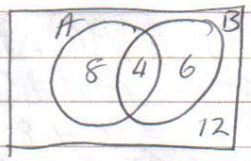
Ex 3.4

Q1



$U = 30$

(i) $P(A) = \frac{12}{30} = \frac{2}{5}$

(ii) $P(B) = \frac{10}{30} = \frac{1}{3}$

(iii) $P(A \cap B) = 4/30 = 2/15$

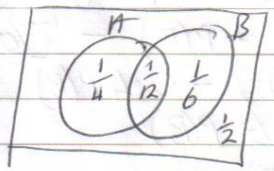
Independent $\Rightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

$\frac{2}{15} = \frac{2}{5} \times \frac{1}{3}$

$2/15 = 2/15$

True $\therefore$ events are Independent

Q2



$U = \frac{1}{4} + \frac{1}{12} + \frac{1}{6} + \frac{1}{2} = \frac{3+1+2+6}{12} = 12/12 = 1$

(i) $P(A) = \frac{1}{4} + \frac{1}{12} = \frac{3+1}{12} = 4/12 = \frac{1}{3}$

(ii) $P(B) = \frac{1}{6} + \frac{1}{12} = 3/12 = 1/4$

$P(A \cap B) = P(A) \cdot P(B)$

$1/12 = \frac{1}{3} \cdot \frac{1}{4}$

$\frac{1}{12} = \frac{1}{12}$ True $\Rightarrow$ are Indep

Q3 $P(A) = 0.8$ $P(B) = 0.6$ $P(A \cap B) = 0.48$

Independent if $P(A \cap B) = P(A) \cdot P(B)$

$0.48 = (0.8)(0.6)$

$0.48 = 0.48$

True $\Rightarrow$ are Independent.

Q4 $P(A) = 0.4$ $P(B) = 0.25$

Indep $\Rightarrow$ $P(A \cap B) = P(A) \cdot P(B)$

$\therefore P(A \cap B) = (0.4)(0.25)$

$= 0.1$

Q5.

$P(A) = 0.4$ $P(A \cup B) = 0.7$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.4 + P(B) - [P(A) \cdot P(B)]$

$0.7 = 0.4 + P(B) - [(0.4)(P(B))]$

$0.3 = P(B) - 0.4\,P(B)$

$0.3 = 0.6\,P(B)$

$\frac{0.3}{0.6} = P(B)$

$0.5 = P(B)$

Q6 $P(A) = 0.45$ $P(B) = 0.35$ $P(A \cup B) = 0.7$

(i) Find $P(A \cap B)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.45 + 0.35 - P(A \cap B)$$

$$0.7 = 0.8 - P(A \cap B)$$

$$-0.1 = -P(A \cap B)$$

$$0.1 = P(A \cap B)$$

(ii) Not Indep.

If Indep

$$P(A \cap B) = P(A) \cdot P(B)$$

$$0.1 = (0.45)(0.35)$$

$$0.1 \neq 0.1575$$

Not equal ∴ Not Indep.

(iii) $$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.1}{0.35} = \frac{2}{7}.$$

Q7 $P(A) = 0.8$ $P(B) = 0.7$ $P(A/B) = 0.8$

(i) $$P(A/B) = \frac{P(A \cap B)}{P(A)}$$

$$0.8 = \frac{P(A \cap B)}{0.7}$$

$$0.56 = P(A \cap B)$$

(ii) Indep if $$P(A \cap B) = P(A) \cdot P(B)$$

$$0.56 = (0.8)(0.7)$$

$$0.56 = 0.56$$

True ∴ are Indep.

Q8 $P(A) = 2/5 \quad P(B) = 1/6 \quad P(A \cup B) = 13/30$

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$13/30 = 2/5 + 1/6 - P(A \cap B)$

$13/30 = 17/30 - P(A \cap B)$

$P(A \cap B) = 17/30 - 13/30$

$P(A \cap B) = 4/30 = 2/15$

(ii) Indep if $P(A \cap B) = P(A) \cdot P(B)$

$2/15 = 2/5 \cdot 1/6$

$2/15 = 2/30$

$2/15 \neq 1/15$

Not equal ∴ Not Indep.

