

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

| Axioms and Theorems <small>(Supported by 46 definitions, 20 propositions) * proof required for JCHL and LCHL ** proof required for LCHL only</small> | | Junior Certificate | | | Leaving Certificate | | |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|----|----|---------------------|----|----|
| | | CC | OL | HL | FL | OL | HL |
| | Axiom 1: There is exactly one line through any two given points | ● | ● | ● | ● | ● | ● |
| | Axiom 2: [Ruler Axiom]: The properties of the distance between points. | ● | ● | ● | ● | ● | ● |
| | Axiom 3: Protractor Axiom (The properties of the degree measure of an angle). | ● | ● | ● | ● | ● | ● |
| 1 | Vertically opposite angles are equal in measure. | ● | ● | ● | ● | ● | ● |
| | Axiom 4: Congruent triangles conditions (SSS, SAS, ASA) | ● | ● | ● | ● | ● | ● |
| 2 | In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles. | ● | ● | ● | ● | ● | ● |
| | Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l. | ● | ● | ● | ● | ● | ● |
| 3 | If a transversal makes equal alternate angles on two lines then the lines are parallel (and converse). | ● | ● | ● | ● | ● | ● |
| 4* | The angles in any triangle add to 180° . | ● | ● | ● | ● | ● | ● |
| 5 | Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal. | ● | ● | ● | ● | ● | ● |
| 6* | Each exterior angle of a triangle is equal to the sum of the interior opposite angles. | ● | ● | ● | ● | ● | ● |
| 7 | The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle. | | | | ● | ● | |
| 8 | Two sides of a triangle are together greater than the third. | | | | ● | ● | |
| 9* | In a parallelogram, opposite sides are equal, and opposite angles are equal. | | ● | ● | ● | ● | ● |
| | Corollary 1. A diagonal divides a parallelogram into two congruent triangles. | | | ● | | | ● |
| 10 | The diagonals of a parallelogram bisect each other. | | ● | ● | ● | ● | ● |
| 11** | If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal. | | | ● | | ● | ● |
| 12** | Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. | | | ● | | ● | ● |
| 13** | If two triangles are similar, then their sides are proportional, in order. | | ● | ● | ● | ● | ● |
| 14* | [Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides. | | ● | ● | ● | ● | ● |
| 15 | [Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle. | | ● | ● | ● | ● | ● |
| | Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent. | | ● | ● | ● | ● | ● |
| 16 | For a triangle, base x height does not depend on the choice of base. | | | | | ● | ● |

| | | | | | | | | |
|-----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--|---|---|---|---|---|---|
| | Definition 38: The area of a triangle is half the base by the height. | | | | | | • | • |
| 17 | A diagonal of a parallelogram bisects the area. | | | | | | • | • |
| 18 | The area of a parallelogram is the base x height. | | | | | | • | • |
| 19* | The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. | | | • | | | | • |
| | Corollary 2†: All angles at points of a circle, standing on the same arc are equal | | | • | | | | • |
| | Corollary 3: Each angle in a semi-circle is a right angle. | | • | • | • | • | • | • |
| | Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter. | | • | • | • | • | • | • |
| | Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180° . | | | • | | | | • |
| 20 | (i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S. | | | | | | • | • |
| | Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear. | | | | | | • | • |
| 21 | (i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre. | | | | | | • | • |

† The corollaries are numbered as in the appendix; corollary 2 is the first one relating to theorem 9

| Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems) | | Junior Certificate | | | Leaving Certificate | | |
|-------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|--------------------|----|----|---------------------|----|----|
| | | CIC | OL | HL | FL | OL | HL |
| 1 | Bisector of an angle, using only compass and straight edge. | ● | ● | ● | ● | ● | ● |
| 2 | Perpendicular bisector of a segment, using only compass and straight edge. | ● | ● | ● | ● | ● | ● |
| 3 | Line perpendicular to a given line l, passing through a given point not on l. | | ● | ● | | | ● |
| 4 | Line perpendicular to a given line l, passing through a given point on l. | ● | ● | ● | ● | ● | ● |
| 5 | Line parallel to given line, through a given point. | ● | ● | ● | ● | ● | ● |
| 6 | Division of a line segment into 2 or 3 equal segments without measuring it. | ● | ● | ● | ● | ● | ● |
| 7 | Division of a line segment into any number of equal segments, without measuring it. | | | ● | | | ● |
| 8 | Line segment of a given length on a given ray. | ● | ● | ● | ● | ● | ● |
| 9 | Angle of a given number of degrees with a given ray as one arm. | | ● | ● | ● | ● | ● |
| 10 | Triangle, given lengths of 3 sides. | | ● | ● | ● | ● | ● |
| 11 | Triangle, given SAS data. | | ● | ● | ● | ● | ● |
| 12 | Triangle, given ASA data | | ● | ● | ● | ● | ● |
| 13 | Right-angled triangle, given length of hypotenuse and one other side | | ● | ● | ● | ● | ● |
| 14 | Right-angled triangle, given one side and one of the acute angles. | | ● | ● | ● | ● | ● |
| 15 | Rectangle given side lengths. | | | ● | ● | ● | ● |
| 16 | Circumcentre and circumcircle of a given triangle, using only straight edge and compass. | | | | | ● | ● |
| 17 | Incentre and incircle of a triangle of a given triangle, using only straight edge and compass. | | | | | ● | ● |
| 18 | Angle of 60° without using a protractor or set square. | | | | ● | ● | ● |
| 19 | Tangent to a given circle at a given point on it. | | | | ● | ● | ● |
| 20 | Parallelogram, given the length of the sides and the measure of the angles. | | | | ● | ● | ● |
| 21 | Centroid of a triangle. | | | | | ● | ● |
| 22 | Orthocentre of a triangle. | | | | | | ● |