



**Pre-Leaving Certificate Examination, 2017**  
**Triailscrúdú na hArdteistiméireachta, 2017**

# Mathematics

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	
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Grade
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## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

**Section A****Concepts and Skills****150 marks**

Answer **all six** questions from this section.

**Question 1****(25 marks)**

- (a) Factorise  $64x^3 - 125$ .

- (b) Show that for all real numbers  $a \geq 2\sqrt{ab} - b$ .

(c) Solve the simultaneous equations

$$x + 3y = -3$$

$$x^2 + y^2 = 13.$$

A large grid of squares, approximately 20 columns by 30 rows, intended for working space or calculations.

**Question 2****(25 marks)**

- (a) By letting  $x = 5^y$ , show that the equation  $p5^y + 5^{-y} = 5$  can be written in the form  $px^2 - 5x + 1 = 0$ .

- (b) (i) Find the value of  $p$  for which the equation  $p5^y + 5^{-y} = 5$  has equal roots.

- (ii) Hence, or otherwise, solve the equation  $p5^y + 5^{-y} = 5$ , correct to two decimal places.

A large grid of squares, approximately 20 columns by 30 rows, intended for考生 to show their working for the problem.

**Question 3****(25 marks)**

- (a) Solve the inequality  $\frac{7-2x}{x+3} \leq 5$ , for  $x \in \mathbb{R}$ ,  $x \neq -3$ .

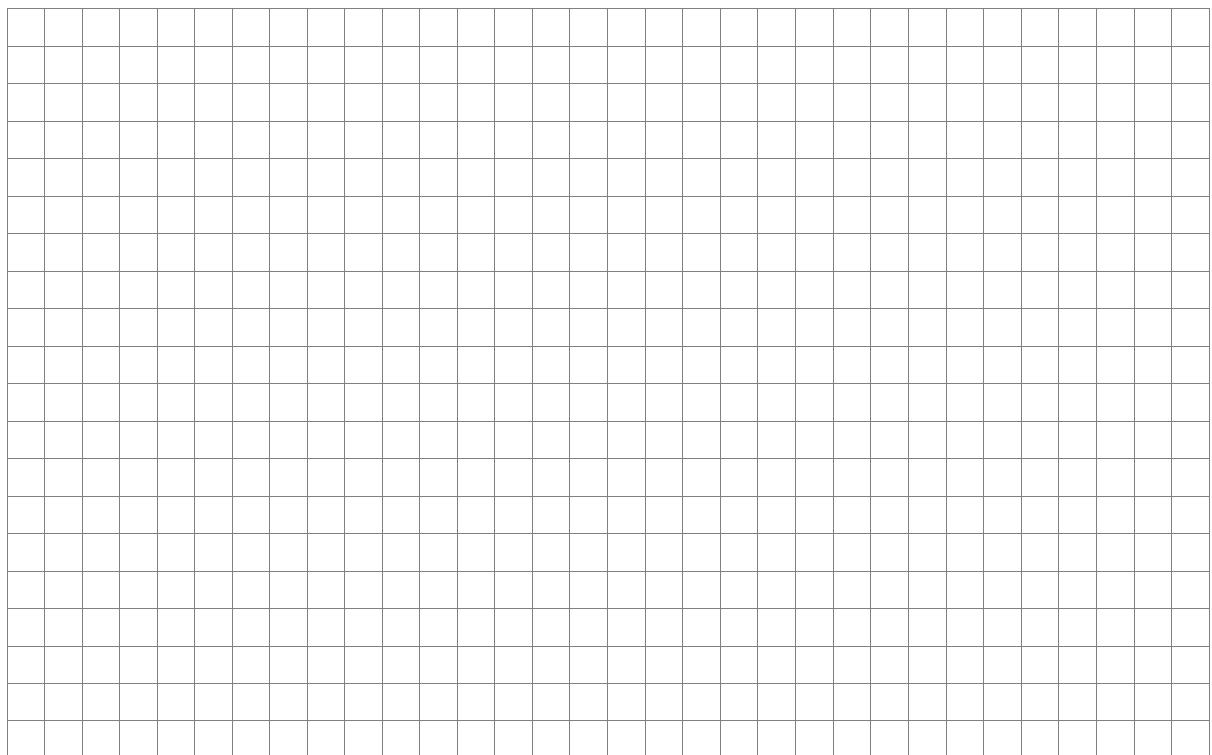
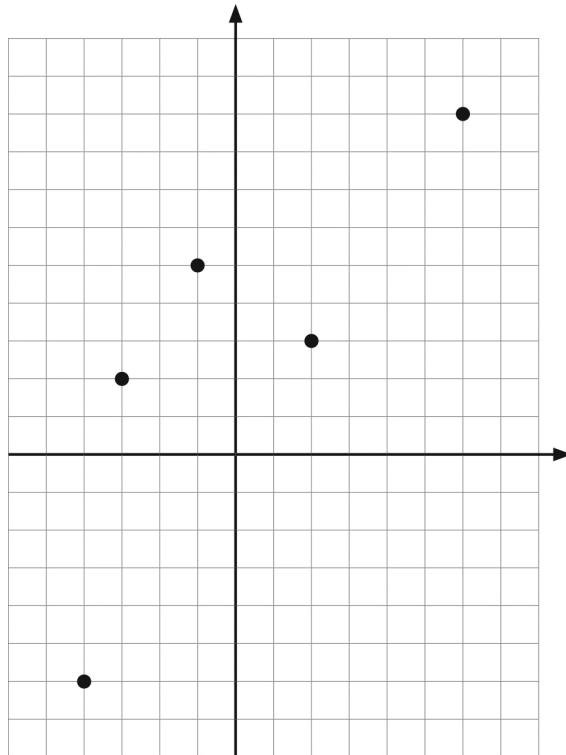
- (b) Prove by induction that  $7^n - 1$  is divisible by 3, for all  $n \in \mathbb{N}$ .

