



Pre-Leaving Certificate Examination, 2017
Triailscrúdú na hArdteistiméireachta, 2017

Mathematics

Paper 1

Higher Level

2½ hours

300 marks

Name:
School:
Address:
Class:
Teacher:

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Running total	
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Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. You may ask the superintendent for more paper. Label any extra work clearly with the question number and part.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write down the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1**(25 marks)**

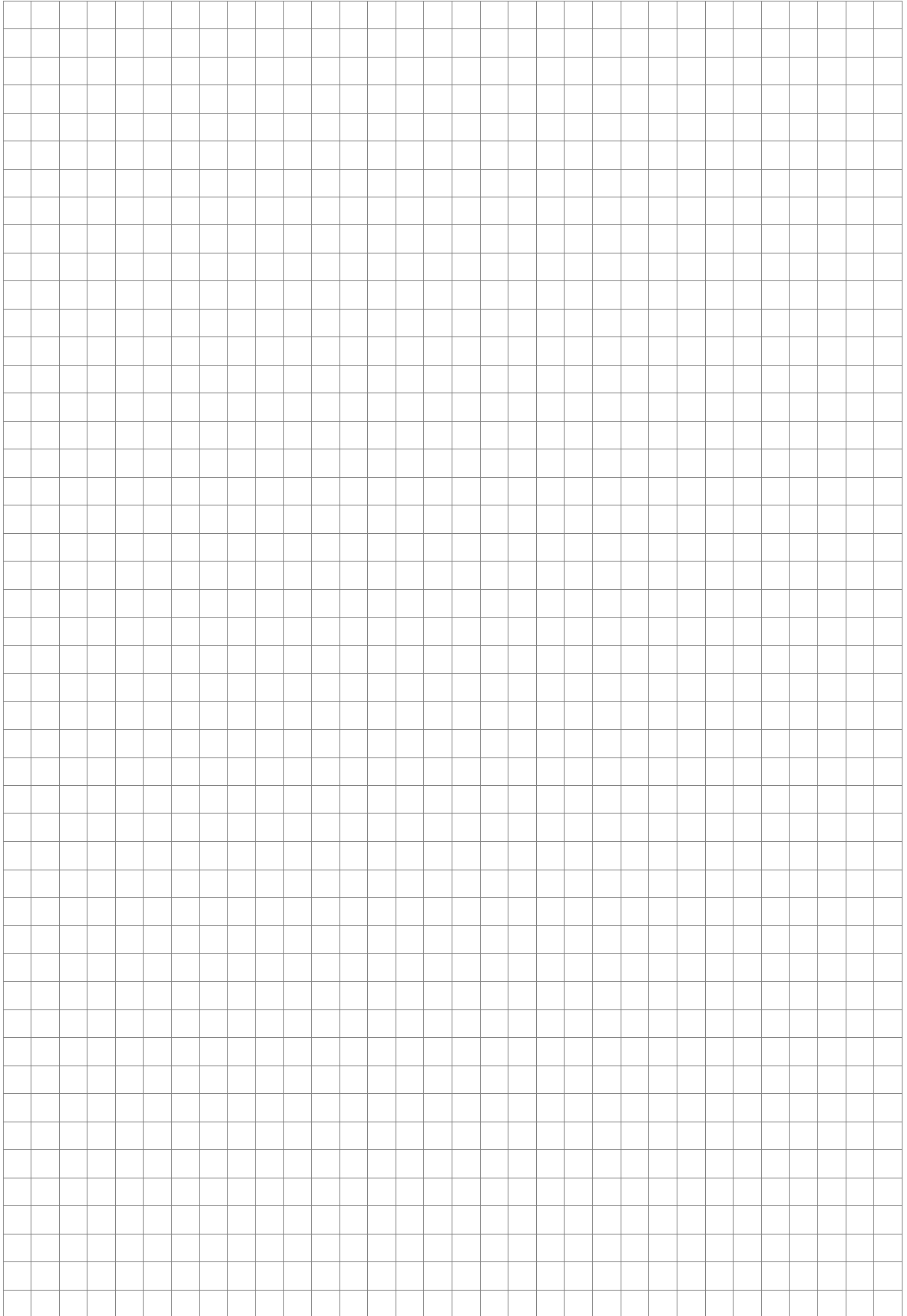
- (a) Factorise $64x^3 - 125$.

- (b) Show that for all real numbers $a \geq 2\sqrt{ab} - b$.

