Ex 4.2

Q1 (i) $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$

(ii) $\int 3e^{x}\,dx = 3e^{x} + C$

(iii) $\int 2e^{4x}\,dx = \left(\frac{1}{4}\right)2e^{4x} + C = \frac{1}{2}e^{4x} + C$

(iv) $\int e^{-3x}\,dx = \frac{1}{-3}e^{-3x} + C$

Q2 (i) $\int \left(e^{3x} + 4\right)dx = \frac{1}{3}e^{3x} + 4x + C$

(ii) $\int 4e^{\frac{1}{2}x}\,dx = \left(\frac{1}{\frac{1}{2}}\right)4e^{\frac{1}{2}x} + C = 8e^{\frac{1}{2}x} + C$

(iii) $\int \left(e^{4x} + \frac{1}{e^{4x}}\right)dx = \int e^{4x} + e^{-4x} = \frac{1}{4}e^{4x} - \frac{1}{4}e^{-4x} + C$

$= \frac{e^{4x}}{4} - \frac{1}{4e^{4x}} + C$

Q3 $y = e^{x^2}$

$\frac{dy}{dx} = 2xe^{x^2}$

$\Rightarrow \int 2xe^{x^2} = e^{x^2} + C$

Q4 (i) $\int \cos 3x \, dx = \frac{\sin 3x}{3} + C$

(ii) $\int \sin 4x \, dx = -\frac{\cos 4x}{4} + C$

(iii) $\int -\sin 5x \, dx = -\left(\frac{-\cos 5x}{5}\right) + C = \frac{\cos 5x}{5} + C$

(iv) $\int \cos kx \, dx = \frac{\sin kx}{k} + C$

Q5

(i) $\int 3\cos 6x \, dx = 3 \cdot \frac{\sin 6x}{6} + C = \frac{\sin 6x}{2} + C$

(ii) $\int (\cos 2x - \sin 5x) \, dx = \frac{\sin 2x}{2} + \frac{\cos 5x}{5} + C$

(iii) $\int 3\cos(-9x) \, dx = 3 \cdot \frac{\sin(-9x)}{-9} + C$

$= \frac{\sin(-9x)}{-3} + C$

Q6 $\int 3(e^x - 4\sin 3x + 2) \, dx$

$= \int (3e^x - 12\sin 3x + 6) \, dx$

$= 3e^x + \frac{12\cos 3x}{3} + 6x + C$

$= 3e^x + 4\cos 3x + 6x + C$

Q7 (i) $\int (4e^{2x} + 4\sin 3x)\,dx$

$= 4\left(\frac{1}{2}\right)e^{2x} - 4\frac{\cos 3x}{3} + C$

$2e^{2x} - \frac{4\cos 3x}{3} + C$

(ii) $\int (3\cos x - 2\cos 4x)\,dx$

$= 3\sin x - \frac{2\sin 4x}{4} + C$

$= 3\sin x - \frac{\sin 4x}{2} + C$

Q8 $y = \cos 4x^2$.

$\frac{dy}{dx} = -8x \sin 4x^2$

$\Rightarrow \int -8x \sin 4x^2 = \cos 4x^2 + C$

Q9 $\boxed{\frac{x+y}{z} = \frac{x}{z} + \frac{y}{z}}$

(i) $\int \frac{e^{2x}+4}{e^x}\,dx = \int \left(\frac{e^{2x}}{e^x} + \frac{4}{e^x}\right) dx$

$= \int (e^x + 4e^{-x})\,dx = e^x + \frac{4e^{-x}}{-1} + C$

$= e^x - 4e^{-x} + C$

$= e^x - \frac{4}{e^x} + C$

(ii) $\int \left(\frac{e^{x+2}+3}{e^x}\right) dx = \int \left(\frac{e^{x+2}}{e^x} + \frac{3}{e^x}\right) dx$

$= \int (e^2 + 3e^{-x})\,dx = xe^2 + \frac{3e^{-x}}{-1} + C$

$= xe^2 - \frac{3}{e^x} + C$

(iii) $\int \left(\frac{1+3e^x}{e^{2x}}\right) dx = \int \left(\frac{1}{e^{2x}} + \frac{3e^x}{e^{2x}}\right) dx$

