Algebraic Fractions

Ex 1.4

Q1 (ii) $\frac{7a^6b^3}{14a^5b^4} = \frac{1a}{2b}$

(iv) $\frac{5ax}{15a+10a^2} = \frac{5ax}{5a(3+2a)} = \frac{x}{3+2a}$

Q2 (d) $\frac{x+1}{4} + \frac{2x-1}{5}$

$\frac{5(x+1)+4(2x-1)}{20}$

$\frac{5x+5+8x-4}{20}$

$\frac{13x+1}{20}$

(h) $\frac{3x+5}{6} - \frac{2x+3}{4} - \frac{1}{12}$

$\frac{2(3x+5)-3(2x+3)-1}{12}$

$\frac{6x+10-6x-9-1}{12}$

$\frac{0}{12} = 0$

(l) $\frac{1}{x} + \frac{1}{x+3}$

$\frac{x+3+x}{(x)(x+3)} \Rightarrow \frac{2x+3}{x^2+3x}$

(m) $\frac{2}{x+2} + \frac{3}{x+4}$

$\frac{2(x+4)+3(x+2)}{(x+2)(x+4)} \Rightarrow \frac{2x+8+3x+6}{(x+2)(x+4)}$

$\frac{5x+14}{(x+2)(x+4)}$

(n) $\frac{2}{x-2} + \frac{3}{2x-1}$

$\frac{2(2x-1)+3(x-2)}{(x-2)(2x-1)} \Rightarrow \frac{4x-2+3x-6}{(x-2)(2x-1)}$

$\frac{7x-8}{(x-2)(2x-1)}$

(q) $\frac{2}{3x-5} - \frac{1}{4}$

$\frac{8-1(3x-5)}{4(3x-5)} \Rightarrow \frac{8-3x+5}{12x-20} = \frac{-3x+13}{12x-20}$

(r) $\frac{5}{2x-1} - \frac{3}{x-2}$

$$\frac{5(x-2) - 3(2x-1)}{(2x-1)(x-2)} \Rightarrow \frac{5x-10-6x+3}{(2x-1)(x-2)}$$

$$\frac{-x-7}{(2x-1)(x-2)}$$

(t) $\frac{3}{x} + \frac{4}{3y} - \frac{2}{3xy}$

$$\frac{3(3y)+4(x)-2(1)}{3xy} \Rightarrow \frac{9y+4x-2}{3xy.}$$

(u) $\frac{3}{x} - \frac{2}{x-1} - \frac{4}{x(x-1)}$

$$\frac{3(x-1)-2(x)-4(1)}{x(x-1)} \Rightarrow \frac{3x-3-2x-4}{x(x-1)}$$

$$\frac{x-7}{x(x-1)}$$

factorising with fractions

Q3 (i) $\frac{2z^2-4z}{2z^2-10z} \Rightarrow \frac{2z(z-2)}{2z(z-5)} = \frac{z-2}{z-5}$

(ii) $\frac{y^2+7y+10}{y^2-25} \Rightarrow \frac{(y+5)(y+2)}{(y+5)(y-5)} = \frac{y+2}{y-5}$

(iii) $\frac{t^2+3t-4}{t^2-3t+2} \Rightarrow \frac{(t+4)(t-1)}{(t-2)(t-1)} = \frac{t+4}{t-2}$

(iv) $\frac{x}{x^2-4} - \frac{1}{x+2} \Rightarrow \frac{x-1(x-2)}{(x+2)(x-2)}$

$\frac{x-x+2}{(x+2)(x-2)} \Rightarrow \frac{2}{x^2-4}$.