**Question 4****(25 marks)**

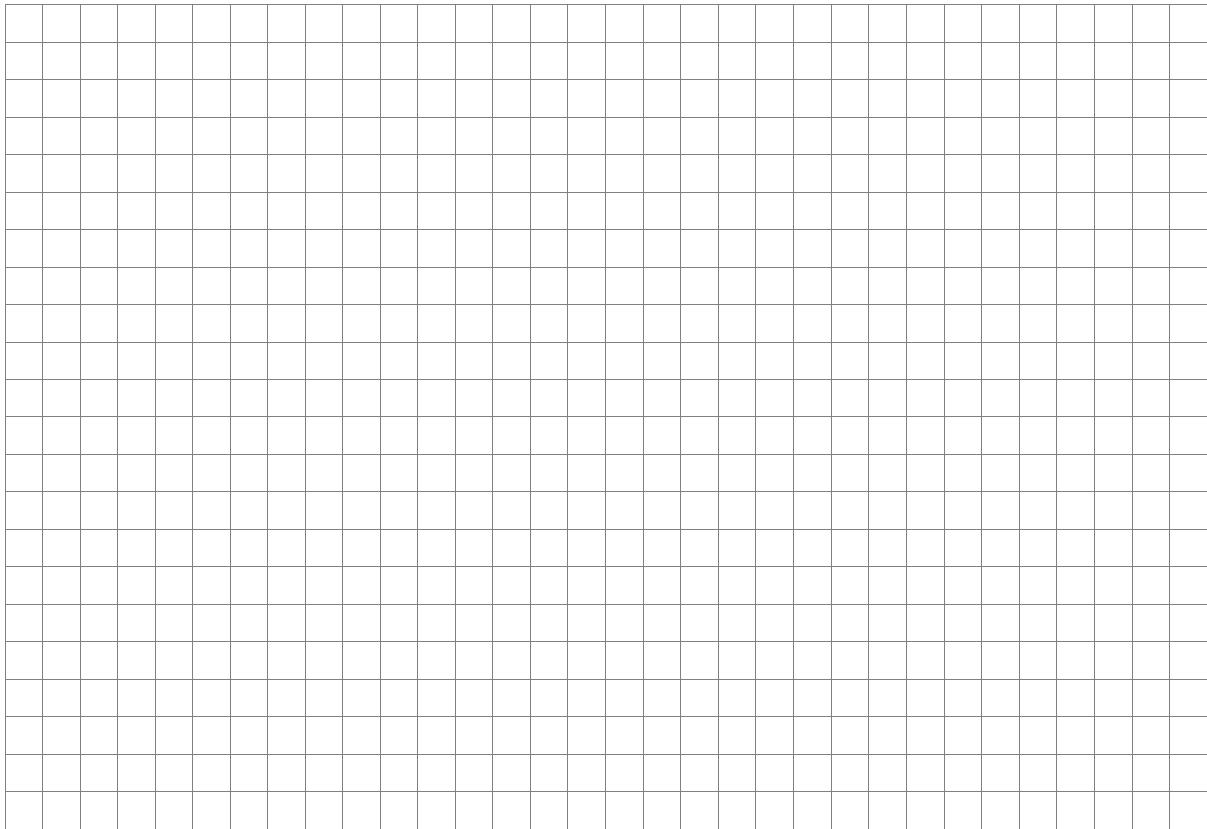
- (a) The complex numbers  $z_1, z_2, z_3, z_4, z_5$  are shown below, where

$$z_2 = -2z_3, \quad z_1 = 3z_3, \quad z_4 = iz_3, \quad z_5 = z_3 + z_4.$$

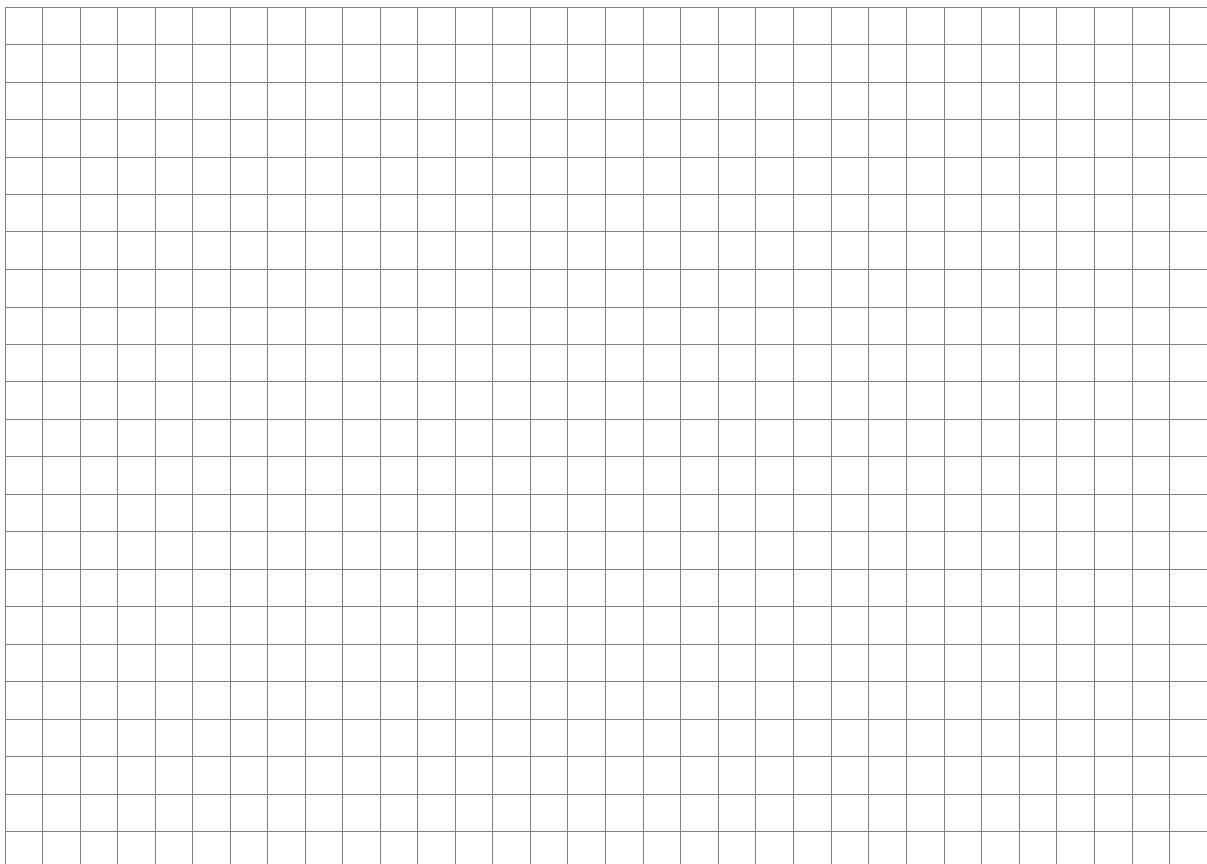
Identify each complex number on the Argand diagram.