Q9 $P(C|B) = 3/3 \quad P(C \cap D) = 1/3$

(i) $P(C|D) = \frac{P(C \cap D)}{P(D)}$

$2/3 = 1/3 / P(D)$

$P(D) = 1/3 / 2/3 = 1/3 \times 3/2 = 1/2$

$P(D) = 1/2.$

Since Indep $P(C|D) = P(C) \cdot P(D)$

$1/3 = P(C) \cdot 1/2$

$1/3 \cdot 2/1 = P(C) = 2/3$

Q10 $P(B) = 0.7$ $P(C) = 0.6$ $P(C|B) = 0.7$.

find $P(B \cap C)$ $\quad P(C|B) = \frac{P(C \cap B)}{P(B)}$

$$0.7 = \frac{P(C \cap B)}{0.7}$$

$$0.49 = P(B \cap C)$$

If Indep $P(B \cap C) = P(B) \cdot P(C)$

$0.49 = (0.7)(0.6)$

$\neq 0.42$

Not equal $\Rightarrow$ Not Indep.

Q11 $P(A) = 0.2$ $P(B) = 0.15$ are Indep.

(i) $P(A \cap B) = P(A) \cdot P(B)$ as are Indep

$P(A \cap B) = (0.2)(0.15)$

$P(A \cap B) = 0.03$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A|B) = \frac{0.03}{0.15} = 0.2$.

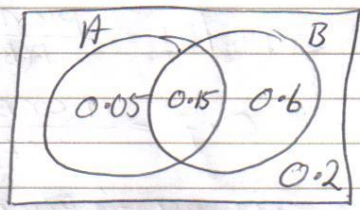
(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.2 + 0.15 - 0.03$

$P(A \cup B) = 0.32$

Q12 $P(A) = 0.2$ $P(A \cap B) = 0.15$ $P(A' \cap B) = 0.6$

(i)



$0.05 + 0.15 + 0.6$
$= 0.8$

(ii) $P(\text{weather}) = 0.2$

(iii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.75} = 0.2$

(iv) Indep if $P(A \cap B) = P(A) \cdot P(B)$

$0.15 = (0.2)(0.75)$

$0.15 = 0.15$

are equal $\Rightarrow$ are Indep

Q13 $P(A) = 8/15$ $P(B) = 1/3$ $P(A|B) = 1/5$

(i) Both! $P(A \cap B)$ $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$1/5 = \frac{P(A \cap B)}{1/3}$

$\frac{1}{5} \times \frac{1}{3} = P(A \cap B)$

$1/15 = P(A \cap B)$

(ii) Only A or B occurs

$P(A) + P(B) = \frac{8}{15} + \frac{1}{3} = \frac{8+5}{15} = \frac{13}{15}$

(iii) Neither $= 1 - 13/15 = 2/15$.

Q14 (i) $P(A \text{ and } B) = P(A) \times P(B)$

$\Rightarrow$ Events are Independent

ie the outcome of A does not affect the outcome of B.

Possible for B: Getting a head when tossing a coin

(ii) $P(C \text{ or } D) = P(C) \times P(D)$

They are mutually exclusive

$\Rightarrow P(C \text{ and } D) = 0$.

Q15

$P(A|B) = 0.4 \qquad P(B|A) = 0.25$

$P(A \cap B) = 0.12$

(i) $P(A)$ and $P(B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$0.4 = \frac{0.12}{P(B)}$

$P(B) = \frac{0.12}{0.4} = 0.3$

$P(B|A) = \frac{P(B \cap A)}{P(A)}$

$0.25 = \frac{0.12}{P(A)}$

$P(A) = \frac{0.12}{0.25} = 0.48$

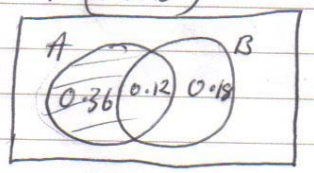
(ii)

$P(A \cap B) = P(A) \cdot P(B)$

$0.12 = (0.48)(0.3)$

$0.12 \neq 0.144 \Rightarrow$ Not Indep.

(iii) $P(A \cap B') = P(A) - P(A \cap B)$



$P(A \cap B') = 0.36$.

Q16 $P(E) = 2/5$ $P(F) = 1/6$ $P(E \cup F) = 13/30$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$\frac{13}{30} = \frac{2}{5} + \frac{1}{6} - P(E \cap F)$$
$$\frac{13}{30} = \frac{12+5}{30} - P(E \cap F)$$
$$P(E \cap F) = \frac{17}{30} - \frac{13}{30}$$
$$P(E \cap F) = \frac{4}{30} = \frac{2}{15}$$

$$P(E \cap F) = P(E) \cdot P(F)$$
$$\frac{2}{15} = \left(\frac{2}{5}\right)\left(\frac{1}{6}\right)$$
$$\frac{2}{15} = \frac{2}{30}$$
$$\frac{2}{15} \neq \frac{1}{15}$$

Not equal
$\Rightarrow$ Not Indep.

Also Since $P(E \cap F) \neq 0$
events are not Mutually Exclusive.