(v) $\frac{2}{a+3} - \frac{a+2}{a^2-9} \Rightarrow \frac{2(a-3)-(a+2)}{(a+3)(a-3)}$

$\frac{2a-6-a-2}{(a+3)(a-3)} = \frac{a-8}{(a+3)(a-3)}$

(vi) $\frac{x-1}{x^2-4} + \frac{1}{x-2} \Rightarrow \frac{x-1+1(x+2)}{(x+2)(x-2)}$

$\frac{x-1+x+2}{(x+2)(x-2)} = \frac{2x+1}{(x+2)(x-2)}$

Q4

(i) $\frac{10}{2x^2 - 3x - 2} - \frac{2}{x-2}$

$\frac{10 - 2(2x+1)}{(2x+1)(x-2)} \Rightarrow \frac{10 - 4x - 2}{(2x+1)(x-2)} = \frac{8 - 4x}{(2x+1)(x-2)}$

$\frac{-1}{4(2-x)}$

$\frac{4(2-x)}{(2x+1)(x-2)} \Rightarrow \frac{-4}{2x+1}$

(ii) $\frac{x+2}{2x^2 - x - 1} - \frac{1}{x-1}$

$\frac{x+2-1(2x+1)}{(2x+1)(x-1)} \Rightarrow \frac{x+2-2x-1}{(2x+1)(x-1)}$

$\frac{-x+1}{(2x+1)(x-1)} = \frac{-1(x-1)}{(2x+1)(x-1)}$

$= \frac{-1}{2x+1}$

Q5

(i) $\frac{1}{x^2-9} - \frac{2}{x^2-x-6}$

$\frac{1}{(x+3)(x-3)} - \frac{2}{(x-3)(x+2)}$

$\frac{1(x+2) - 2(x+3)}{(x+3)(x-3)(x+2)}$

$\frac{x+2-2x-6}{(x+3)(x-3)(x+2)} = \frac{-x-4}{(x+3)(x-3)(x+2)}$

Ex 1.4 Pg 18. Contd.

Q5 (ii) $\frac{3}{x^2+x-2} - \frac{2}{x^2+3x+2}$

$\frac{3}{(x+2)(x-1)} - \frac{2}{(x+2)(x+1)}$

$\frac{3(x+1)-2(x-1)}{(x+2)(x-1)(x+1)} \Rightarrow \frac{3x+3-2x+2}{(x+2)(x-1)(x+1)}$

$\Rightarrow \frac{x+5}{(x+2)(x-1)(x+1)}$

(iii) $\frac{2}{6x^2-5x-4} - \frac{3}{9x^2-16}$

$\frac{2}{(3x-4)(2x+1)} - \frac{3}{(3x+4)(3x-4)}$

$\frac{2(3x+4)-3(2x+1)}{(3x-4)(2x+1)(3x+4)} \Rightarrow \frac{6x+8-6x-3}{(3x-4)(2x+1)(3x+4)}$

$\frac{5}{(3x-4)(2x+1)(3x+4)}$

(iv) $\frac{1}{xy-x^2} - \frac{1}{y^2-xy}$

$\frac{1}{x(y-x)} - \frac{1}{y(y-x)}$

$\frac{1y-1x}{xy(y-x)} = \frac{y-x}{xy(y-x)} = \frac{1}{xy}$

Q6

(ii) $\dfrac{\frac{2}{3}+\frac{5}{6}}{\frac{3}{8}} \Rightarrow \dfrac{\frac{4+5}{6}}{\frac{3}{8}} = \dfrac{9^{3}}{6_{3}} \times \dfrac{8^{4}}{3_{1}} = 4.$

(iii) $\dfrac{x-\frac{1}{x}}{1+\frac{1}{x}} \Rightarrow \dfrac{\frac{x^2-1}{x}}{\frac{x+1}{x}} \Rightarrow \dfrac{(x+1)(x-1)}{x^2-1} \times \dfrac{x}{x+1}$

$\Rightarrow x-1.$

Q7 (ii) $\dfrac{\frac{1}{x^2}-4}{\frac{1}{x}-2} \Rightarrow \dfrac{\frac{1-4x^2}{x^2}}{\frac{1-2x}{x}} \Rightarrow \dfrac{(1-2x)(1+2x)}{x^2} \times \dfrac{x}{1-2x}$

$\Rightarrow \dfrac{1+2x}{x}$

(iii) $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}} \Rightarrow \dfrac{x+y}{\frac{y+x}{xy}} \Rightarrow x+y \times \dfrac{xy}{x+y} = xy$

