## Section 7.10 The Graph of $y = \log_a(x)$


Graphs of logs are the inverse of graphs of exponential functions.

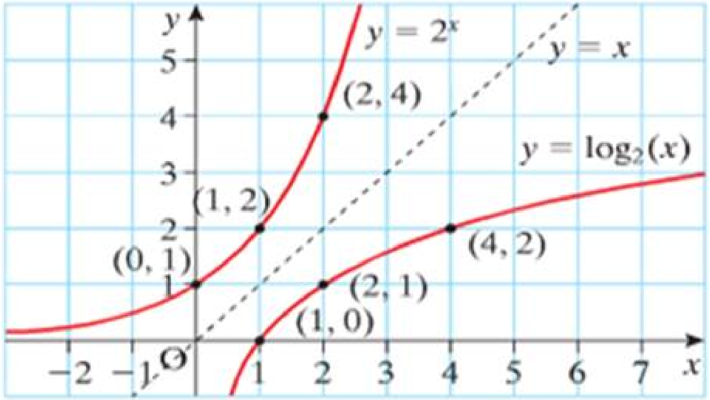
$y = \log_a x$ - All pass through (1, 0), $\log_{\text{any base}} 1 = 0$

- All pass through (a, 1) , $\log_2 2 = \log_e e = \log_{10} 10 = 1$
- All graphs of log functions are increasing.
- All tend towards the negative y-axis but never reach it.
- The larger the base number 'a', the flatter the curve.
- $y = \log_a(0)$ is not defined.

## Comparing $\log_a(x)$ with $y = a^x$

If we let $a = 2$; comparing $y = 2^x$ and $y = \log_2(x)$, we have

| $x$ | $y = 2^x$ | $x$ | $y = \log_2 x$ |
|---|---|---|---|
| 0 | $y = 2^0 = 1$ | 1 | $y = \log_2 1 = 0$ |
| 1 | $y = 2^1 = 2$ | 2 | $y = \log_2 2 = 1$ |
| 2 | $y = 2^2 = 4$ | 4 | $y = \log_2 4 = 2$ |
| 3 | $y = 2^3 = 8$ | 8 | $y = \log_2 8 = 3$ |
| 4 | $y = 2^4 = 16$ | 16 | $y = \log_2 16 = 4$ |




The points $(0, 1), (1, 2), (2, 4), (3, 8), (4, 16), \ldots$ are on the curve $y = 2^x$.
The points $(1, 0), (2, 1), (4, 2), (8, 3), (16, 4), \ldots$ are on the curve $y = \log_2(x)$.

$\therefore \quad y = \log_2(x)$ is the inverse function of $y = 2^x$.

Also, $y = \log_{10} x$ is the inverse function of $y = 10^x$.

The graphs of $y = 2^x$ and $y = \log_2(x)$ reflect in the line $x = y$.

~~Section~~

Exercise 7.10.

Q1. 1. $\log_{(\text{any base})} 1 = 0$.

$\text{Any Base}^0 = 1$ True

2. $\log_2 2 = \log_e e = \log_{10} 10 = 1$

$\text{Any Base}^{1} = \text{The } N^o$

$2^1 = 2 \quad e^1 = e \quad 10^1 = 10$

5. $y = \log_a(0)$ is not defined.

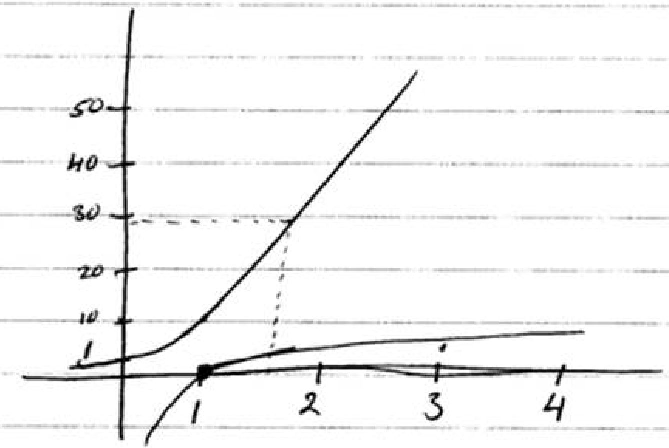
$a^y = 0$ Any base to a power $= 0$ does not exist

