Section 7.6- Indices

4^5 5 is the index and 4 is the base number

Rules – In tables pg 21. Must know how to calculate on a calculator.

Revision of the rules of indices.

	Example	Rule
1.	$6^3 \times 6^4 = 6^7$	$a^m . a^n = a^{m+n}$
2.	$\frac{4^5}{4^2} = 4^3$, also $\frac{4^5}{4^7} = 4^{-2}$	$\frac{a^m}{a^n} = a^{m-n}$
3.	$(2^4)^3 = 2^{12}$	$(a^m)^n = a^{m.n}$
4.	$3^0 = 5^0 = 9^0 = (-3)^0 = \left(\frac{1}{4}\right)^0 = 1$	$a^0 = 1$
5.	$3^{-1} = \frac{1}{3}$, $3^{-4} = \frac{1}{3^4}$	$a^{-n} = \frac{1}{a^n}$
6.	$7^{\frac{1}{2}} = \sqrt[2]{7}$, $5^{\frac{1}{3}} = \sqrt[3]{5}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
7.	$8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$ or $(8^2)^{\frac{1}{3}}$	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
8.	$(2 \times 5)^2 = 2^2 \times 5^2$ $\left(\frac{5}{6}\right)^4 = \frac{5^4}{6^4}$	$(a.b)^n = a^n.b^n$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Note: $\sqrt{x} = \sqrt[2]{x} = x^{\frac{1}{2}} =$ the square root of x.

$\sqrt[3]{x} = x^{\frac{1}{3}} =$ the cube root of x.

$\sqrt[4]{x} = x^{\frac{1}{4}} =$ the fourth root of x … etc.

Note: $25^{\frac{3}{2}} = \left(25^{\frac{1}{2}}\right)^3 = (5)^3 = 125$ (most often, it is easier to get the root first and then raise to the power.)

Example 1

Evaluate each of the following.

(i) $27^{\frac{1}{3}}$ (ii) $36^{\frac{3}{2}}$ (iii) $64^{-\frac{2}{3}}$ (iv) $\left(\frac{27}{125}\right)^{-\frac{2}{3}}$

(i) $\sqrt[3]{27} = 3$

(ii) $36^{3/2} = \sqrt[2]{36^3} = (6)^3 = 216$

(iii) $64^{-2/3} = \frac{1}{64^{2/3}} = \frac{1}{\sqrt[3]{64^2}} = \frac{1}{4^2} = \frac{1}{16}$

(iv) $\left(\frac{27}{125}\right)^{-2/3} = \left(\frac{125}{27}\right)^{2/3} = \sqrt[3]{\frac{125}{27}}^{\,2} = \left(\frac{5}{3}\right)^2 = 25/9$

Example 2

Simplify each of the following. (i) $\left(\frac{x^2y^{-3}}{x^{-4}y^5}\right)^{\frac{1}{2}}$ (ii) $\frac{\sqrt{a^3}}{\sqrt[4]{a}\times\sqrt[3]{a^2}}$

(i) $\left(\frac{x^2y^{-3}}{x^{-4}y^5}\right)^{1/2} = \sqrt{x^6y^{-8}} = x^3y^{-4} = \frac{x^3}{y^4}$

(Div sub powers)

(ii) $\frac{\sqrt{a^3}}{\sqrt[4]{a}\times\sqrt[3]{a^2}} = \frac{a^{3/2}}{a^{1/4}\times a^{2/3}} = \frac{a^{3/2}}{a^{11/12}} = a^{7/12}$

mult ⇒ add powers

Div Sub powers

Example 3

Show that $\dfrac{5^{n+1} - (4)5^n}{5^{n-2} + 5^n} = \dfrac{25}{26}$.

$$\frac{(5^n)(5^1) - (4)5^n}{\left(\frac{5^n}{5^2}\right) + 5^n}$$

$$\frac{(5)5^n - (4)5^n}{\frac{5^n}{25} + 5^n} = \frac{5^n}{\frac{5^n + 25(5^n)}{25}}$$

$$\frac{5^n}{\frac{(26)5^n}{25}} = \cancel{5^n} \times \frac{25}{(26)\cancel{5^n}} = \frac{25}{26}$$

