(b) The heptathlon is an Olympic competition. It consists of seven events including the 200 m race and the javelin. The scoring system uses formulas to calculate a score for each event. The table below shows the formulas for two of the events and the values of constants used in these formulas, where x is the time taken (in seconds) or distance achieved (in metres) by the competitor and y is the number of points scored in the event.

Event	x	Formula	a	b	c
200 m race	Time (s)	$y = a(b-x)^c$	4·99087	42·5	1·81
Javelin	Distance (m)	$y = a(x-b)^c$	15·9803	3·8	1·04

(i) In the heptathlon, Jessica ran 200 m in 23·8 s and threw the javelin 58·2 m. Use the formulas in the table to find the number of points she scored in each of these events, correct to the nearest point.

200 m	Javelin
$y = 4{\cdot}99087(42{\cdot}5-23{\cdot}8)^{1{\cdot}81}$	$y = 15{\cdot}9803(58{\cdot}2-3{\cdot}8)^{1{\cdot}04}$
$= 1000{\cdot}482\ldots$	$= 1020{\cdot}017\ldots$
$= 1000$ points	$= 1020$ points

(ii) The world record distance for the javelin, in the heptathlon, would merit a score of 1295 points. Find the world record distance for the javelin, in the heptathlon, correct to two decimal places.

$$1295 = 15{\cdot}9803(x-3{\cdot}8)^{1{\cdot}04}$$

$$\frac{1295}{15{\cdot}9803} = (x-3{\cdot}8)^{1{\cdot}04}$$

$$\sqrt[1{\cdot}04]{\frac{1295}{15{\cdot}9803}} = x - 3{\cdot}8$$

$$\sqrt[1{\cdot}04]{\frac{1295}{15{\cdot}9803}} + 3{\cdot}8 = x$$

$$72{\cdot}2343\ldots = x$$

$$72{\cdot}23\ (2dp) = x$$

(iii) The formula used to calculate the points for the 800 m race, in the heptathlon, is the same formula used for the 200 m race but with different constants.
Jessica ran the 800 m race in 2 minutes and 1·84 seconds which merited 1087 points.
If $a = 0{\cdot}11193$ and $b = 254$ for the 800 m race, find the value of c for this event, correct to two decimal places.

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$y = a(b-x)^c$ 2m and 1·84 sec = $\underset{}{\overset{x}{121{\cdot}84}}$ sec

$1087 = 0{\cdot}11193(254 - 121{\cdot}84)^c$

$\frac{1087}{0{\cdot}11193} = 132{\cdot}16^c$

$\ln \frac{1087}{0{\cdot}11193} = \ln 132{\cdot}16^c$

$\ln \frac{1087}{0{\cdot}11193} = c \ln 132{\cdot}16$

$\frac{\ln \frac{1087}{0{\cdot}11193}}{\ln 132{\cdot}16} = C$

$1{\cdot}8798\ldots = C$

$1{\cdot}88 = C$ (2d.p.)

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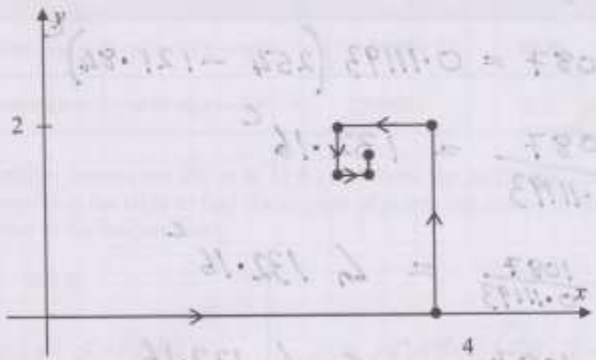
Question 9 **(55 marks)**

(a) At the first stage of a pattern, a point moves 4 units from the origin in the positive direction along the x-axis. For the second stage, it turns left and moves 2 units parallel to the y-axis. For the third stage, it turns left and moves 1 unit parallel to the x-axis.
At each stage, after the first one, the point turns left and moves half the distance of the previous stage, as shown.



(i) How many stages has the point completed when the total distance it has travelled, along its path, is 7·9375 units?

$4 + 2 + 1 + \frac{1}{2} + \cdots$ $\quad a = 4 \quad r = \frac{1}{2}$

$S_n = \frac{a(1-r^n)}{1-r}$

$7{\cdot}9375 = \frac{4(1-0{\cdot}5^n)}{1-0{\cdot}5}$

$\frac{7{\cdot}9375 \times 0{\cdot}5}{4} - 1 = -0{\cdot}5^n$

$+\,-0{\cdot}0078125 = +\,-0{\cdot}5^n$

$\ln 0{\cdot}0078125 = n \ln 0{\cdot}5$

$\frac{\ln 0{\cdot}0078125}{\ln 0{\cdot}5} = n$

$7 = n$

(ii) Find the maximum distance the point can move, along its path, if it continues in this pattern indefinitely.

$S_\infty = \frac{a}{1-r} = \frac{4}{1-0{\cdot}5} = \frac{4}{0{\cdot}5} = 8$ metres

(iii) Complete the second row of the table below showing the changes to the x co-ordinate, the first nine times the point moves to a new position. Hence, or otherwise, find the x co-ordinate and the y co-ordinate of the final position that the point is approaching, if it continues indefinitely in this pattern.

Stage	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Change in x	+4	0	−1	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	0	$\frac{1}{64}$
Change in y	0	2	0	$-\frac{1}{2}$	0	$\frac{1}{8}$	0	$-\frac{1}{32}$	0

x axis : $4 - 1 + \frac{1}{4} - \frac{1}{16} + \frac{1}{64}$ - - - - -

$a = 4 \quad r = -\frac{1}{4}$

$S_\infty = \frac{a}{1-r} = \frac{4}{1-(-0.25)} = 3.2$

y axis $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} +$ - - - - $a = 2 \quad r = -\frac{1}{4}$

$S_\infty = \frac{a}{1-r} = \frac{2}{1-(-0.5)} = 1.6$

$(3.2, 1.6)$

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(b) A male bee comes from an unfertilised egg, i.e. he has a female parent but he does not have a male parent. A female bee comes from a fertilised egg, i.e. she has a female parent and a male parent.

(i) The following diagram shows the ancestors of a certain male bee. We identify his generation as G_1 and our diagram goes back to G_4. Continue the diagram to G_5.

G_1	G_2	G_3	G_4	G_5
				Female
			Female	
		Female		Male
Male →	Female		Male	→ Female
		Male →	Female	Female
				Male

(ii) The number of ancestors of this bee in each generation can be calculated by the formula

$$G_{n+2} = G_{n+1} + G_n,$$

where $G_1 = 1$ and $G_2 = 1$, as in the diagram.
Use this formula to calculate the number of ancestors in G_6 and in G_7.

G_6	G_7
$G_{4+2} = G_{4+1} + G_4$ $= 5 + 3 = 8$	$G_7 = G_{5+2} = G_{5+1} + G_5$ $= 8 + 5$ $= 13$

(iii) The number of ancestors in each generation can also be calculated by using the formula

$$G_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}.$$

Use this formula to verify the number of ancestors in G_3.

$G_3 = 2$

$$G_3 = \frac{(1+\sqrt{5})^3 - (1-\sqrt{5})^3}{2^3\sqrt{5}}$$

$= 2$ (Calculator)

✓ True