$= \int (e^{-2x} + 3e^{-x})\,dx = \frac{e^{-2x}}{-2} + \frac{3e^{-x}}{-1} + C$

$= -\frac{e^{-2x}}{2} - 3e^{-x} + C$

$= -\frac{1}{2e^{2x}} - \frac{3}{e^x} + C$

Q10 (i) $\int (e^x - e^{-x})^2 \, dx$

$= \int (e^{2x} - 2e^x \cdot e^{-x} + e^{-2x}) \, dx$

$= \int (e^{2x} - 2 + e^{-2x}) \, dx$

$= \frac{e^{2x}}{2} - 2x + \frac{e^{-2x}}{-2} + C$

$= \frac{e^{2x}}{2} - 2x - \frac{1}{2e^{2x}} + C$

(ii) $\int (3 + e^x)(2 + e^{-x}) \, dx$

$= \int (6 + 3e^{-x} + 2e^x + e^x \cdot e^{-x}) \, dx$

$= \int (6 + 3e^{-x} + 2e^x + 1) \, dx$

$= \int (3e^{-x} + 2e^x + 7) \, dx$

$= \frac{3e^{-x}}{-1} + 2e^x + 7x + C$

$= -\frac{3}{e^x} + 2e^x + 7x + C$

Q11 $y = 7^x$

$\ln y = x \ln 7$

$\frac{\ln y}{\ln 7} = x$

(i) $\frac{dx}{dy} = \frac{1}{y} \cdot \frac{1}{\ln 7} = \frac{1}{y \ln 7}$

(ii) $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = y \ln 7$ but $y = 7^x$ (ii)

$\frac{dy}{dx} = 7^x \ln 7$.

(iii) $\int 7^x dx = \frac{7^x}{\ln 7} + C$

Q12 $\frac{dy}{dx} = ae^{-x} + 2$ $\frac{dy}{dx} = 5$ when $x = 0$

$\Rightarrow ae^{-0} + 2 = 5$

$a + 2 = 5$

$a = 3$. $\Rightarrow \frac{dy}{dx} = 3e^{-x} + 2$

$y = \int 3e^{-x} + 2$

$y = \frac{3e^{-x}}{-1} + 2x + C$ $y = -3$ when $x = 0$.

$-3 = -3e^{-0} + 2(0) + C$

$-3 = -3 + C$

$0 = C$

$\Rightarrow y = -3e^{-x} + 2x$

Q13 $\frac{dy}{dx} = e^{kx}$ Tangent at $(1, e^2)$ through $(0,0)$

$$m = \frac{e^2 - 0}{1 - 0} = e^2.$$

(i) Slope $= \frac{dy}{dx}$ at $x = 1$

$$e^{k(1)} = e^2$$

$$\Rightarrow k = 2.$$

(ii) eqn $= \int e^{2x}\, dx$

$$y = \frac{e^{2x}}{2} + C$$

Pt $(1, e^2)$

$$e^2 = \frac{e^{2(1)}}{2} + C$$

$$e^2 - \frac{e^2}{2} = C$$

$$\frac{e^2}{2} = C$$

$$\Rightarrow y = \frac{e^{2x}}{2} + \frac{e^2}{2} = \frac{1}{2}\left(e^{2x} + e^2\right)$$

Q14 $f(x) = 2x e^x$

(i) $f'(x) = (2x)(e^x) + (e^x)(2)$ Product Rule.

$= 2xe^x + 2e^x$

(ii) $\int (2x e^x)\, dx$

$\int (2xe^x + 2e^x)\, dx = 2xe^x + C$

$\int (2xe^x)dx + \int (2e^x)dx = 2xe^x + C$

$\int (2xe^x)dx + 2e^x = 2xe^x + C$

$\int (2xe^x)dx = 2xe^x - 2e^x + C$

Q15 $f(x) = x \sin x$

$f'(x) = x\cos x + \sin x(1)$

$\int x \cos x\, dx$.

$\int (x\cos x + \sin x)dx = x\sin x + C$

$\int (x\cos x)dx + \int \sin x\, dx = x\sin x + C$

$\int (x\cos x)dx - \cos x = x\sin x + C$

$\int (x\cos x)dx = x\sin x + \cos x + C$.

Q16 $f(x) = 4xe^{2x}$

(i) $f'(x) = (4x)(2e^{2x}) + e^{2x}(4)$

$= 8xe^{2x} + 4e^{2x}$

(ii) $\int (8xe^{2x})\,dx$

$\int (8xe^{2x} + 4e^{2x})\,dx = 4xe^{2x} + C$

$\int (8xe^{2x})\,dx + \int (4e^{2x})\,dx = 4xe^{2x} + C$

$\int (8xe^{2x})\,dx + 4\frac{e^{2x}}{2} = 4xe^{2x} + C$

$\int (8xe^{2x})\,dx = 4xe^{2x} - 2e^{2x} + C$

Q17 $y = 2xe^{3x} + \cos x$

$\frac{dy}{dx} = 2x(3)e^{3x} + e^{3x}(2) - \sin x$

$\frac{dy}{dx} = 6xe^{3x} + 2e^{3x} - \sin x$

$\int (6xe^{3x})\,dx$

$\int (6xe^{3x} + 2e^{3x} - \sin x)\,dx = 2xe^{3x} + \cos x + C$

$\int (6xe^{3x})\,dx + \int (2e^{3x})\,dx - \int \sin x\,dx = 2xe^{3x} + \cos x + C$

$\int (6xe^{3x})\,dx + \frac{2e^{3x}}{3} + \cos x = 2xe^{3x} + \cos x + C$

$\int (6xe^{3x})\,dx = 2xe^{3x} - \frac{2}{3}e^{3x} + C$