- (b)** Let  $z_1 = 2 - 5i$  and let  $z_2 = -1 - 3i$ . Investigate if  $\frac{|z_1|}{|z_2|} = \left| \frac{z_1}{z_2} \right|$ .

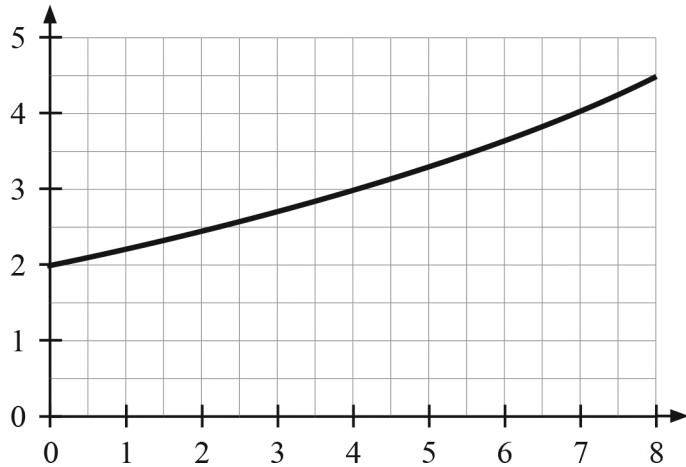


- (c)** Use de Moivre's Theorem to prove the identity  $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$ .



**Question 5****(25 marks)**

The graph of the function  $g(x) = 2e^{0.1x}$  in the domain  $0 \leq x \leq 8$  is shown below.