Theres is $N^o$ Solution

does not exist
Theres is No Solution

Q2

$y = 10^x$




$10^{1.5} = 30.$

**Q3** $y = \log_3 x$

(i)

| $x$ | $1/9$ | $1/3$ | $1$ | $3$ | $9$ |
|---|---|---|---|---|---|
| $y=\log_3 x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |

$\log_3 \frac{1}{9} = y$
$3^y = \frac{1}{9}$
$3^y = 3^{-2}$
$y = -2$

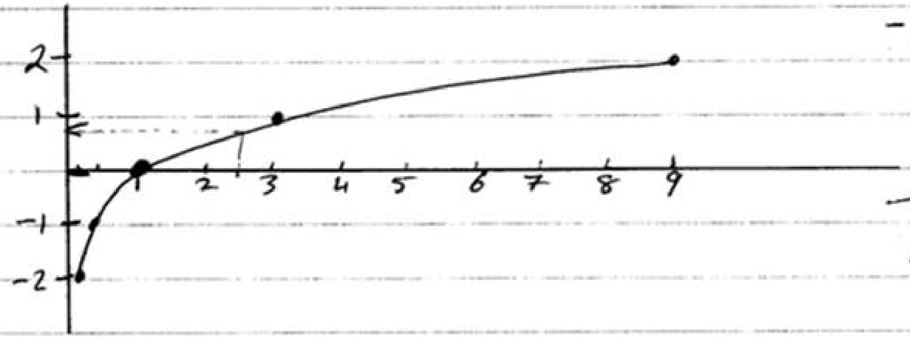
$-1 = \log_3 x$
$3^{-1} = x$
$\frac{1}{3} = x$

$\log_3 1 = y$
$3^y = 1$
$3^y = 3^0$
$y = 0$

$\log_3 3 = y$
$3^y = 3^1$
$y = 1$

$\log_3 x = 2$
$3^2 = x$
$9 = x$

(ii)




(iii) $\log_3 2.5 = 0.8$ from Graph.

(iv) $\log_3 2.5 = \dfrac{\log_{10} 2.5}{\log_{10} 3} = 0.834$

**Q4** $y = 5^x \quad 0 \le x \le 2$ | $y = \log_5 x$

$\log_3 x = \dfrac{\log_{10} x}{\log_{10} 3}$

$y = 1$

$\log_3 x = 2$

$3^2 = x$

$9 = x$

**Q4** $y = 5^x \quad 0 \le x \le 2$

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| 1 | 5 |
| 2 | 25 |

$y = \log_5 x$

| $x$ | $y$ |
|---|---|
| 0 | undefined |
| 5 | 1 |
| 10 | 1.4 |
| 15 | 1.68 |
| 20 | 1.86 |
| 25 | 2 |