(c) Solve the simultaneous equations

$$x + 3y = -3$$

$$x^2 + y^2 = 13.$$



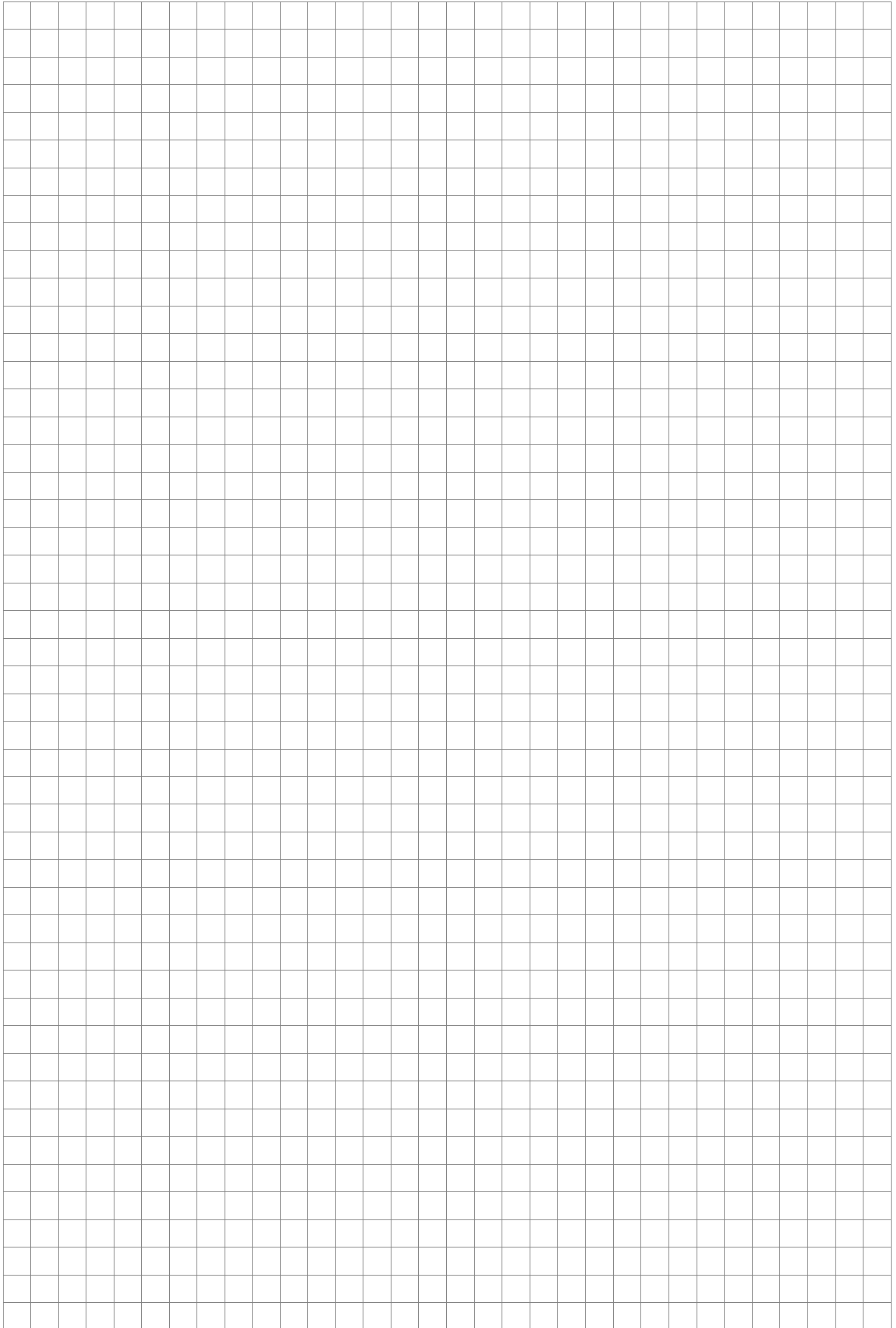
Question 2

(25 marks)

- (a) By letting $x = 5^y$, show that the equation $p5^y + 5^{-y} = 5$ can be written in the form $px^2 - 5x + 1 = 0$.

- (b) (i) Find the value of p for which the equation $p5^y + 5^{-y} = 5$ has equal roots.

- (ii) Hence, or otherwise, solve the equation $p5^y + 5^{-y} = 5$, correct to two decimal places.



Question 3

(25 marks)

- (a) Solve the inequality $\frac{7-2x}{x+3} \leq 5$, for $x \in \mathbb{R}$, $x \neq -3$.

- (b) Prove by induction that $7^n - 1$ is divisible by 3, for all $n \in \mathbb{N}$.

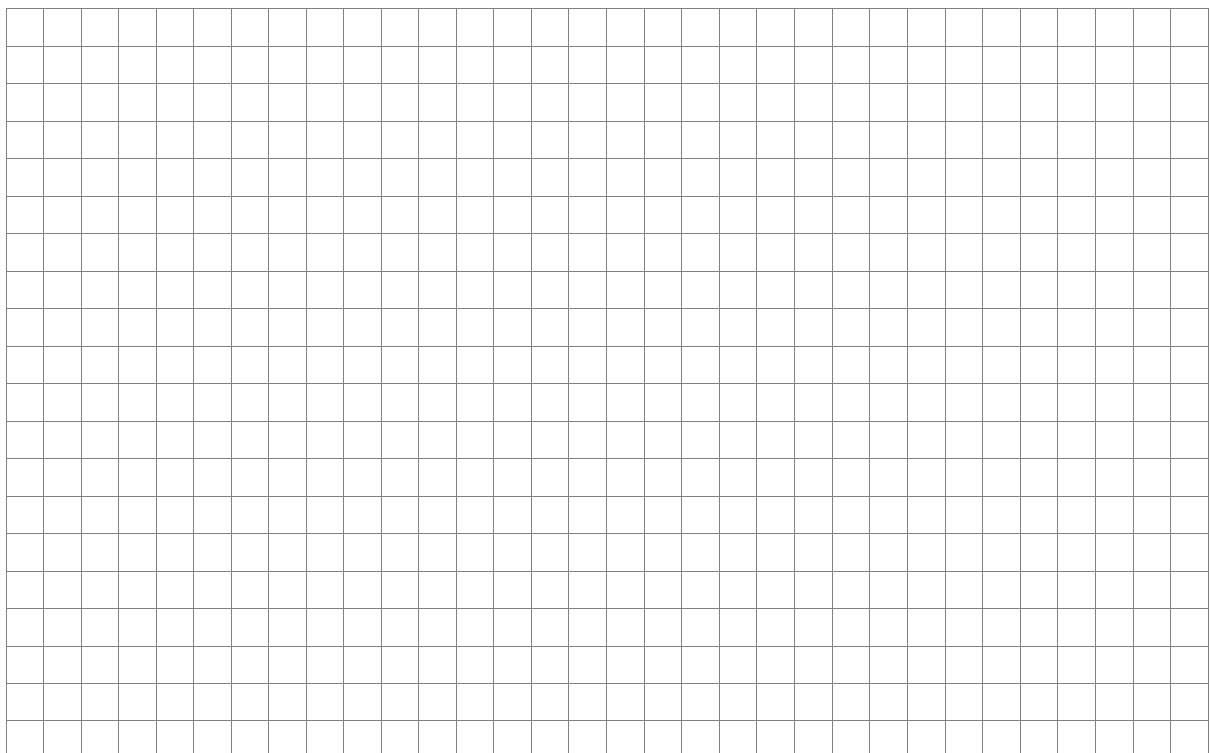
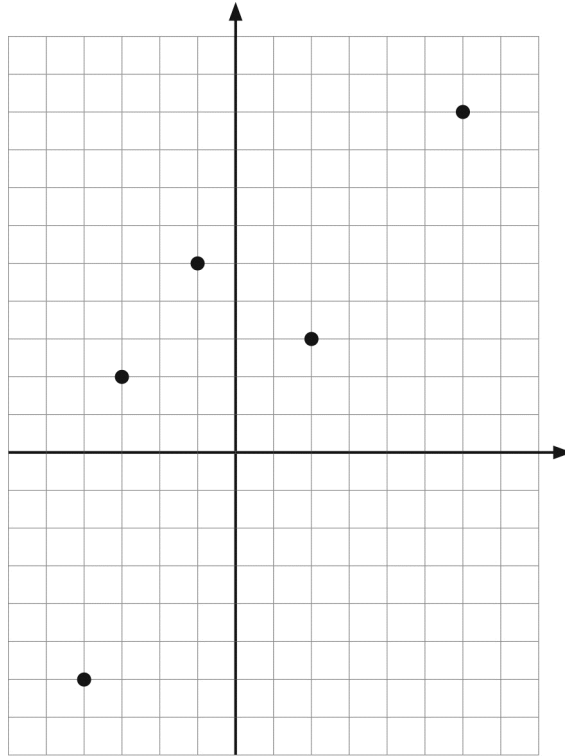
Question 4

(25 marks)