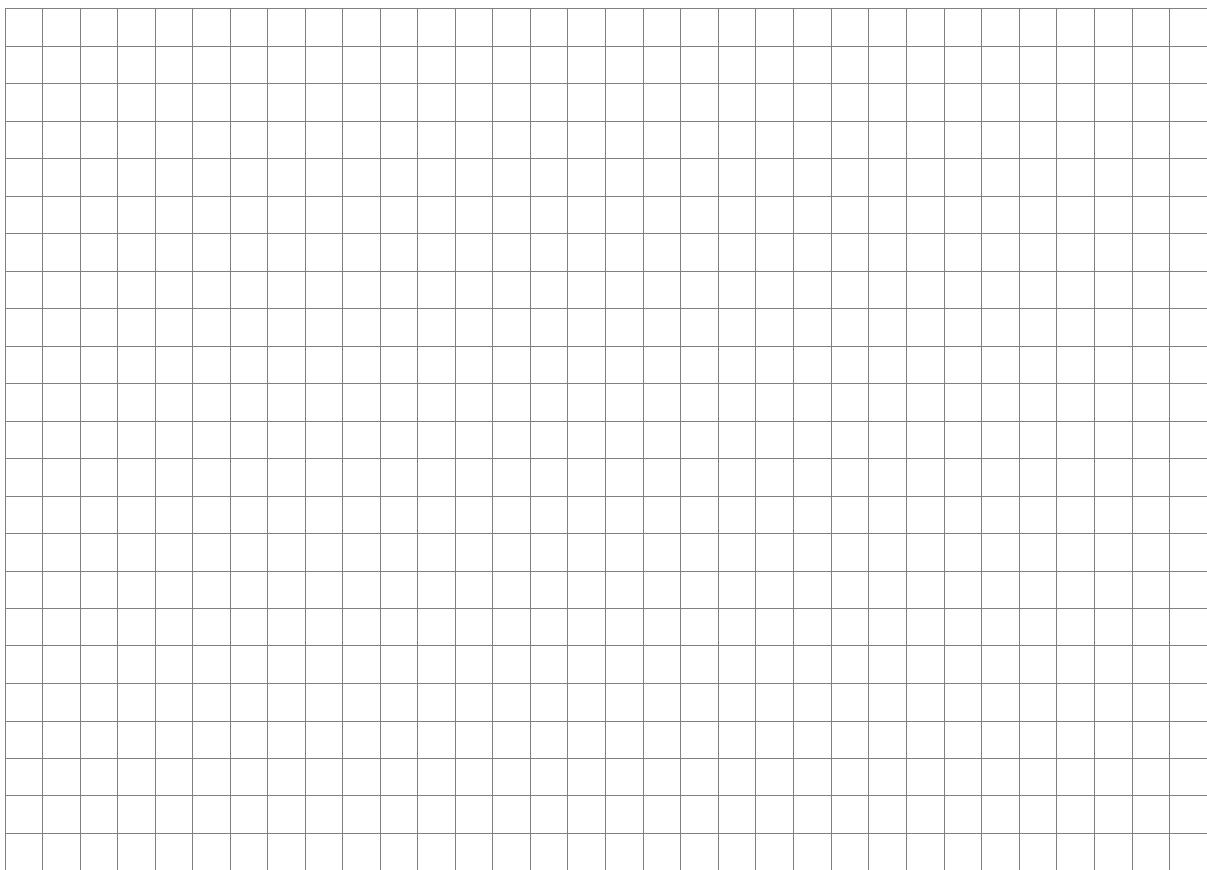
- (a) (i) Fill in the following table

$x$	0	1	2	3	4	5	6	7	8
$g(x)$									

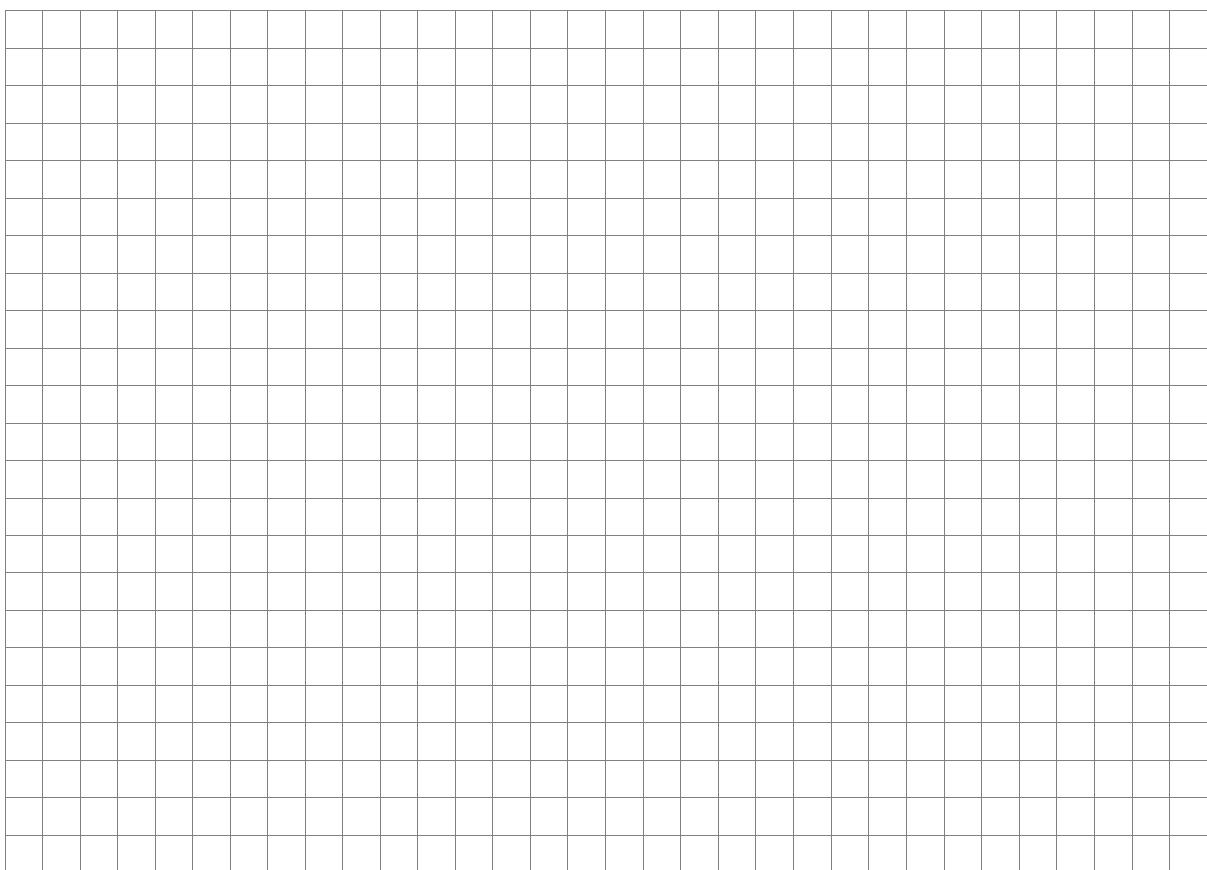
- (ii) Use the trapezoidal rule to find an approximation for the area bounded by the curve  $g(x)$  and the  $x$  axis, correct to one decimal place.

[A large grid area for working space.]

- (iii)** Use integration methods to find the area under the curve, correct to one decimal place.

