Revision Exercise Core.

Q1

$$x^2 - 6x + 5 = 0$$
$$(x-5)(x-1) = 0$$
$$x = 5 \quad x = 1$$

$$\left(t - \frac{6}{t}\right)^2 - 6\left(6 - \frac{6}{t}\right) + 5 = 0$$

$t - \frac{6}{t} = 5 \quad (\times t)$	$t - \frac{6}{t} = 1 \quad (\times t)$
$t^2 - 6 = 5t$	$t^2 - 6 = t$
$t^2 - 5t - 6 = 0$	$t^2 - t - 6 = 0$
$(t-6)(t+1) = 0$	$(t-3)(t+2) = 0$
$t = 6 \quad t = -1$	$t = 3 \quad t = -2$

2 $2(x+1)(x-4) - (x-2)^2 = 0$

$(2x+2)(x-4) - (x^2-4x+4) = 0$

$2x^2 - 8x + 2x - 8 - x^2 + 4x - 4 = 0$

$x^2 - 2x - 12 = 0$

~~$(x-6)(x+2) = 0$~~] In surd form
~~$x = 6, x = -2$~~] ⇒ Use formula

$a = 1 \quad b = -2 \quad c = -12$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4+48}}{2} = \frac{2 \pm \sqrt{52}}{2}$$

$$= \frac{2 \pm \sqrt{4 \times 13}}{2} = \frac{2 \pm 2\sqrt{13}}{2}$$

$$= 1 \pm \sqrt{13}$$

Q3 ~~$px^2 + 2x + p$~~ .

$$px^2 + 2x + 1 = 0$$

No real roots $\Rightarrow$ $b^2 - 4ac < 0$

$$2^2 - 4(p)(1) < 0$$
$$4 - 4p < 0$$
$$-4p < -4$$
$$4p > 4$$
$$p > 1$$

* Not inequality is also changed!

Q4 $x^2 - (a+d)x + (ad - b^2) = 0$

Real Roots $\Rightarrow$ $b^2 - 4ac \geq 0$

$$[-(a+d)]^2 - 4(1)(ad - b^2) \geq 0$$
$$a^2 + 2ad + d^2 - 4ad + 4b^2 \geq 0$$
$$a^2 - 2ad + d^2 + 4b^2$$
$$(a - d)^2 + 4b^2 \geq 0$$

Any variable squared is Positive, hence has real roots.

Q5 $6x^4 - x^3 + ax^2 - 6x + b$

$(x+1)$ a factor $\Rightarrow$ $6(-1)^4 - (-1)^3 + a(-1)^2 - 6(-1) + b = 0$

$6 + 1 + a + 6 + b = 0$

$\boxed{a + b = -13}$

$(x-2)$ a factor $\Rightarrow$ $6(2)^4 - (2)^3 + a(2)^2 - 6(2) + b = 0$

$96 - 8 + 4a - 12 + b = 0$

$\boxed{4a + b = -76}$

$a + b = -13$

$-4a + b = -76$ $(\times -1)$

$-3a = 63$

$\boxed{a = -21}$

$a + b = -13$

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$\boxed{b = 8}$

Q6 $f(x) = x^3 - 4x^2 - 11x + 30$ (by observation $x = 1$ and $x = -1$ won't work)

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(i) Try $x = 2$. $(2)^3 - 4(2)^2 - 11(2) + 30$

$8 - 16 - 22 + 30 = 0 \Rightarrow (x-2)$ a factor

(ii)

$$x-2 \overline{)\, x^3 - 4x^2 - 11x + 30}$$

$x^2 - 2x - 15$

$-x^3 \pm 2x^2$

$-2x^2 - 11x$

$\mp 2x^2 \mp 4x$

$-15x + 30$

$-15x + 30$

$x^2 - 2x - 15$

$(x+3)(x-5)$ factors are $(x-2)(x+3)(x-5)$

(iii) Solutions are $x = 2 \quad x = -3 \quad x = 5$.

Q7 (i) $x^2 - 2x - 5 = 0$

$b^2 - 4ac$

$(-2)^2 - 4(1)(-5)$

$4 + 20$

$24 \Rightarrow > 0 \Rightarrow$ real roots

(ii) $x^2 - 4x + 6 = 0$

$b^2 - 4ac$

$(-4)^2 - 4(1)(6)$

$16 - 20$

$-4 < 0 \Rightarrow$ Imaginary roots

(iii) $-6 + 4x - x^2 = 0$

$x^2 - 4x + 6 = 0$

$b^2 - 4ac$

$(-4)^2 - 4(1)(6)$

$16 - 24$

$-8 < 0 \Rightarrow$ Imaginary roots

$16 - 24$

$-8 < 0 \Rightarrow$ Imaginary roots

Q8 $y = 3^x$

$3^{2x} - 12(3^x) + 27 = 0$

$(3^x)^2 - 12(3^x) + 27 = 0$

$y^2 - 12y + 27 = 0$

$(y - 3)(y - 9) = 0$

$y = 3 \quad y = 9$

$3^x = 3^1$ $\qquad$ $3^x = 9$

$x = 1$ $\qquad$ $3^x = 3^2$

$x = 2.$