Q8

(i) $\dfrac{4y-\frac{3}{2}}{2} \Rightarrow \dfrac{\frac{8y-3}{2}}{2} \Rightarrow \dfrac{8y-3}{2} \times \dfrac{1}{2}.$

$\dfrac{8y-3}{4}$

(ii) $\dfrac{2-\frac{1}{x}}{2} \Rightarrow \dfrac{\frac{2x-1}{x}}{2} \Rightarrow \dfrac{2x-1}{x} \times \dfrac{1}{2} = \dfrac{2x-1}{2x}$

$= \left(1-\dfrac{1}{2x}\right)$

(iii) $\dfrac{3x + \frac{1}{2x}}{2} \Rightarrow \dfrac{\frac{3x^2+1}{2x}}{2} \Rightarrow \dfrac{3x^2+1}{x} \times \dfrac{1}{2} \Rightarrow \dfrac{3x^2+1}{2x}$

(iv) $\dfrac{y + \frac{1}{4}}{\frac{1}{2}} \Rightarrow \dfrac{\frac{4y+1}{4}}{\frac{1}{2}} \Rightarrow \dfrac{4y+1}{4_2} \times \dfrac{2}{1} = \dfrac{4y+1}{2}$

Q9

(iii) $\dfrac{z - \frac{1}{2z}}{z - \frac{1}{3z}} \Rightarrow \dfrac{\frac{2z^2-1}{2z}}{\frac{3z^2-1}{3z}}$

$\dfrac{2z^2-1}{2z} \times \dfrac{3z}{3z^2-1} = \dfrac{6z^2-3}{6z^2-2}$

(iv) $\dfrac{x - \frac{1}{x+1}}{x-1} \Rightarrow \dfrac{\frac{x^2+x-1}{x+1}}{x-1}$

$\dfrac{x^2+x-1}{x+1} \times \dfrac{1}{x-1} = \dfrac{x^2+x-1}{x^2-1}$

Q10

$\dfrac{x + \frac{2x}{x-2}}{1 + \frac{4}{(x+2)(x-2)}} \Rightarrow \dfrac{\frac{x^2-2x+2x}{x-2}}{\frac{(x+2)(x-2)+4}{(x+2)(x-2)}}$

$\dfrac{x^2}{(x-2)} \times \dfrac{(x+2)(x-2)}{(x+2)(x-2)+4} = \dfrac{x^2(x+2)}{(x+2)(x-2)+4}$

$\dfrac{x^2(x+2)}{x^2-4+4} = x+2.$

Q11

(i) $$\frac{\frac{a+b}{a-b} - \frac{a-b}{a+b}}{1 + \frac{a-b}{a+b}} = \frac{\frac{(a+b)^2 - (a-b)^2}{(a-b)(a+b)}}{\frac{a+b+a-b}{a+b}}$$

$$\frac{a^2 + 2ab + b^2 - a^2 + 2ab - b^2}{(a-b)(a+b)} \times \frac{(a+b)}{2a}$$

$$\frac{4ab}{a-b} \times \frac{1}{2a} = \frac{2b}{a-b}$$

(ii) $$\frac{x + \frac{3}{x}}{x - \frac{9}{x^3}} = \frac{\frac{x^2+3}{x}}{\frac{x^4-9}{x^3}}$$

$$\frac{x^2+3}{x} \times \frac{x^3}{(x^2+3)(x^2-3)} = \frac{x^2}{x^2-3}$$

(iii) $$\frac{9 - \frac{1}{y^2}}{9 + \frac{6}{y} + \frac{1}{y^2}} = \frac{\frac{9y^2-1}{y^2}}{\frac{9y^2+6y+1}{y^2}} = \frac{(3y+1)(3y-1)}{\frac{y^2}{(3y+1)(3y+1)}}$$

$$\frac{(3y+1)(3y-1)}{y^2} \times \frac{y^2}{(3y+1)(3y+1)} = \frac{3y-1}{3y+1}$$