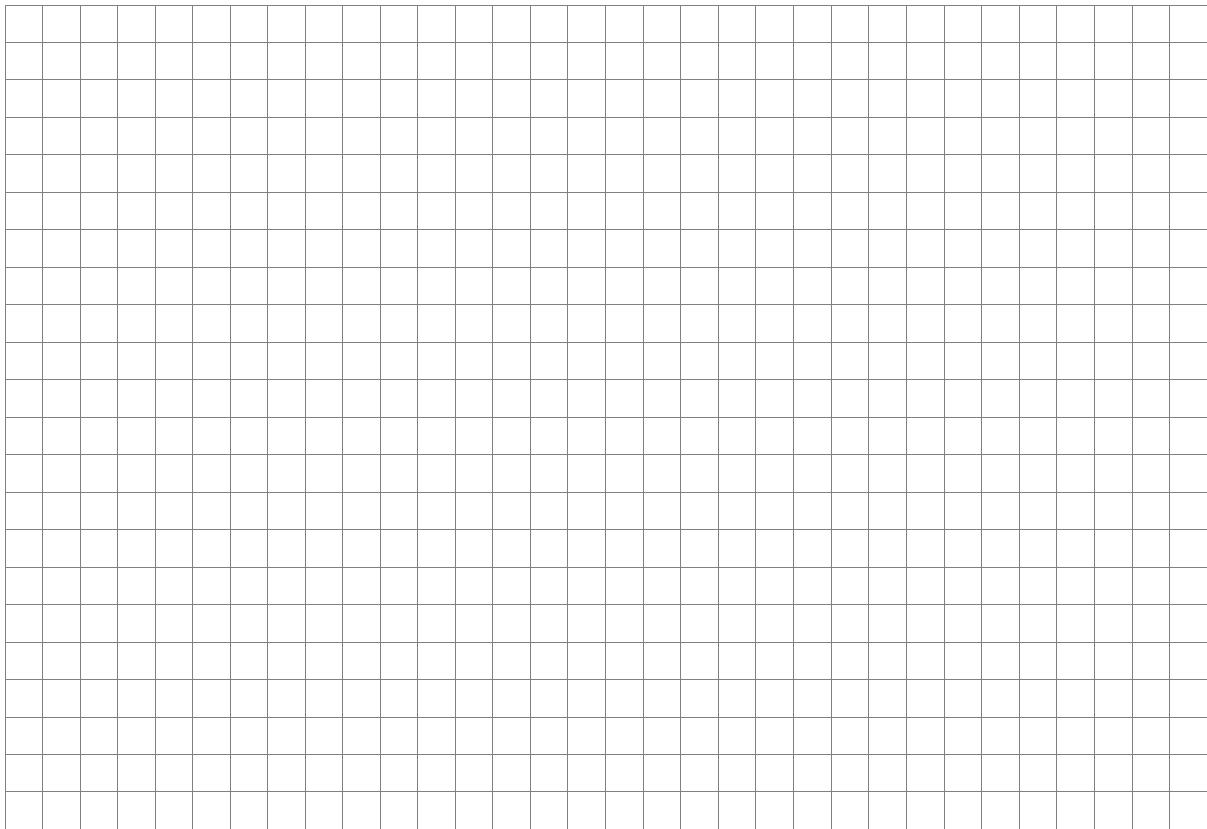
A large grid of squares, approximately 20 columns by 20 rows, intended for working space or calculations.

- (b)** Differentiate  $2x^2 + 3x + 3$  from first principles.

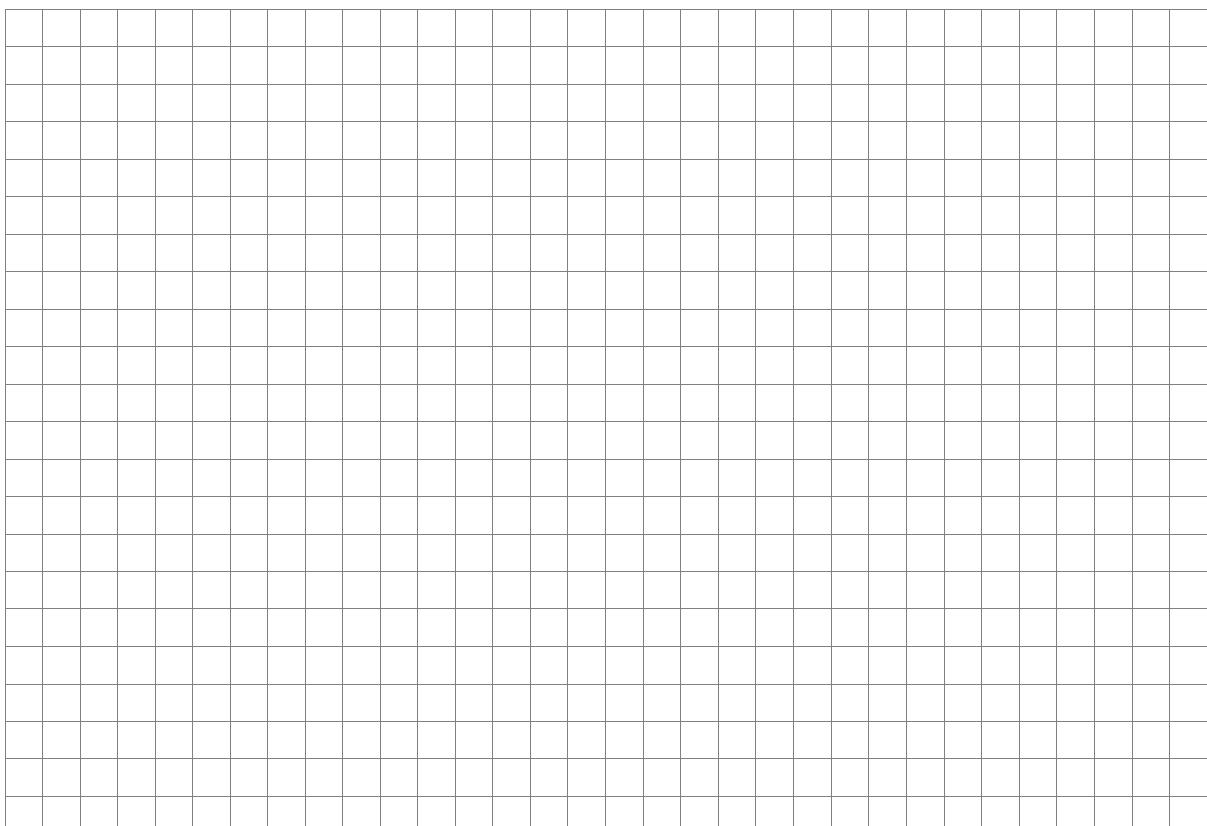
A large grid of squares, approximately 20 columns by 20 rows, intended for working space or calculations.