Exercise 7·6

Q1

(i) $a^2 \times a^3 = a^5$ (ii) $x.x.x^2 = x^4$

(iii) $2x^3 \times 3x^3 = 6x^6$ (iv) $x^5/x^2 = x^3$

(v) $x^4/x^5 = x^{-1}$ (vi) $a^0 = 1$

(vii) $\sqrt[3]{27} = 3$ (viii) $(a^3)^2 = a^6$

(ix) $\dfrac{(x^3)^2}{x^3} = \dfrac{x^6}{x^3} = x^3$ (x) $(3ab)^2 = 9a^2b^2$

Q2

(i) $\sqrt[3]{64} = 4$ (ii) $3^{-2} = 1/3^2 = 1/9$

(iii) $1/2^{-3} = 2^3 = 8$ (iv) $2^{-2}/3^{-2} = 3^2/2^2 = 9/4$

(v) $1/4^{-1/2} = 4^{1/2} = \sqrt{4} = 2.$

Q3 (i) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$

(ii) $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$

(iii) $27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$

(iv) $81^{3/4} = (\sqrt[4]{81})^3 = 3^3 = 27$

(v) $125^{2/3} = (\sqrt[3]{125})^2 = 5^2 = 25$

Q4 (i) $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = 9/4$

(ii) $(4/9)^{-1/2} = \sqrt{9/4} = 3/2$

(iii) $\left(9/25\right)^{-3/2} = \left(25/9\right)^{3/2} = \left(\sqrt{25/9}\right)^3 = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$

(iv) $\left(\frac{27}{125}\right)^{-2/3} = \left(\sqrt[3]{\frac{125}{27}}\right)^2 = \left(5/3\right)^2 = 25/9$

(v) $\left(3\,3/8\right)^{1/3} = \sqrt[3]{\frac{27}{8}} = 3/2$

Q5 $\dfrac{4^2 \times 16^{1/2}}{64^{2/3} \times 4^3}$ as 4^n

$$= \frac{4^2 \times 4}{(\sqrt[3]{64})^2 \times 4^3} = \frac{4^3}{4^2 \times 4^3} = \frac{4^3}{4^5} = 4^{-2}$$

Q6 $$\frac{3^{1/4} \times 3 \times 3^{1/6}}{\sqrt{3}} = \frac{3^{\frac{1}{4}+1+\frac{1}{6}}}{3^{1/2}} = \frac{3^{17/12}}{3^{6/12}} = 3^{11/12}$$

$$3^p = 3^{11/12} \Rightarrow p = 11/12$$

Q7 (i) $$\frac{(xy^2)^3 \times (x^2y)^{-2}}{xy} = \frac{x^3y^6 \times x^{-4}y^{-2}}{xy}$$

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$$= \frac{x^{-1}y^4}{xy} = x^{-2}y^3 = \frac{y^3}{x^2}$$

(ii) $$\left(\frac{p^2q}{p^{-1}q^3}\right)^4 = \frac{p^8q^4}{p^{-4}q^{12}} = p^{12}q^{-8} = \frac{p^{12}}{q^8}$$

(iii) $$a^{1/4} \times a^{-5/4} = a^{-1} = 1/a$$

(iv) $$\left(\frac{y^{-2}}{y^{-3}}\right)^{2/3} = (y^1)^{2/3} = y^{2/3}$$

(v) $$\frac{(a\sqrt{b})^{-3}}{\sqrt{a^3b}} = \frac{a^{-3}\,b^{-3/2}}{a^{3/2}\,b^{1/2}} = a^{-9/2}\,b^{-4/2} = \frac{1}{a^{9/2}b^2}$$

(vi) $$\frac{\sqrt[4]{x^7}}{\sqrt{x^3}} = \frac{x^{7/4}}{x^{3/2}} = x^{1/4}$$

Q8 (i) $\dfrac{x^{1/2} + x^{-1/2}}{x^{1/2}} = \dfrac{x^{1/2}}{x^{1/2}} + \dfrac{x^{-1/2}}{x^{1/2}} = x^0 + x^{-1}$

