

Ex 1.2

Q2 $f: x \rightarrow 2x+1$ $g: x \rightarrow 4x-3$

(i) $f(3) = 2(3) + 1 = 7$

(ii) $g(f(3))$: $f(3) = 7$
 $g(7) = 4(7) - 3 = 25$

(iii) $f(g(-2))$
 $g(-2) = 4(-2) - 3 = -11$
 $f(-11) = 2(-11) + 1 = -21$

(iv) $g \circ f(x)$
 $f(x) = 2x+1$
 $g(2x+1) = 4(2x+1) - 3$
 $8x+4-3 = 8x+1$

Solve $f \circ g(x) = 19$
 $g(x) = 4x-3$
 $f(4x-3) = 2(4x-3) + 1$
 $8x-6+1 = 8x-5$

$$\begin{aligned} 8x-5 &= 19 \\ 8x &= 24 \\ x &= 3 \end{aligned}$$

Q4 $f(x) = 2^{x-1}$ $g(x) = 3 + 4x$

(i) $f \circ g(x)$
 $g(x) = 3 + 4x$
 $f(3+4x) = 2^{3+4x-1} = 2^{4x+2}$

(ii) $g \circ f(x)$
 $f(x) = 2^{x-1}$
 $g(2^{x-1}) = 3 + 4(2^{x-1})$
 $3 + 2^2(2^{x-1})$
 $3 + 2^{2+x-1}$
 $3 + 2^{x+1}$ *Mult \Rightarrow add Powers*

Q6 $f(x) = 2x + 3$ $g(x) = 2x - 3$

(i) $f \circ g(x)$:
 $g(x) = 2x - 3$
 $f(2x-3) = 2(2x-3) + 3$
 $= 4x - 6 + 3 = 4x - 3$

$g \circ f(x)$:
 $f(x) = 2x + 3$
 $g(2x+3) = 2(2x+3) - 3$
 $4x + 6 - 3 = 4x + 3$

(ii) $f \circ g(x) \times g \circ f(x)$
 $(4x-3)(4x+3) = 16x^2 - 9$

Least value is at $x=0$.
 $\Rightarrow 16(0)^2 - 9 = -9$.

$$\text{Def } f(x) = s+tx, \quad g(x) = x^2-4, \quad h(x) = 3x+1$$

$$t \in \mathbb{N}$$

$$hg f(x) = 4(3x^2 + 3x - 2)$$

$$f(x) = s+tx$$

$$g(s+tx) = (s+tx)^2 - 4 = s^2 + 2stx + t^2x^2 - 4$$

$$h(s^2 + 2stx + t^2x^2 - 4) = 3(s^2 + 2stx + t^2x^2 - 4) + 1$$

$$= 3s^2 + 6stx + 3t^2x^2 - 12 + 1$$

$$= 3s^2 + 6stx + 3t^2x^2 - 11$$

$$3s^2 + 6stx + 3t^2x^2 - 11 = 4(3x^2 + 3x - 2)$$

$$3t^2x^2 + 6stx + 3s^2 - 11 = 12x^2 + 12x - 8$$

Equate Co-efficients

$$3t^2 = 12$$

$$t^2 = 4$$

$$t = 2 \text{ as } t \in \mathbb{N}$$

$$6st = 12$$

$$6s(2) = 12$$

$$12s = 12$$

$$s = 1$$

Q11 $f(x) = 2x + 3$

$f^n(x)$ in terms of n .

(i) $f^2(x) = f \circ f(x)$

$$\begin{aligned} f(x) &= 2x + 3 \\ f(2x+3) &= 2(2x+3) + 3 \\ &= 4x + 6 + 3 = 4x + 9 \\ &= 2^2x + 3(2^2 - 1) \end{aligned}$$

(ii) $f^3(x) = f \circ f \circ f(x)$

$$\begin{aligned} f(4x+9) &= 2(4x+9) + 3 \\ &= 8x + 18 + 3 = 8x + 21 \\ &= 2^3x + 3(2^3 - 1) \end{aligned}$$

(iii) $f^4(x) = f \circ f \circ f \circ f(x)$

$$\begin{aligned} f(8x+21) &= 2(8x+21) + 3 \\ &= 16x + 42 + 3 = 16x + 45 \\ &= 2^4x + 3(2^4 - 1) \end{aligned}$$

$$\Rightarrow f^n(x) = 2^n x + 3(2^n - 1)$$

Q14 $p(x) = (3x-4)^3$

$$f(x) = 3x, \quad g(x) = x-4, \quad h(x) = x^3$$

$$hg f(x)$$

$$f(x) = 3x$$

$$g(3x) = 3x-4$$

$$h(3x-4) = (3x-4)^3 = p(x)$$

Q15 (i) $h(x) = (3x-1)^2$ $fg(x) = h(x)$

$$\Rightarrow f(x) = x^2 \quad g(x) = 3x-1$$

(ii) $h(x) = \frac{1}{5x+3}$ $gf(x) = h(x)$

$$\Rightarrow g(x) = \frac{1}{x} \quad f(x) = 5x+3$$

(iii) $h(x) = \sin^2(3x)$ $fgk(x) = h(x)$

$$f(x) = x^2 \quad g(x) = \sin x \quad k(x) = 3x$$

(iv) $b(x) = \cos \sqrt{2x}$ $hg f(x) = b(x)$

$$f(x) = 2x, \quad g(x) = \sqrt{x} \quad h(x) = \cos x$$

Q17

$$T = \frac{D}{S} \quad \left(\begin{array}{l} D = 5T \\ r = 5 = 4T \end{array} \right)$$

$$\Rightarrow f(r) = \frac{5}{4}t$$

$$(i) f(\text{Area}) = \pi r^2$$

$$(ii) f(A) = \pi \left(\frac{5}{4}t \right)^2$$

Q18 (i) $f(x) = €0.04x$ This is 4% of Sales.

(ii) $g(x) = €(x - 4000)$ Is sales in excess of 4000.

Commission is 4% on sales after €4000.

\Rightarrow Is represented by function $fg(x)$

$$fg(x) = f(x - 4000) = 0.04(x - 4000)$$

$$x = 8000$$

$$\Rightarrow fg(8000) = 0.04(8000 - 4000) = 0.4(4000) \\ = €160$$