**Question 6****(25 marks)**

- (a) Differentiate  $y = x^2 \sin(3x + 5)$ , with respect to  $x$ .



- (b) Differentiate  $\cos^{-1} \frac{2}{x^2}$ , with respect to  $x$ .



(c) If  $y = x^4 e^{3x}$ , show that  $\frac{dy}{dx} - 3y = \frac{4y}{x}$ .

**Section B****Contexts and Applications****150 marks**

Answer **all three** questions from this section.

**Question 7****(75 marks)**

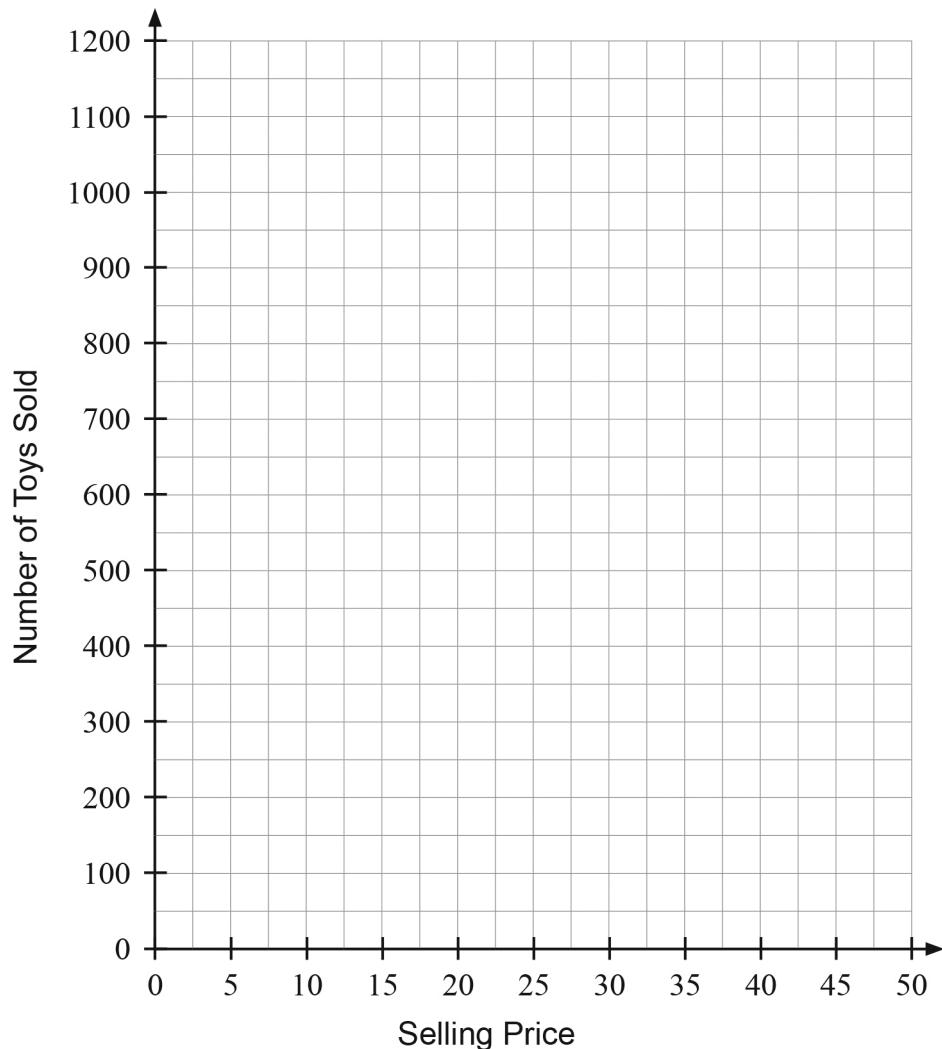
DigiAgeToys, a digital toy manufacturer, is trying to determine the optimum selling price for their newly developed toy.

From previous years' data of new toys, they have the following data.

Selling Price € ( $x$ )	Number of Toys Sold in 1 year ( $n$ ) (000's)
10	1000
15	900
20	800
25	700

- (a) (i) Describe the relationship between the selling price and the quantity of the toys sold.

- (ii) Draw a graph to show the relationship between the selling price and the quantity of the toys sold.



- (iii) How many toys will the company initially manufacture from this model?

- (iv) Find the equation of the line that represents the number of toys sold.

- (b)** Each toy costs €10 to manufacture. The profit ( $P$ ) of the toy can be calculated as

$$P = \text{Total Income} - \text{Production Costs}$$

where the

$$\text{Total Revenue} = \text{Price} \times \text{Quantity Sold}$$

and

$$\text{Production Costs} = \text{Cost Per Item} \times \text{Quantity Sold}.$$

- (i)** Show that the profit can be represented by the formula  $P(x) = xn - 10n$ .