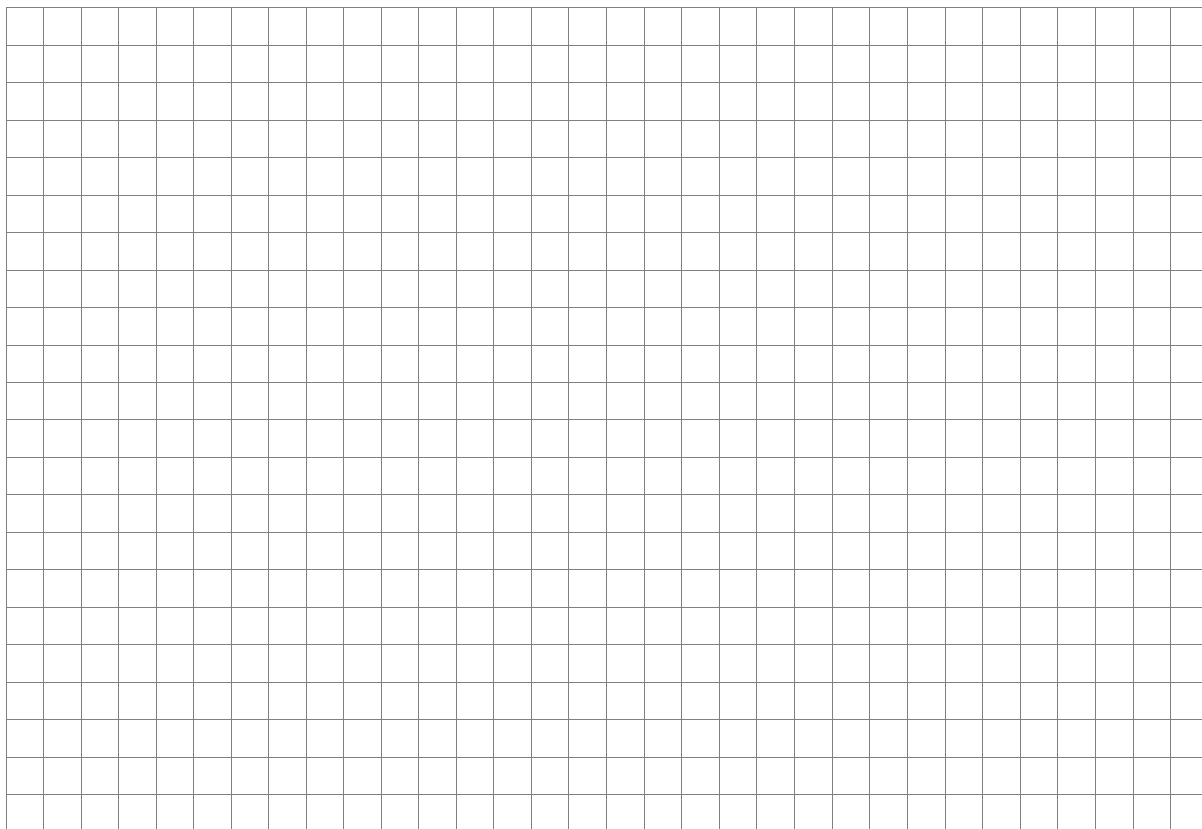
(a) The complex numbers z_1, z_2, z_3, z_4, z_5 are shown below, where

$$z_2 = -2z_3, \quad z_1 = 3z_3, \quad z_4 = iz_3, \quad z_5 = z_3 + z_4.$$

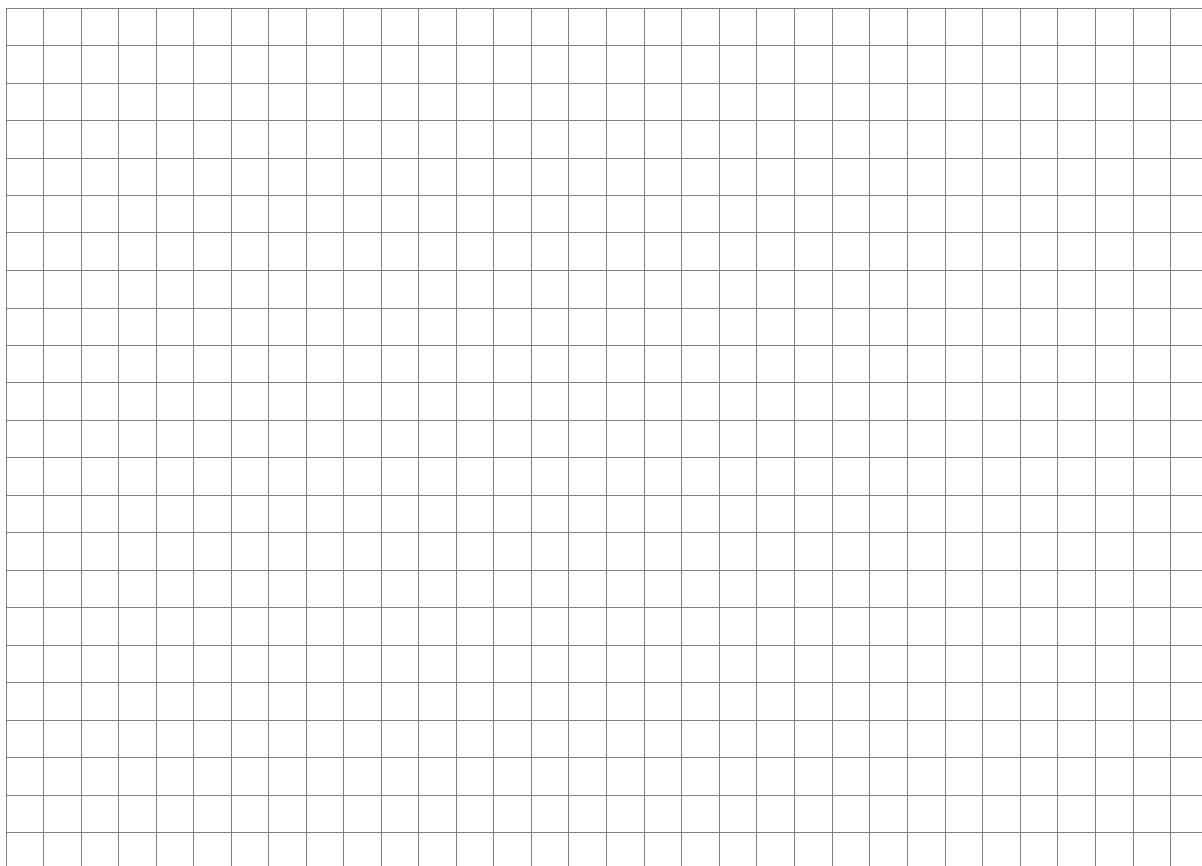
Identify each complex number on the Argand diagram.



- (b) Let $z_1 = 2 - 5i$ and let $z_2 = -1 - 3i$. Investigate if $\frac{|z_1|}{|z_2|} = \frac{|z_1|}{|z_2|}$.



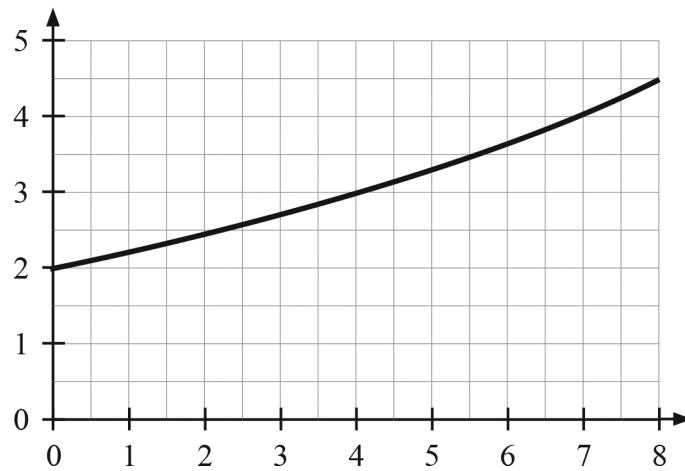
- (c) Use de Moivre's Theorem to prove the identity $\cos 3\theta = 4\cos^3 \theta - 3\cos \theta$.



