Exercise 1.5

1. $ax^2 + bx + c = (2x-3)(3x+4)$

$ax^2 + bx + c = 6x^2 + 8x - 9x - 12$

$ax^2 + bx + c = 6x^2 - x - 12$

$a = 6, \quad b = -1, \quad c = -12$

2. $(3x-2)(x+5) = 3x^2 + px + q$

$3x^2 + 15x - 2x - 10 = 3x^2 + px + q$

$3x^2 + 13x - 10 = 3x^2 + px + q$

$p = 13 \qquad q = -10$

3. $x^2 + 6x + 16 = (x+a)^2 + b$

$x^2 + 6x + 16 = x^2 + 2ax + a^2 + b$

$2a = 6 \qquad a^2 + b = 16$

$a = 3 \qquad 9 + b = 16$

$b = 7$

4. $x^2 + 4x - 6 = (x+a)^2 + b$

$x^2 + 4x - 6 = x^2 + 2ax + a^2 + b$

$4 = 2a \qquad -6 = a^2 + b$

$2 = a \qquad -6 = 4 + b$

$-10 = b$

8. $(x+5)(x+3)(x+2)$

$(x^2 + 8x + 15)(x+2)$

$x^3 + 10x^2 + 31x + 30 = ax^3 + bx^2 + cx + d.$

$a = 1 \quad b = 10 \quad c = 31 \quad d = 30$

Q9 $3(x-p)^2+q = 3x^2-12x+7$

$3(x^2-2px+p^2)+q = 3x^2-12x+7$

$3x^2-6px+3p^2+q = 3x^2-12x+7$

$\Rightarrow -6p = -12$ and $3p^2+q = 7$

$\boxed{p=2}$ but $p=2$.

$3(2)^2+q = 7$

$12+q = 7$

$\boxed{q=-5}$

Q11

$(x-4)^3 = x^3+px^2+qx-64$

$(x-4)(x-4)^2$

$(x-4)(x^2-8x+16)$

$x^3-8x^2+16x-4x^2+32x-64$

$x^3-12x^2+48x-64 = x^3+px^2+qx-64$

$\Rightarrow \boxed{-12=p}$ and $\boxed{48=q}$

Q14 $(5a-b)x+b+2c = 0$

$(5a-b)x+b+2c = 0x+0$

$\Rightarrow 5a-b=0$ $\quad b+2c=0$

$\Rightarrow (b=-2c)$

$5a+2c=0$

$5a=2c$

$a=\frac{-2}{5}c$

Q15 $(4x+r)(x^2+s) = 4x^3+px^2+qx+2.$

$4x^3+4sx+rx^2+rs = 4x^3+px^2+qx+2.$

$\Rightarrow r = p$ and $4s = q$ and $rs = 2.$

$\therefore pq = (r)(4s) = 4rs$

but $rs = 2$

$\Rightarrow pq = 4(2) = 8.$

Q17

$$\frac{1}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}.$$

$$\frac{1}{(x+1)(x-1)} = \frac{A(x-1)+B(x+1)}{(x+1)(x-1)}$$

$$1 = Ax - A + Bx + B.$$
$$1 = (A+B)x + (B-A)$$
$$0x+1 = (A+B)x + (B-A)$$

$\Rightarrow A+B = 0$ and $B-A = 1.$

Sim Eqns $A+B=0$

$-A+B=1$

$2B = 1$

$\boxed{B = 1/2}$

Sub in: $A+B=0$

$A+1/2=0$

$\boxed{A = -1/2}$

Q18 $\frac{1}{(x+2)(x-3)} = \frac{C}{x+2} + \frac{D}{x-3}$

$1 = C(x-3) + D(x+2)$

$0x + 1 = Cx - 3C + Dx + 2D$

$0x + 1 = (C+D)x - 3C + 2D$

$\Rightarrow \quad C + D = 0 \quad$ and $\quad -3C + 2D = 1$

Sim Eqn

$C + D = 0 \quad (\times 3)$

$-3C + 2D = 1$

$3C + 3D = 0$

$-3C + 2D = 1$

$5D = 1$

$\boxed{D = 1/5}$

sub in $C + D = 0$

$C + 1/5 = 0$

$\boxed{C = -1/5}$

Q20 $(x-3)^2$ a factor of x^3+ax+b

x^2-6x+9

$$x^2-6x+9 \;\overline{)\; x^3+0x^2+ax+b \;}\;\; x+6$$

$$\ominus x^3 \oplus 6x^2 \ominus 9x$$

$$6x^2+(a-9)x+b$$

$$\ominus 6x^2 \oplus 36x \ominus 54$$

$$(a-9+36)x + b-54$$

$$(a+27)x + b-54.$$

But is a factor $\Rightarrow$ remainder $=0$.

$$(a+27)x + b-54 = 0x + 0$$

$\Rightarrow$ $a+27=0$ and $b-54=0$

$a=-27$ $\quad b=54$.

Q21 $(x-2)^2$ a factor of x^3+px+q.

$\Rightarrow (x-2)^2(x+k) = x^3+px+q$

$(x^2-4x+4)(x+k)$

$x^3+kx^2-4x^2-4kx+4x+4k$

$x^3+(k-4)x^2+(4-4k)x+4k = x^3+px+q.$

$\Rightarrow$ $k-4=0$ $\quad 4-4k=P$ $\quad 4k=q$

$\boxed{k=4}$ $\quad$ But $k=4$ $\quad$ but $k=4$

$4-16=P$ $\quad 4(4)=q$

$\boxed{-12=P}$ $\quad \boxed{16=q}$

Q23 (x^2+b) a factor of $x^3-3x^2+bx-15$

$$
\begin{array}{r|l}
 & x-3 \\
\hline
x^2+b & x^3-3x^2+bx-15 \\
 & \ominus x^3 \mp bx \\
\hline
 & -3x^2-15 \\
 & \oplus 3x^2 \pm 3b \\
\hline
 & -15+3b.
\end{array}
$$

Is a factor $\Rightarrow$ remainder $=0$.

$$-15+3b=0$$
$$3b=15$$
$$\boxed{b=5}$$

Q24

$$
\begin{array}{r|l}
 & x+p \\
\hline
x^2-px+9 & x^3+0x^2+ax+b \\
 & \ominus x^3 \pm px^2 \mp 9x \\
\hline
 & px^2+(a-9)x+b \\
 & \ominus px^2 \pm p^2x \mp 9p \\
\hline
 & (a-9+p^2)x+b-9p
\end{array}
$$

Is a factor $\Rightarrow$ remainder $=0$

$$(a-9+p^2)x+b-9p = 0x+0$$

$\Rightarrow$ $a-9+p^2=0$ $\qquad$ $b-9p=0$

$\boxed{a=9-p^2}$ $\qquad$ $\boxed{b=9p}$

$a+b=17 \Rightarrow 9-p^2+9p=17$

$$p^2-9p+8=0$$
$$(p-8)(p-1)=0$$
$$p=8 \quad p=1$$

Q28

$$2x-\sqrt{3}\;\overline{)\;4x^2-2x-2\sqrt{3}x+\sqrt{3}}\;\;2x-1$$

$$\ominus\; 4x^2 \mp 2\sqrt{3}x$$

$$-2x+\sqrt{3}$$

$$\pm 2x \mp \sqrt{3}$$

$$0$$

There is no remainder $\Rightarrow$ Is a factor.

2nd factor is $2x-1$.