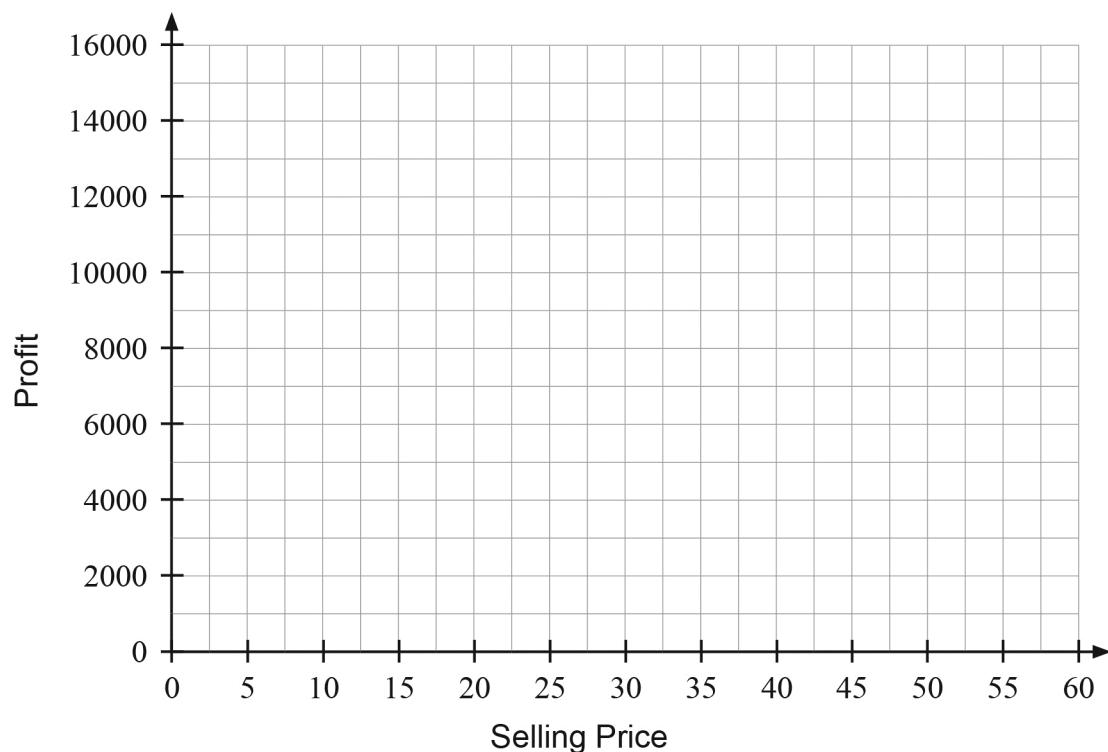
- (ii)** Hence, or otherwise, show that the profit can be expressed as:

$$P(x) = -20x^2 + 1,400x - 12,000.$$

(iii) Find the selling price that will maximize the profit earned from the sale of the toy.

(iv) Find the maximum profit made by the company.

(v) Draw a sketch of the function  $P(x)$ .



## Question 8

(40 marks)

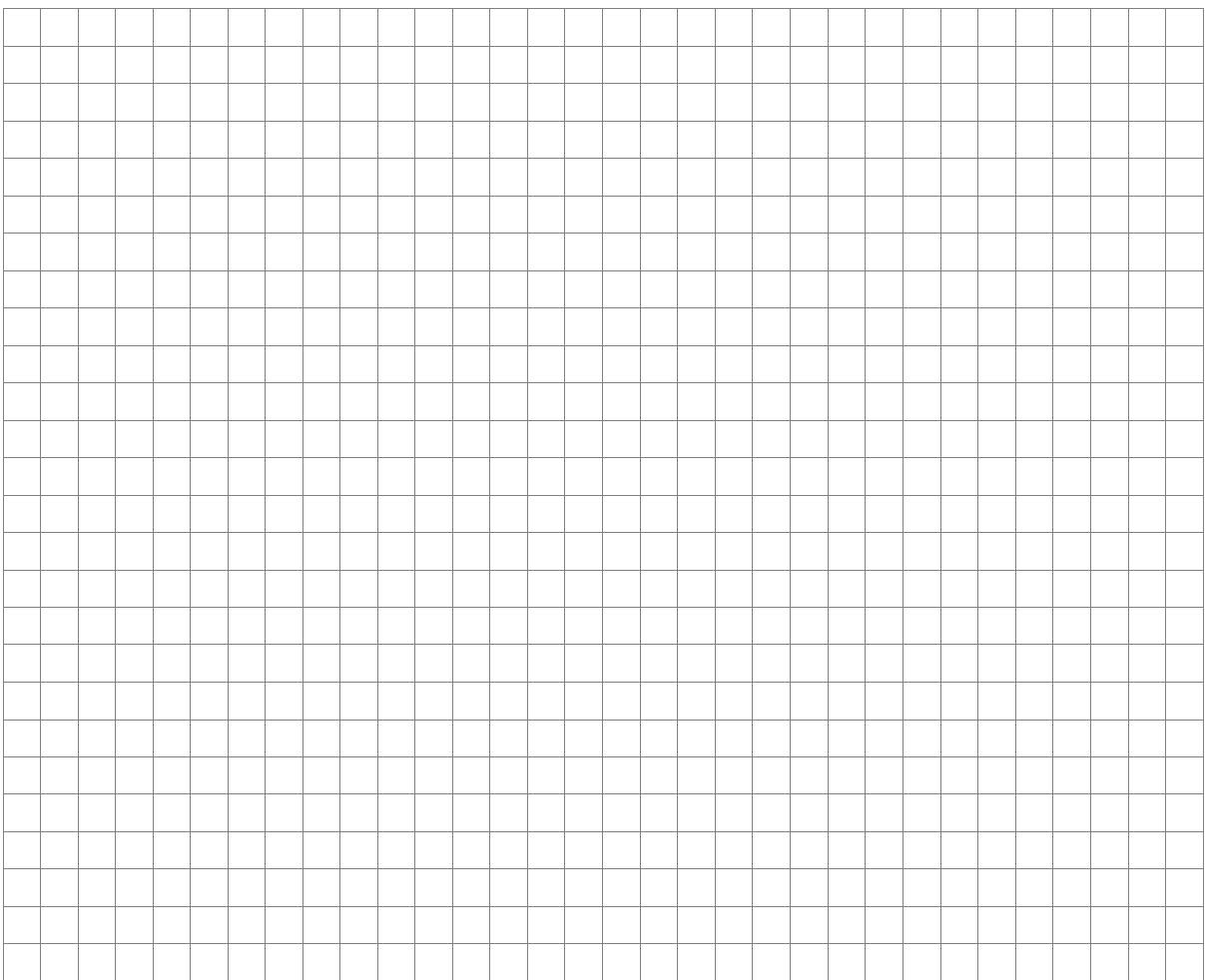
A dog food company is launching a new brand of food. The food will be sold in cylindrical shaped tins of radius  $r$  cm and height  $h$  cm. Each tin will have a volume of  $98\pi \text{ cm}^3$ .

