Section 7.7 - Exponential Equations

An unknown in the power

$3^{x} = 27$ is an exponential equation

To solve: Make the base numbers the same.

$3^{x} = 3^{3}$ $\rightarrow$ $x = 3$

$3^{x} = 27$

$3^{x} = 3^{3}$

$x = 3$

Example 1

Solve these equations. (i) $\frac{1}{8^x} = 16^{\frac{1}{3}}$ (ii) $27^{x-3} = 3 \times 9^{x-2}$

(i) $\frac{1}{8^x} = 16^{1/3}$

$\frac{1}{2^{3x}} = \left(2^4\right)^{1/3}$

$2^{-3x} = 2^{4/3}$

$-3x = 4/3 \Rightarrow x = -4/9$

$27^{x-3} = 3 \times 9^{x-2}$

$\left(3^3\right)^{x-3} = 3^1 \times \left(3^2\right)^{x-2}$

$3^{3x-9} = 3^1 \times 3^{2x-4}$

$3^{3x-9} = 3^{2x-4+1}$

$3x - 9 = 2x - 4 + 1$

$x = 6$

By using a suitable change of variable an exponential function can be transformed into a quadratic equation and then solved.

Note: If $2^x = y$,

$$\Rightarrow 3.2^x = 3y$$
$$\Rightarrow 2^{2x} = (2^x)^2 = y^2$$
$$\Rightarrow 2^{x+2} = 2^x.2^2 = 4y$$

Example 2

If $y = 3^x$, express 3^{2x} in terms of y.

Hence solve the equation $3^{2x} - 4.3^x + 3 = 0$.

$$3^{2x} = (3^x)^2 = y^2$$

Hence

$$3^{2x} - 4.3^x + 3 = 0$$

$$y^2 - 4y + 3 = 0$$

$$(y - 3)(y - 1) = 0$$

$y = 3$ $\quad$ $y = 1$

$3^x = 3^1$

$x = 1$

$3^x = 1$

$3^x = 3^0$

$x = 0$

Exercise 7.7

Q1 (i) $2^x = 32$

$2^x = 2^5$

$\Rightarrow x = 5$

(ii) $16^x = 64$

$4^{2x} = 4^3$

$2x = 3$

$\Rightarrow x = 3/2$

(iii) $25^x = 125$

$5^{2x} = 5^3$

$2x = 3$

$\Rightarrow x = 3/2$

(iv) $3^x = 1/27$

$3^x = 3^{-3}$

$\Rightarrow x = -3$

Q2 (i) $9^x = \frac{1}{27}$

$3^{2x} = 3^{-3}$

$2x = -3$

$x = -3/2$

(ii) $4^x = 1/32$

$2^{2x} = 2^{-5}$

$2x = -5$

$x = -5/2$

(iii) $4^{x-1} = 2^{x+1}$

$2^{2(x-1)} = 2^{x+1}$

$2x - 2 = x + 1$

$x = 3$

(iv) $\frac{1}{9^x} = 27$

$3^{-2x} = 3^3$

$-2x = 3$

$x = -3/2$

Q3 (i) $2^x = \frac{\sqrt{2}}{2}$

(ii) $25^x = 125$

$x = 3/2$

(iii) $4^{x-1} = 2^{x+1}$

$2^{2(x-1)} = 2^{x+1}$

$2x - 2 = x + 1$

$x = 3$

(iv) $\frac{1}{9^x} = 27$

$3^{-2x} = 3^3$

$-2x = 3$

$x = -3/2$

Q3 (i) $2^x = \frac{\sqrt{2}}{2}$

$2^x = 2^{-1/2}$

$x = -1/2$

(ii) $25^x = \frac{125}{\sqrt{5}}$

$5^{2x} = 5^3 / 5^{1/2}$

$5^{2x} = 5^{2\frac{1}{2}}$

$2x = 2\frac{1}{2}$

$x = 5/4$

(iii) $\frac{1}{8^x} = \sqrt{2}$

$2^{-3x} = 2^{1/2}$

$-3x = 1/2$

$x = -1/6$

(iv) $7^x = \frac{1}{\sqrt[3]{7}}$

$7^x = 7^{-1/3}$

$x = -1/3$

Q4 $\sqrt{32} = 2^{5/2}$

Hence solve $16^{x-1} = 2\sqrt{32}$

$2^{4(x-1)} = 2(2^{5/2})$

$2^{4x-4} = 2^{1+5/2}$

$4x-4 = 7/2$

$4x = 15/2$

$x = 15/8$

Q5 $27^x = 9$

$3^{3x} = 3^2$

$3x = 2$

$x = 2/3$

$2^{x-y} = 64$

$2^{2/3-y} = 2^6$

$2/3 - y = 6$

$-y = 6 - 2/3$

$-y = \frac{16}{3}$

$y = -16/3$

$y = -16/3$

Q6/ $2^{x+2} = (2^x)(2^2) = 4.2^x$

$2^x + 2^x = 2^x(1+1) = 2.2^x$

Hence Solve

$2^x + 2^x = 2^{x+2}(c-2)$

$2.2^x = 4.2^x(c-2)$

$2 = 4c - 8$

$10 = 4c$

$10/4 = c$

$c = 5/2$.