Question 5

(25 marks)

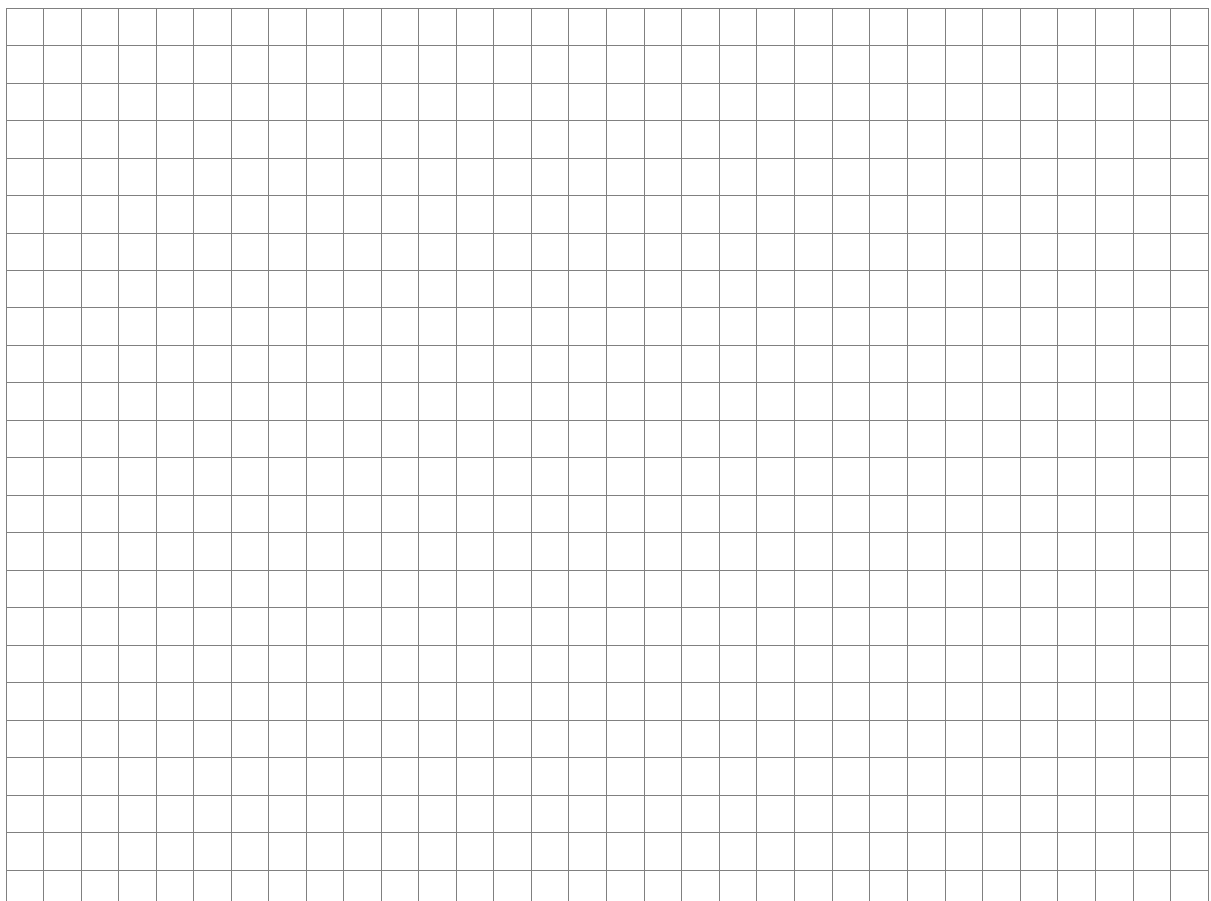
The graph of the function $g(x) = 2e^{0.1x}$ in the domain $0 \leq x \leq 8$ is shown below.