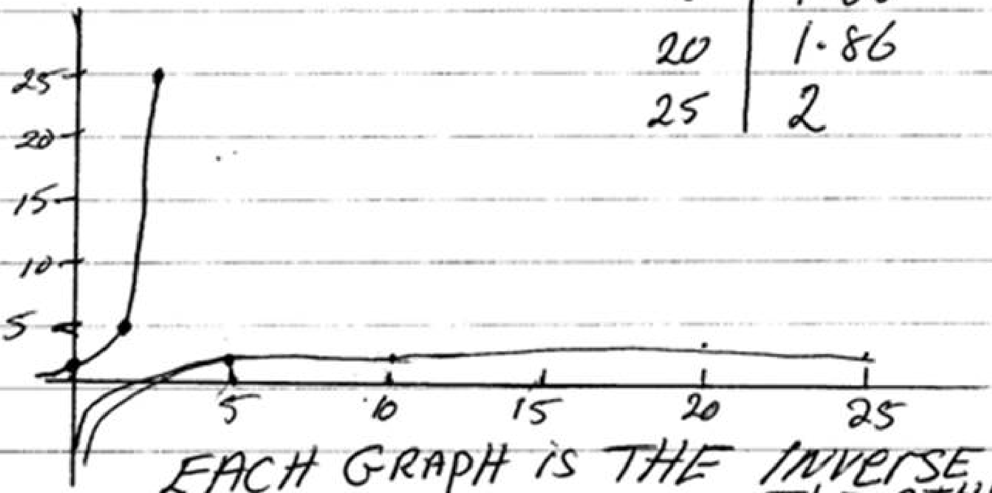
$\log_5 10 = y$

~~$5^y = 10$~~

$\dfrac{\log_{10} 10}{\log_{10} 5} = 1.4$

$\log_5 15 = \dfrac{\log 15}{\log 5} = 1.68$

$\log_5 20 = \dfrac{\log 20}{\log 5} = 1.86$



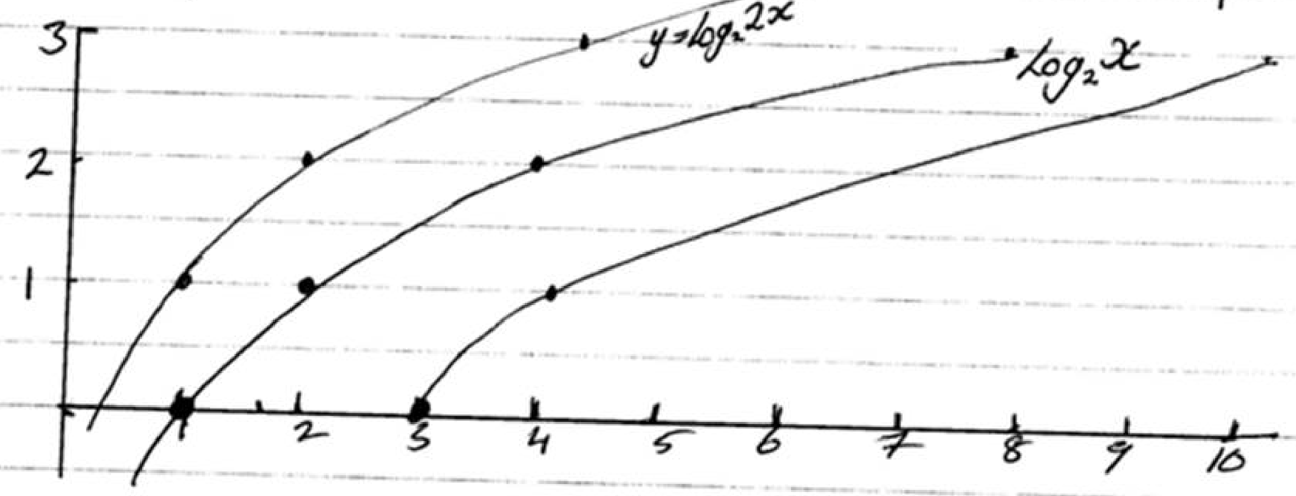

EACH GRAPH IS THE INVERSE OF THE OTHER.

Q5 (i) $y = \log_2 x$ (ii) $y = \log_2 2x$ (iii) $y = \log_2 (x-2)$

| $x$ | $y$ |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 4 | 2 |
| 8 | 3 |

| $x$ | $y$ |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 4 | 3 |
| 8 | 4 |