- (a) Show that the height of the tin can be expressed as  $h = \frac{98}{r^2}$ .

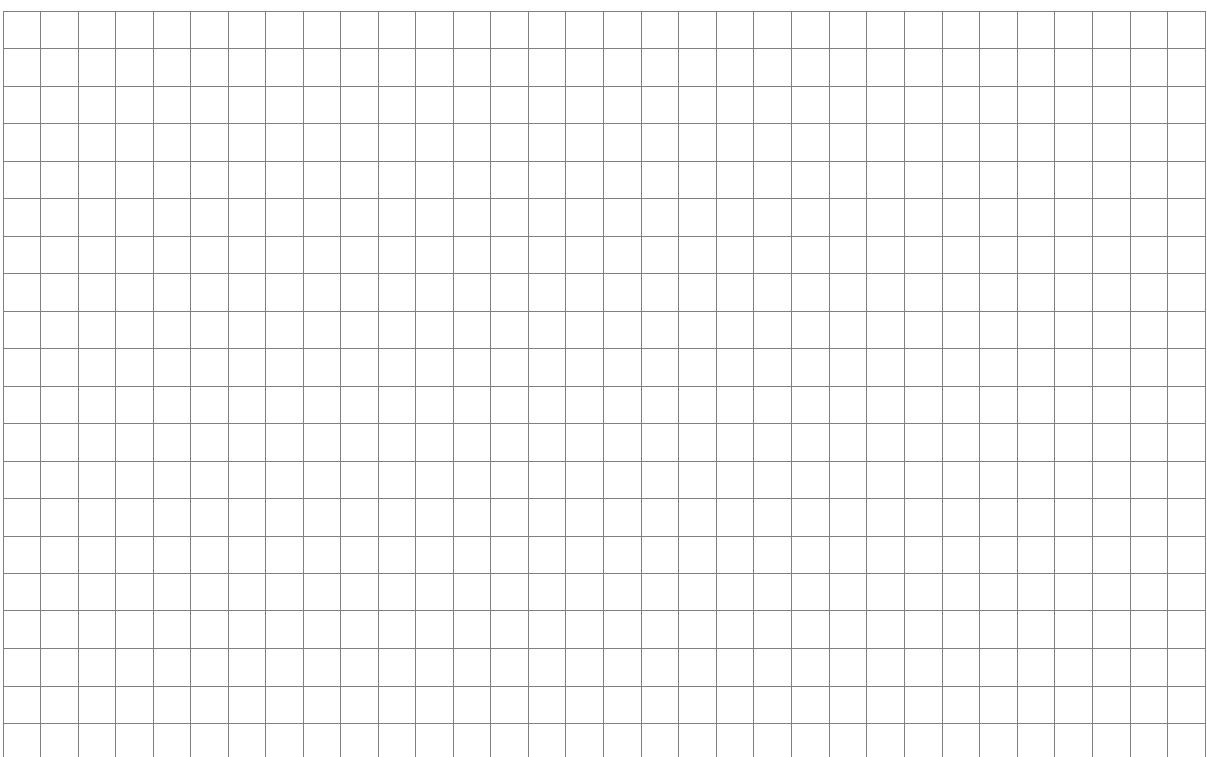


- (b)** Show that the total surface area,  $S$ , of the tin can be written as  $S = 2\pi\left(\frac{r^3 + 98}{r}\right)$ .

- (c) Find the radius of the tin that will minimise the amount of aluminium required to manufacture the tin, correct to one decimal place.

A large grid of squares, approximately 20 columns by 20 rows, intended for考生 to work out their calculations for part (c).

- (d) Hence, or otherwise, find the total surface area of the tin, correct to two decimal places.

A large grid of squares, approximately 20 columns by 20 rows, intended for考生 to work out their calculations for part (d).

## Question 9

(35 marks)

Paul and Michelle are planning on taking their children to Disneyland in 6 years' time. They estimate that the cost of the trip will be €15,000. The expected interest rate over the 6 years is 2.5% APR.

- (a)** Find the equivalent interest rate if Paul and Michelle intend to save monthly.



- (b)** How much will they need to save at the end of each month in order to reach €15,000, correct to the nearest euro?

- (c) After returning from their trip, Paul's car breaks down. He decides to replace the car. He borrows €12,000 over 3 years at an expected interest rate of 6% APR. Calculate his yearly repayment, correct to the nearest euro.



**You may use this page for extra work.**



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