Q7 $3^x = y$

$3^{2x} - 12(3^x) + 27 = 0$

$(3^x)^2 - 12(3^x) + 27 = 0$

$y^2 - 12y + 27 = 0$

$(y-3)(y-9) = 0$

$y = 3 \qquad y = 9$

$\Rightarrow 3^x = 3 \qquad 3^x = 9$

$x = 1 \qquad 3^x = 3^2$

$x = 2$

Q8 $2^{2x} - 3(2^x) - 4 = 0$ Let $y = 2^x$

$y^2 - 3y - 4 = 0$

$(y-4)(y+1) = 0$

$y = 4 \qquad y = -1$

$\Rightarrow 2^x = 4 \qquad 2^x = -1$

$2^x = 2^2$ Not Valid.

$x = 2$

$\Rightarrow$ Ans: $x = 2$

$\Rightarrow 2^x = 4$ $\quad 2^x = -1$

$2^x = 2^2$ $\quad$ Not Valid.

$x = 2$

$\Rightarrow$ Ans: $x = 2$

Q9 (i) $2^{2x} - 9(2^x) + 8 = 0$ $\quad$ Let $y = 2^x$

$y^2 - 9y + 8 = 0$

$(y - 8)(y - 1) = 0$

$y = 8$ $\quad$ $y = 1$

$\Rightarrow$ $2^x = 8$ $\quad$ $2^x = 1$

$2^x = 2^3$ $\quad$ $2^x = 2^0$

$x = 3$ $\quad$ $x = 0$.

Q9 (ii) $3^{2x} - 10(3^x) + 9 = 0$ let $y = 3^x$

$y^2 - 10y + 9 = 0$

$(y - 9)(y - 1) = 0$

$y = 9$ $y = 1$

$\Rightarrow$ $3^x = 9$ $3^x = 1$

$3^x = 3^2$ $3^x = 3^0$

$x = 2$ $x = 0$

Q10 $y = 2^x$

(i) $2^{2x} = y^2$

(ii) $2^{2x+1} = (2^x)^2 \times 2^1 = y^2 \cdot 2 = 2y^2$

(iii) $2^{x+3} = 2^x \cdot 2^3 = 8y$

hence solve

$2^{2x+1} - 2^{x+3} - 2^x + 4 = 0$

$2y^2 - 8y - y + 4 = 0$

$2y^2 - 9y + 4 = 0$

$(2y - 1)(y - 4) = 0$

$y = 1/2$ $y = 4$

$\Rightarrow$ $2^x = 1/2$ $2^x = 4$

$2^x = 2^{-1}$ $2^x = 2^2$

$x = -1$ $x = 2$

$2y^2 - 4y + 4y = 0$

$(2y - 1)(y - 4) = 0$

$y = 1/2 \qquad y = 4$

$\Rightarrow \quad 2^x = 1/2 \qquad 2^x = 4$

$2^x = 2^{-1} \qquad 2^x = 2^2$

$x = -1 \qquad x = 2$

Q11 $\quad y = 3^x \qquad 3 \cdot 3^x + 3^{-x} = 4$

$3y + 1/y = 4$

$3y^2 + 1 = 4y$

$3y^2 - 4y + 1 = 0$

$(3y - 1)(y - 1) = 0$

$y = 1/3 \qquad y = 1$

$\Rightarrow \quad 3^x = 1/3 \qquad 3^x = 1$

$3^x = 3^{-1} \qquad 3^x = 3^0$

$x = -1 \qquad x = 0$

Q12 $2(4^x) + 4^{-x} = 3$ Let $y = 4^x$

$2y + \frac{1}{y} = 3$

$2y^2 + 1 = 3y$

$2y^2 - 3y + 1 = 0$

$(2y - 1)(y - 1) = 0$

$y = \frac{1}{2}$ $y = 1$

$\Rightarrow$ $4^x = \frac{1}{2}$ $4^x = 1$

$2^{2x} = 2^{-1}$ $4^x = 4^0$

$2x = -1$ $x = 0$

$x = -\frac{1}{2}$

Q13

Q13

$3^x - 28 + 27(3^{-x}) = 0$ let $y = 3^x$

$y - 28 + \frac{27}{y} = 0$

$y^2 - 28y + 27 = 0$

$(y - 27)(y - 1) = 0$

$y = 27 \quad y = 1$

$\Rightarrow 3^x = 27 \qquad 3^x = 1$

$3^x = 3^3 \qquad 3^x = 3^0$

$x = 3 \qquad x = 0.$

Q14 $2^x = y$ $\qquad 2^{x+1} + 2(2^{-x}) - 5 = 0$

$$2y + \frac{2}{y} - 5 = 0$$

$$2y^2 + 2 - 5y = 0$$

$$2y^2 - 5y + 2 = 0$$

$$(2y - 1)(y - 2) = 0$$

$y = 1/2 \qquad y = 2$

$\Rightarrow$ $\quad 2^x = 1/2 \qquad 2^x = 2$

$2^x = 2^{-1} \qquad 2^x = 2^1$

$x = -1 \qquad x = 1$

Q15

$3^x + 81(3^{-x}) - 30 = 0$ $\quad$ let $y = 3^x$

Q15

$3^x + 81(3^{-x}) - 30 = 0$ let $y = 3^x$

$y + \frac{81}{y} - 30 = 0$

$y^2 + 81 - 30y = 0$

$y^2 - 30y + 81 = 0$

$(y - 27)(y - 3) = 0$

$y = 27$ $y = 3$

$\Rightarrow$ $3^x = 27$ $3^x = 3$

$3^x = 3^3$ $3^x = 3^1$

$x = 3$ $x = 1$