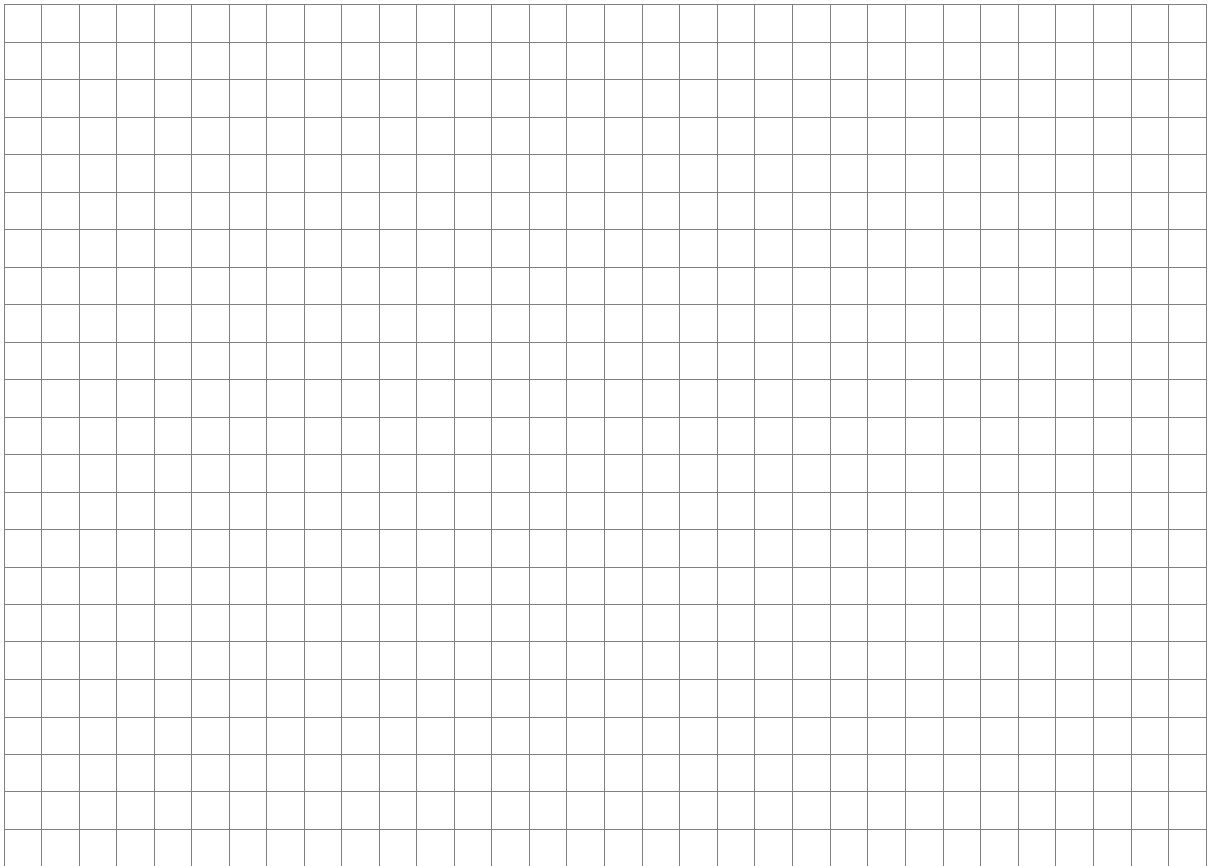
(a) (i) Fill in the following table

x	0	1	2	3	4	5	6	7	8
$g(x)$									

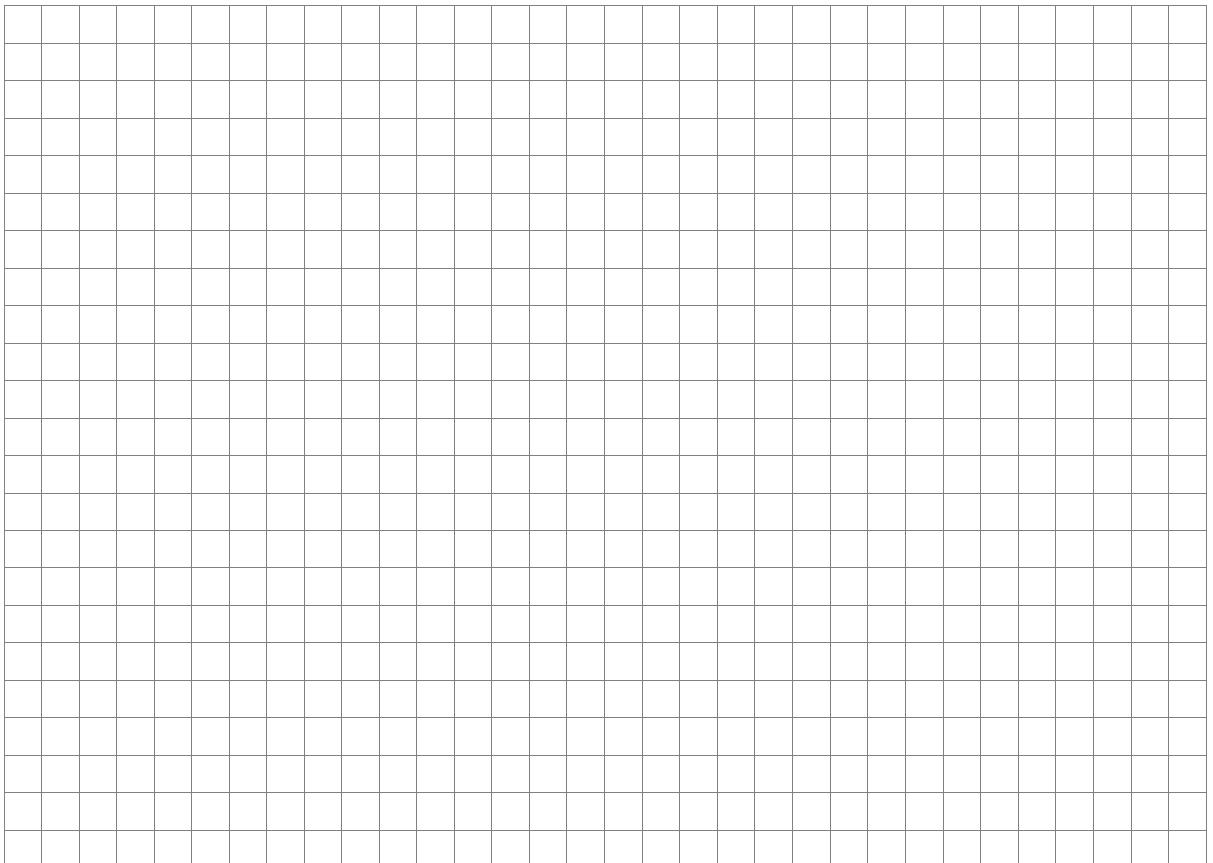
(ii) Use the trapezoidal rule to find an approximation for the area bounded by the curve $g(x)$ and the x axis, correct to one decimal place.



- (iii)** Use integration methods to find the area under the curve, correct to one decimal place.



- (b)** Differentiate $2x^2 + 3x + 3$ from first principles.



Question 6

(25 marks)

- (a) Differentiate $y = x^2 \sin(3x + 5)$, with respect to x .

- (b) Differentiate $\cos^{-1} \frac{2}{x^2}$, with respect to x .

(c) If $y = x^4 e^{3x}$, show that $\frac{dy}{dx} - 3y = \frac{4y}{x}$.



Answer **all three** questions from this section.

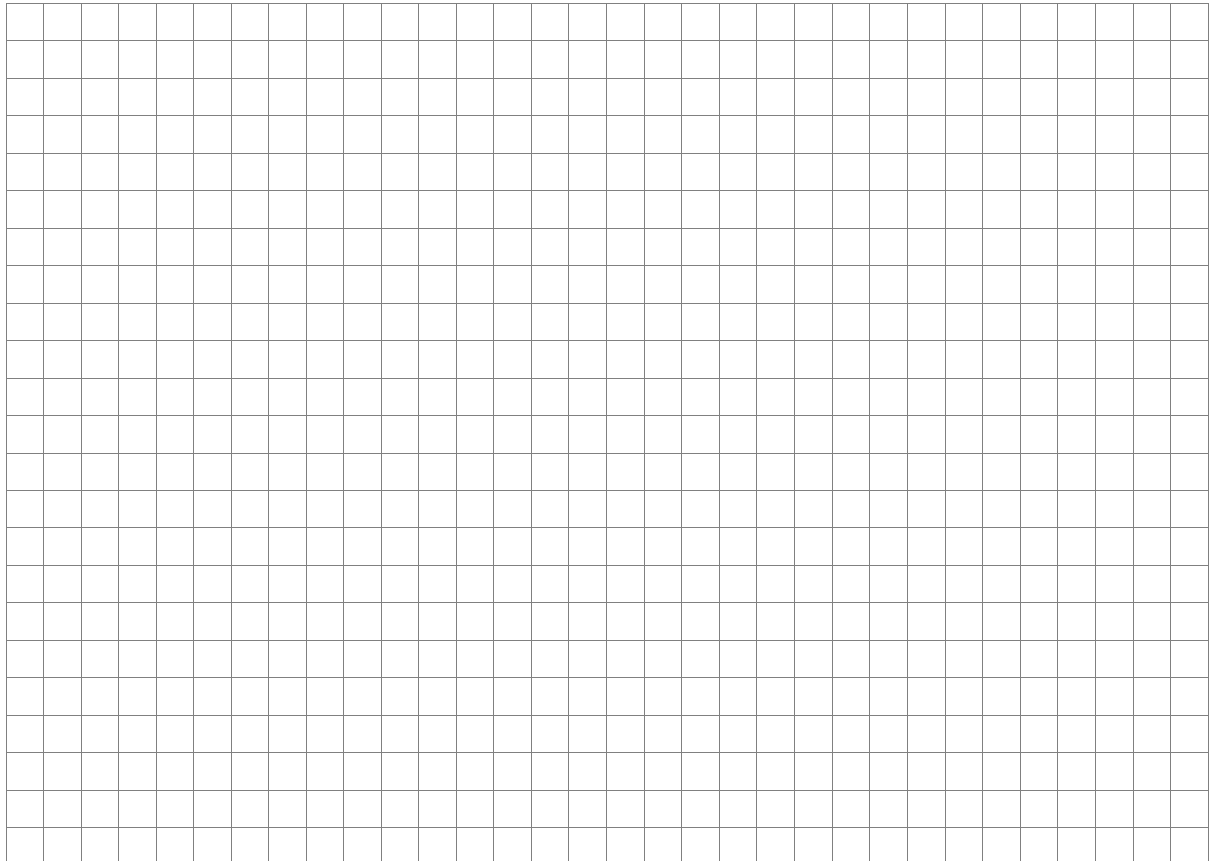
Question 7**(75 marks)**

DigiAgeToys, a digital toy manufacturer, is trying to determine the optimum selling price for their newly developed toy.