$= 1 + \dfrac{1}{x} = \dfrac{x+1}{x}$

(ii) $\left(x + x^{1/2}\right)\left(x - x^{1/2}\right)$

$x^2 \cancel{- x^{3/2}} \cancel{+ x^{3/2}} - x^1 = x^2 - x$

(iii) $\dfrac{\sqrt{x} + \sqrt{x^3}}{\sqrt{x}} = \dfrac{x^{1/2}}{x^{1/2}} + \dfrac{x^{3/2}}{x^{1/2}} = x^0 + x^1$

$= 1 + x$

$= 1 + x$

Q9

$$\frac{(x-1)^{1/2} + (x-1)^{-1/2}}{(x-1)^{1/2}} \times \frac{(x-1)^{\frac{1}{2}}}{(x-1)^{1/2}}$$

$$= \frac{(x-1)^1 + (x-1)^0}{(x-1)^1} = \frac{x-1+1}{x-1} = \frac{x}{x-1}$$

Q10

$$\sqrt{3^{2n+1}} \times \sqrt[3]{3^{-3n}} = \left(3^{2n+1}\right)^{1/2} \times \left(3^{-3n}\right)^{1/3}$$

$$= 3^{\frac{2n+1}{2}} \times 3^{-n} = 3^{\frac{2n+1}{2} - n} = 3^{n+\frac{1}{2}-n} = 3^{1/2}$$

$$3^k = 3^{\frac{1}{2}} \Rightarrow k = \tfrac{1}{2}$$

Q11 $2^{1/12}$

$$220 \times 2^{1/12} \times 2^{1/12} \times 2^{1/12} = 220(2^{1/4})$$

$$= 261.626 = 262 \text{ Hz.}$$

Q12 $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

$$\frac{A_1}{A_2} = \left(\frac{V_1}{V_2}\right)^{2/3}$$

$$\frac{A_1}{A_2} = \frac{\cancel{4\pi} r_1^2}{\cancel{4\pi} r_2^2} = \frac{r_1^2}{r_2^2}$$

$$\left(\frac{V_1}{V_2}\right)^{2/3} = \left(\frac{4/3\ \pi\ r_1^3}{4/3\ \pi\ r_2^3}\right)^{2/3} = \frac{\cancel{(4/3)^{2/3}}\ \cancel{(\pi)^{2/3}}\ (r_1^3)^{2/3}}{\cancel{(4/3)^{2/3}}\ \cancel{(\pi)^{2/3}}\ (r_2^3)^{2/3}} = \frac{r_1^2}{r_2^2}$$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{V_1}{V_2}\right)^{2/3}$$

$$\left(\frac{V_1}{V_2}\right)^{2/3} = \left(\frac{4/3\,\pi\, r_1^{\,3}}{4/3\,\pi\, r_2^{\,3}}\right)^{2/3} = \frac{\cancel{(4/3)^{2/3}}\;\cancel{(\pi)^{2/3}}\;(r_1^{\,3})^{2/3}}{\cancel{(4/3)^{2/3}}\;\cancel{(\pi)^{2/3}}\;(r_2^{\,3})^{2/3}} = \frac{r_1^{\,2}}{r_2^{\,2}}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \left(\frac{V_1}{V_2}\right)^{2/3}$$

Vols are $162\,cm^3$ & $384\,cm^3$

$$\Rightarrow \quad \frac{A_1}{A_2} = \left(\frac{162}{384}\right)^{2/3}$$

$$\frac{A_1}{A_2} = \left(\frac{27}{64}\right)^{2/3} = \left(\sqrt[3]{\frac{27}{64}}\right)^{2} = \left(\frac{3}{4}\right)^{2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{9}{16}$$

$$f(n) = 3^n$$

Q13 (i) $f(n+3) = 3^{n+3}$

(ii) $f(n+1) = 3^{n+1}$

hence: $f(n+3) - f(n+1) = k f(n)$

$3^{n+3} - 3^{n+1} = $

$3^n . 3^3 - 3^n . 3^1$

$3^n (27-3)$

$= 24(3^n)$

$\Rightarrow k = 24$

Q14 $f(n) = 3^{n-1}$

find k

$f(n+3) + f(n) = k f(n)$

$3^{n+3-1} + 3^{n-1} = k(3^{n-1})$

$3^{n-1+3} + 3^{n-1}$

$3^{n-1} . 3^3 + 3^{n-1}$

$3^{n-1}(27 + 1)$

$= 28(3^{n-1})$

$\Rightarrow k = 28$