| $x$ | $y$ |
|---|---|
| 3 | 0 |
| 4 | 1 |
| 10 | 3 |



Q7 (i) $y = 3^{x+2} - 5$

$y+5 = 3^{x+2}$

$\log(y+5) = \log 3^{x+2}$

$\log(y+5) = (x+2)\log 3$

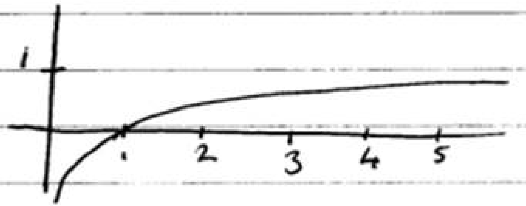
$\frac{\log(y+5)}{\log 3} = x+2$

$\frac{\log(y+5)}{\log 3} - 2 = x$

(ii) $y=30 \Rightarrow x = \frac{\log 35}{\log 3} - 2 = 3.2362 - 2 = 1.2362$

$\Rightarrow x = 1.236.$

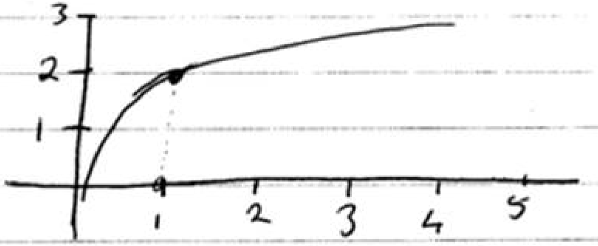
**Q8**




$y = \log_{10} x$

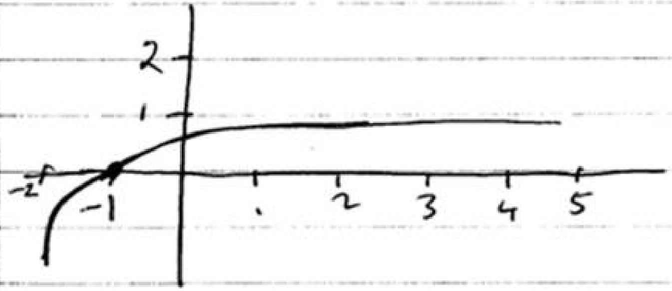
$y = \log_{10} x$
At $x = 1$
$10^y = 1$
$10^0 = 1 \Rightarrow y = 0$
$(1, 0)$

(i) $y = \log_{10} x + 2 \Rightarrow$ shifts up 2.

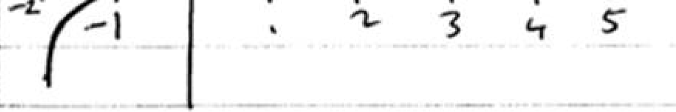



$(\log_{10} x) + 2$
at $x = 1$.
$(0) + 2 = 2$
$(1, 2)$

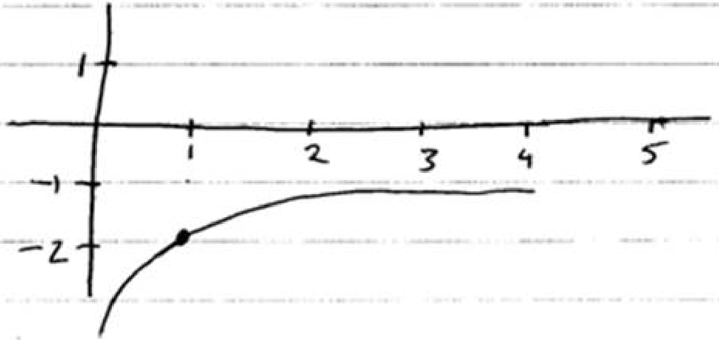
(ii) $y = \log_{10}(x+2) \Rightarrow$ shift left 2.




$y = \log_{10}(x+2)$ at $x = -1$
$y = \log_{10}(1)$
$10^y = 1$
$\Rightarrow y = 0$ $(-1, 0)$




(iii) $y = \log_{10} x - 2 \Rightarrow$ shift down 2



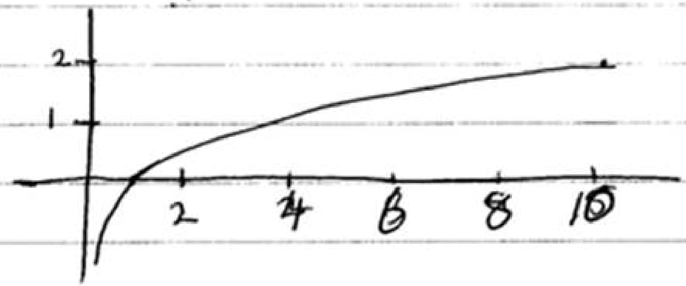

$(\log_{10} x) - 2$ at $x = 1$

$(\log_{10} 1) - 2$

$0 - 2 = -2$

$(1, -2)$

(iv) $y = 2\log_{10} x \Rightarrow$ Spins antiClockwise $\Rightarrow (10, 1) \rightarrow (5, 1\frac{1}{2})$ $\rightarrow (10, 2)$



