ln a \quad n = 100$

$10100 = \frac{\cancel{100}^{50}}{\cancel{2}}[2\ln a + 99\ln a]$

$202 = 101\ln a$

$\frac{202}{101} = \ln a$

$2 = \ln a$

$e^2 = a.$

5

(c) Verify that, for all values of a,

$$(T_1+T_2+T_3+\cdots+T_{10})+100d=(T_{11}+T_{12}+T_{13}+\cdots+T_{20}),$$

where d is the common difference of the sequence.

$S_{10} = \frac{10}{2}[2\ln a + 9\ln a]$

$= 5[11\ln a]$

$= 55\ln a.$

$(T_1+T_2+T_3 \cdots T_{10}) + 100d.$

$55\ln a + 100\ln a$

$= 155\ln a$

$S_{20} - S_{10}.$

$\frac{20}{2}[2\ln a + 19\ln a] - 55\ln a$

$10[21\ln a] - 55\ln a$

$210\ln a - 55\ln a$

$= 155\ln a.$

$\therefore$ True

5

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