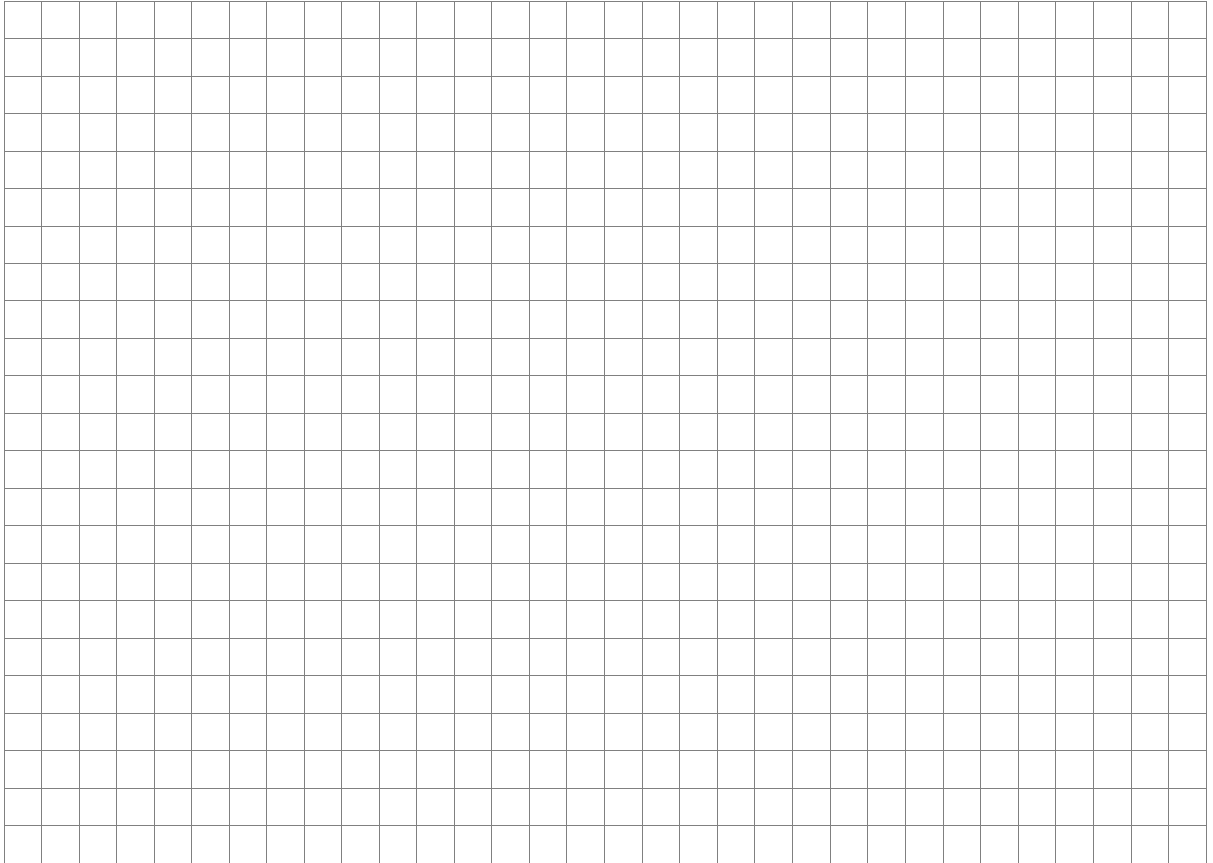
From previous years' data of new toys, they have the following data.

Selling Price € (x)	Number of Toys Sold in 1 year (n) (000's)
10	1000
15	900
20	800
25	700

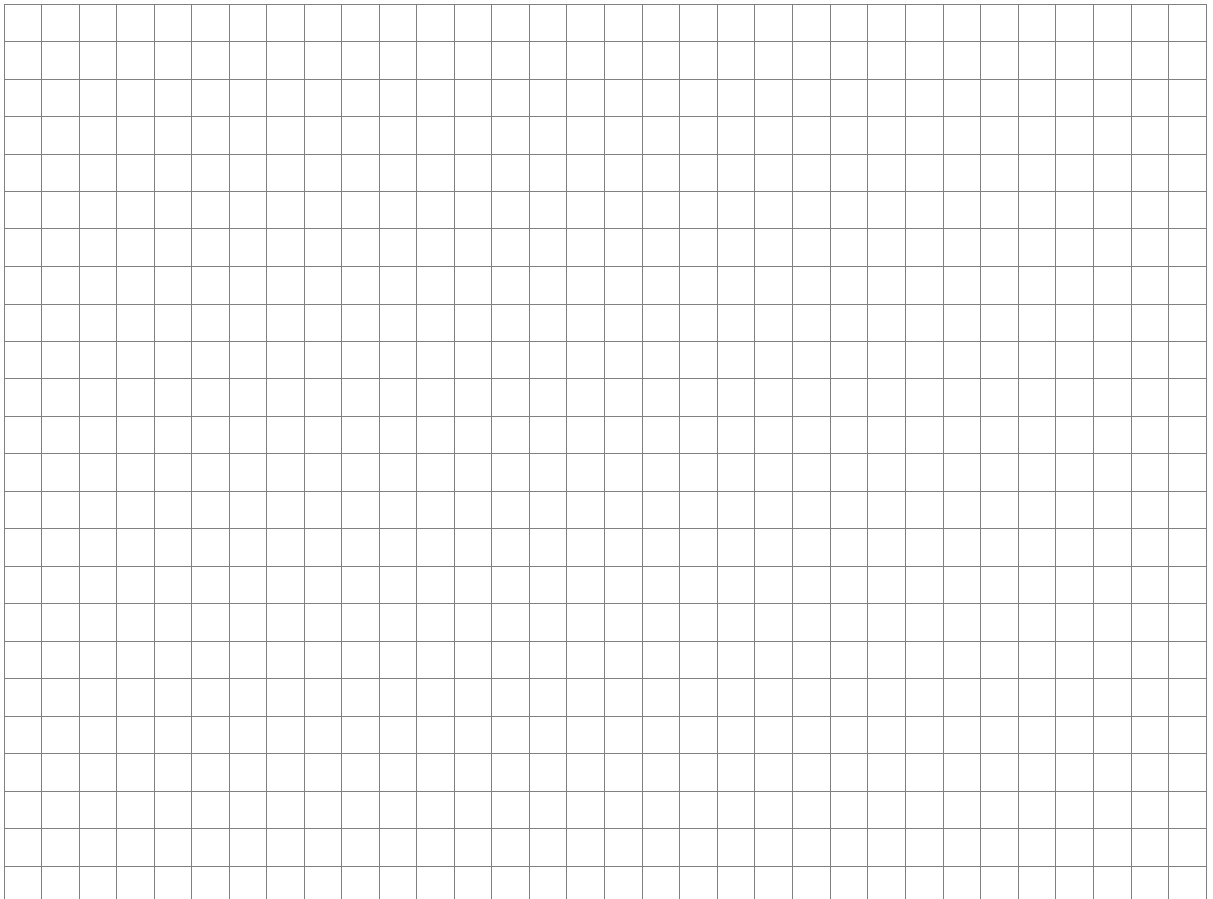
- (a) (i) Describe the relationship between the selling price and the quantity of the toys sold.