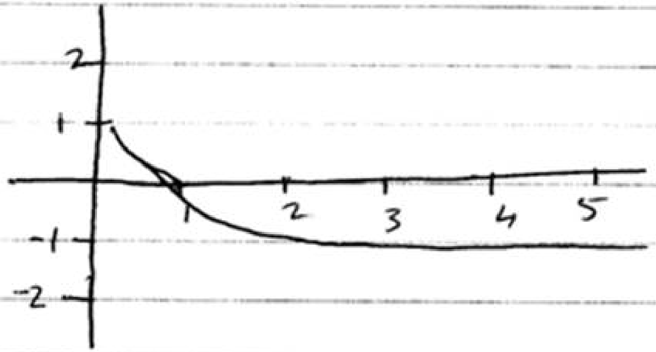

$2(\log_{10} x)$ at $x = 10$

$2(\log_{10} 10)$

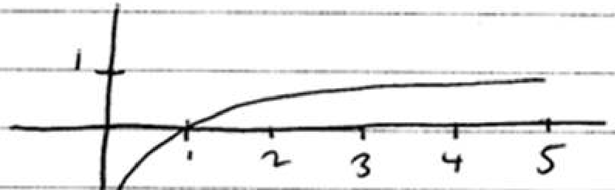
$2(1) = 2$ $(10, 2)$

(v) $y = -\log_{10} x \Rightarrow$ Inverse (power of $-1$)