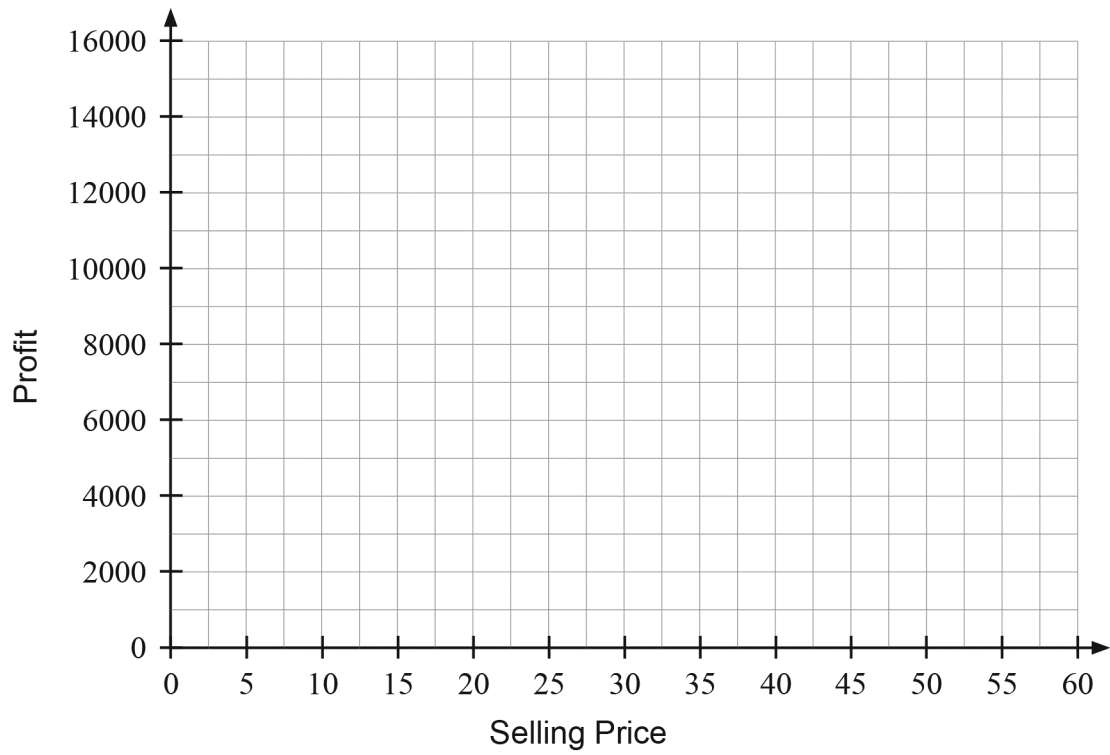
(iii) Find the selling price that will maximize the profit earned from the sale of the toy.



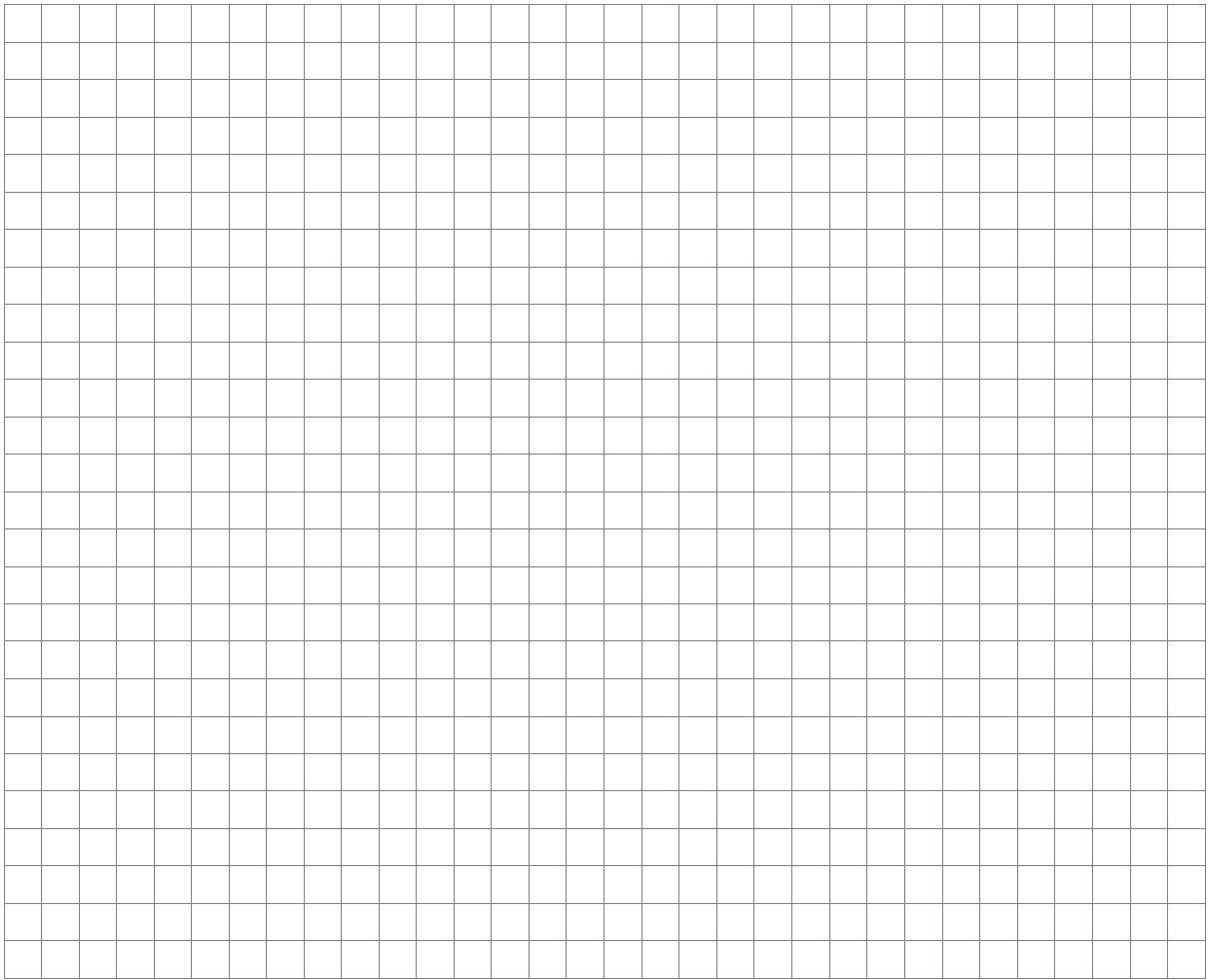
(iv) Find the maximum profit made by the company.



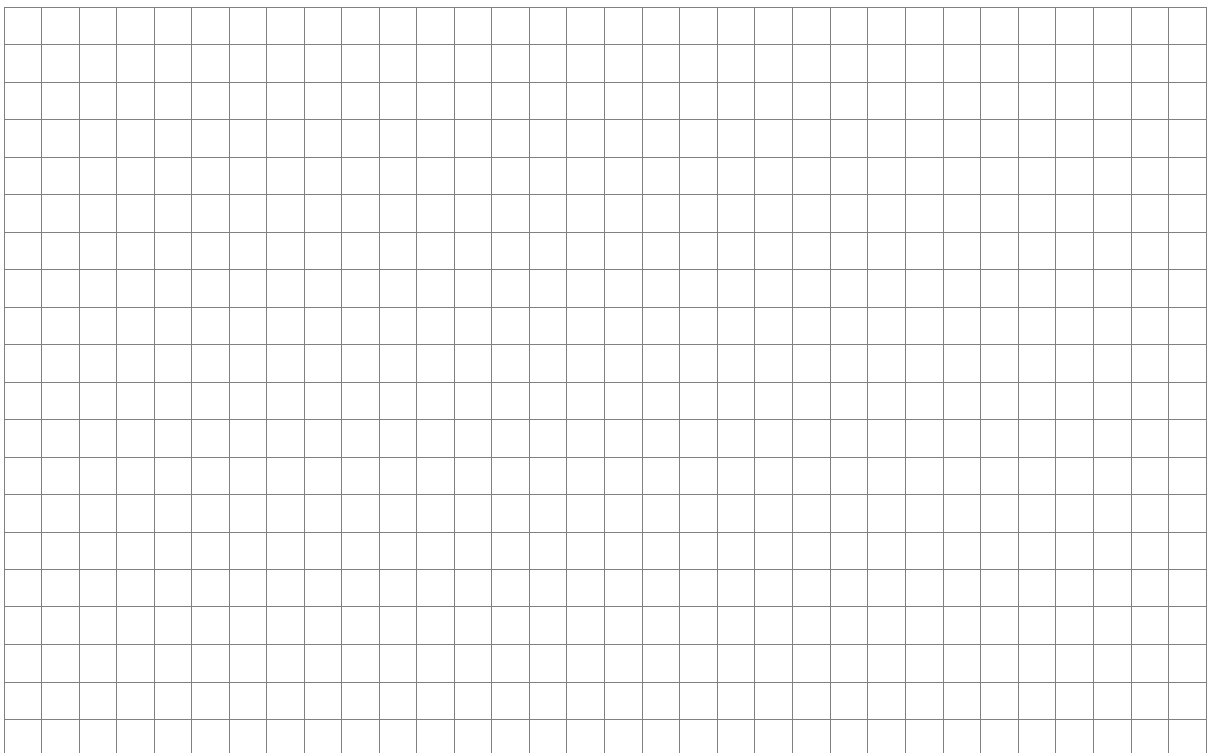
(v) Draw a sketch of the function $P(x)$.



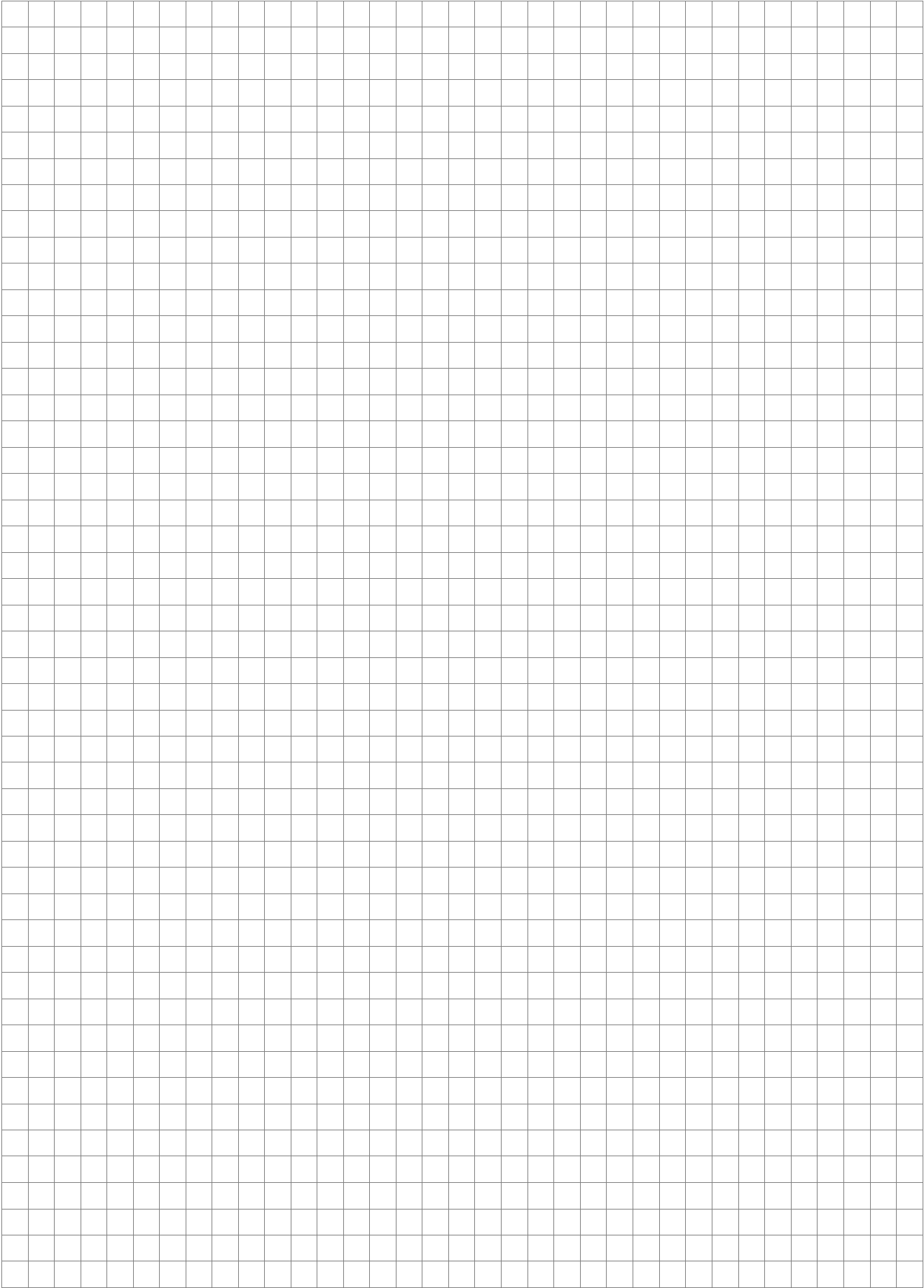
- (c) Find the radius of the tin that will minimise the amount of aluminium required to manufacture the tin, correct to one decimal place.



- (d) Hence, or otherwise, find the total surface area of the tin, correct to two decimal places.



You may use this page for extra work.



You may use this page for extra work.