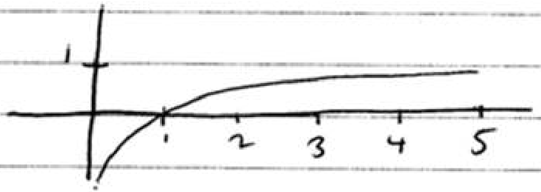
$y = (\log_{10} x)^{-1}$



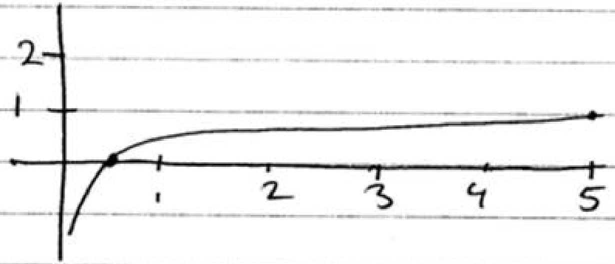

**Q9** $y = \log_{10} x$



**Q9** $y = \log_{10} x$




(i) $y = \log_{10}(2x)$




$y = \log_{10} 2x$ at $x = \frac{1}{2}$

$\log_{10} 2(\frac{1}{2}) = \log_{10} 1$

$\Rightarrow y = 0$

$(\frac{1}{2}, 0)$

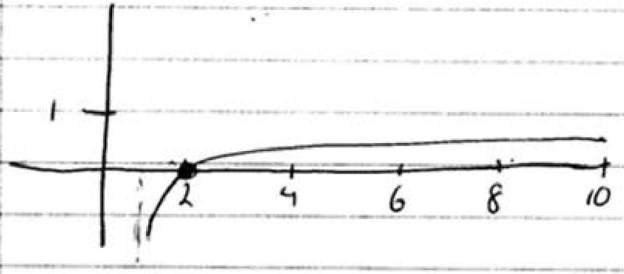
At $x = 5$

$\log_{10} 2(5)$

$= \log_{10} 10 \Rightarrow y = 1$

$(5, 1)$

(ii) $y = \log_{10}\left(\frac{x}{2}\right)$




at $x = 2$

$\log_{10} 1 \Rightarrow y = 0$

$(2, 0)$

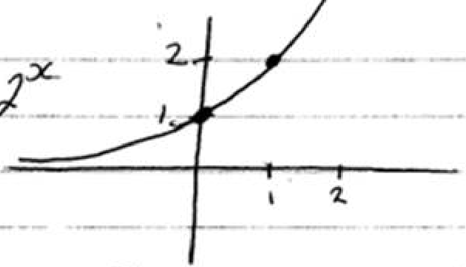
at $x = 20$

$\log_{10} 10 \Rightarrow y = 1$

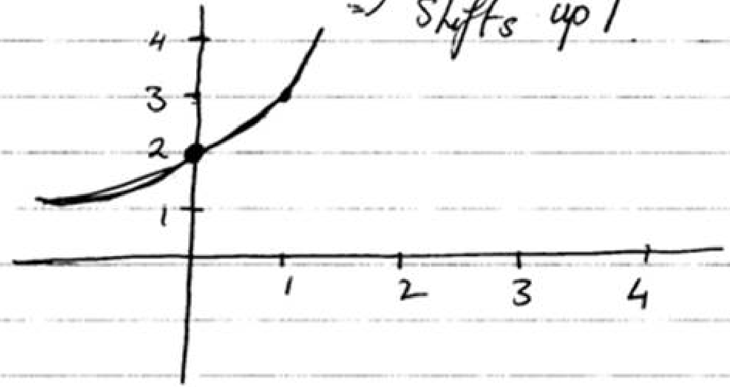
$(20, 1)$

Q10 (i) $y = 2^x + 1$

$y = 2^x$

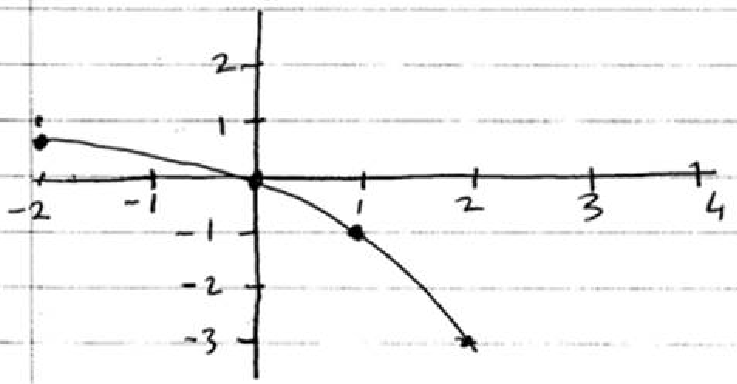



$\Rightarrow$ Shifts up 1




[at $x = 0$ $y = 2$ $(0,2)$
at $x = 1$ $y = 3$ $(1,3)$]

(ii) $y = 1 - 2^x$ $\Rightarrow$ Inverse and shift down 1




at $x = 0$ $y = 0$ $(0,0)$

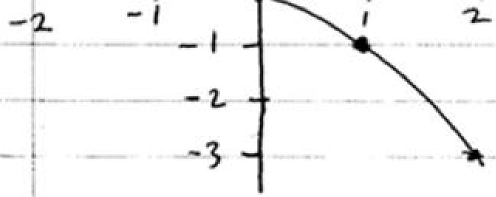
at $x = 1$ $y = -1$ $(1,-1)$

at $x = 2$ $y = -3$ $(2,-3)$

at $x = -2$

$y = 1 - \frac{1}{4} = \frac{3}{4}$

$(-2, \frac{3}{4})$



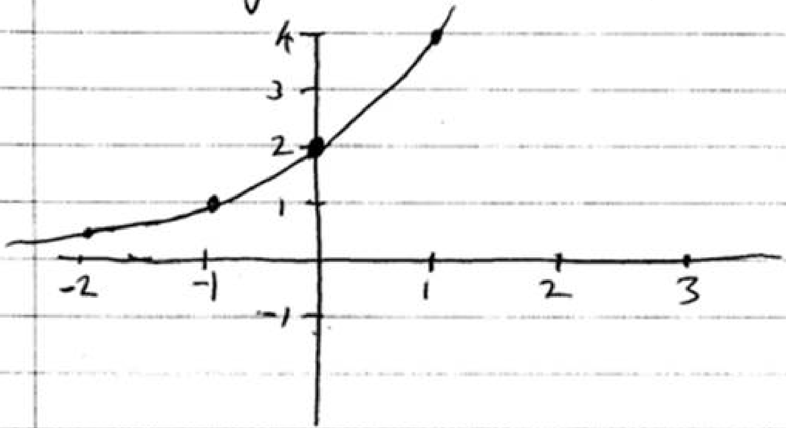

at $x=2$ $y=-3$ $(2,-3)$

at $x=-2$

$y = 1 - \frac{1}{4} = \frac{3}{4}$

$(-2, \frac{3}{4})$

(iii) $y = 2^{x+1}$ $\Rightarrow$ Spin Anti Clockwise




at $x=0$ $y=2$ $(0,2)$

at $x=1$ $y=4$ $(1,4)$

at $x=-1$ $y=1$ $(-1,1)$

at $x=-2$ $y=2^{-1}=\frac{1}{2}$

$(-2, \frac{1}{2})$

(iv) $y = \left(\frac{1}{2}\right) 2^x$ → spin clockwise

at $x = 0$ $y = \frac{1}{2}$ $(0, \frac{1}{2})$
at $x = 1$ $y = 1$ $(1, 1)$
at $x = 2$ $y = 2$ $(2, 2)$
at $x = 3$ $y = 4$ $(3, 4)$
at $x = -1$ $y = \frac{1}{4}$ $(-1\ \frac{1}{4})$
at $x = -2$ $y = \frac{1}{8}$ $(-2, \frac{1}{8})$