Ex 1.6

Q1 $(2,-4)$ $\quad 3x-4y-17=0$

$$d=\frac{|3(2)-4(-4)-17|}{\sqrt{3^2+(-4)^2}}=\frac{|6+16-17|}{\sqrt{9+16}}=\frac{5}{\sqrt{25}}=1$$

Q2 $(1,1)$ $\quad 3x+4y-12=0$

$$d=\frac{|3(1)+4(1)-12|}{\sqrt{3^2+4^2}}=\frac{5}{\sqrt{25}}=1$$

$(1,1)$ $\quad 5x-12y+20=0$

$$d=\frac{|5(1)-12(1)+20|}{\sqrt{5^2+(-12)^2}}=\frac{13}{\sqrt{169}}=\frac{13}{13}=1.$$

$\therefore$ $(1,1)$ is equidistant from each line.

Q3 $(6,2)$ $\quad 5x-3y+10=0$

$$d=\frac{|5(6)-3(2)+10|}{\sqrt{5^2+(-3)^2}}=\frac{34}{\sqrt{34}}=\sqrt{34}$$ ✓ True.

Q4 $(5,-5)$ $\quad x-2y+10=0$

$$d=\frac{|(5)-2(-5)+10|}{\sqrt{5^2+(-2)^2}}=\frac{25}{\sqrt{5}}=5\sqrt{5}$$

$(5,-5)$ $\quad 2x+y-30=0$

$$d=\frac{|2(5)+(-5)-30|}{\sqrt{2^2+1^2}}=\frac{25}{\sqrt{5}}=5\sqrt{5}$$

$\therefore$ $(5,-5)$ is equidistant from each line.

Q5 [values]

$\perp$ d from $(3,1)$ to $4x+3y+c=0$ is 5.

$$5 = \frac{|4(3)+3(1)+c|}{\sqrt{4^2+3^2}}$$

$$5 = \frac{|12+3+c|}{\sqrt{25}}$$

$$5 = \frac{|15+c|}{5}$$

$$25 = |15+c|$$

$$625 = 225+30c+c^2$$

$$c^2+30c-400=0$$

$$(c-10)(c+40)=0$$

$$c=10 \qquad c=-40$$

Q6 $(2,2)$ on $3x-y-4=0$

$$3(2)-2-4=0$$

$0=0$ $\therefore$ $(2,2)$ is on the line.

dis between // Line $3x-y-4=0$ and $6x-2y+7=0$ is $\perp$ dis from $(2,2)$ to $6x-2y+7=0$

$$d = \frac{|6(2)-2(2)+7|}{\sqrt{6^2+(-2)^2}} = \frac{|12-4+7|}{\sqrt{40}} =$$

$$= \frac{15}{\sqrt{40}} = \frac{3\sqrt{10}}{4}$$

Q7 (1,1) $x+7y-3=0$

$$d=\frac{|(1)+7(1)-3|}{\sqrt{1^2+7^2}}=\frac{5}{\sqrt{50}}$$

$$=\frac{\sqrt{2}}{2}$$

(1,1) $x-y+1=0$

$$d=\frac{|(1)-(1)+1|}{\sqrt{(1)^2+(-1)^2}}=\frac{1}{\sqrt{2}}$$

$$=\frac{\sqrt{2}}{2}$$

$\Rightarrow$ Yes (1,1) is equidistants from both lines.

Q8 $(-2,3)$ to $ax+y-7=0$ is $\sqrt{10}$

$$\sqrt{10}=\frac{|a(-2)+(3)-7|}{\sqrt{a^2+1^2}}$$

$$\sqrt{10}=\frac{|-2a-4|}{\sqrt{a^2+1}}$$

$\sqrt{10}\sqrt{a^2+1}=|-2a-4|$ sq both sides.

$\sqrt{10a^2+10}=|-2a-4|$

$10a^2+10=4a^2+16a+16$

$6a^2-16a-6=0$ $(\div 2)$

$3a^2-8a-3=0$

$(3a+1)(a-3)=0$

$a=-1/3$ $a=3$

Q9 $(-2, a)$ equidistant from $4x+3y-3=0$ & $12x+5y-13=0$

$$\frac{|4(-2)+3(a)-3|}{\sqrt{4^2+3^2}} = \frac{|12(-2)+5(a)-13|}{\sqrt{12^2+5^2}}$$

$$\frac{|-8+3a-3|}{\sqrt{16+9}} = \frac{|-24+5a-13|}{\sqrt{144+25}}$$

$$\frac{|3a-11|}{5} = \frac{|-37+5a|}{13}$$

$$13(3a-11) = 5(-37+5a)$$
$$39a-143 = -185+25a$$
$$14a = -42$$
$$a = -3$$

Q10 $(-2, 6)$ and origin $(0,0)$ and $3x+2y-7=0$

$$d = \frac{3(-2)+2(6)-7}{\sqrt{3^2+2^2}}$$
$$= \frac{-6+12-7}{\sqrt{9+4}}$$
$$\frac{-2}{\sqrt{13}}$$

$$d = \frac{3(0)+2(0)-7}{\sqrt{3^2+2^2}}$$
$$= \frac{-7}{\sqrt{13}}$$

Both are Neg $\Rightarrow$ both are on same side

Q11 $(3,4)$ $(9,3)$ $3x + 4y - 36 = 0$

$$d = \frac{3(3) + 4(4) - 36}{\sqrt{3^2 + 4^2}} \qquad d = \frac{3(9) + 4(3) - 36}{\sqrt{3^2+4^2}}$$

$$= \frac{9 + 16 - 36}{\sqrt{25}} \qquad = \frac{27 + 12 - 36}{\sqrt{25}}$$

$$= -\frac{11}{5} \qquad = \frac{3}{5}$$

opp signs $\Rightarrow$ are on opp sides

Q12 $(-3,1)$ $(3,-4)$ $2x - 3y + 7 = 0$

$$d = \frac{2(-3) - 3(1) + 7}{\sqrt{2^2 + (-3)^2}} \qquad d = \frac{2(3) - 3(-4) + 7}{\sqrt{2^2 + (-3)^2}}$$

$$= \frac{-6 - 3 + 7}{\sqrt{13}} \qquad = \frac{6 + 12 + 7}{\sqrt{13}}$$

$$= \frac{-2}{\sqrt{13}} \qquad = \frac{25}{\sqrt{13}}$$

Neg + Pos $\Rightarrow$ Are Not on same side

Q13 $4x + 3y + 1 = 0$

$\parallel$ Line is $4x + 3y + k = 0$

2 units from $4x + 3y + 1 = 0$

require a pt. on this line. ~~$(1, -5/3)$~~ $(-1, 1)$

$(-1,1)$ $4x + 3y + k = 0$ $d = 2$

$$2 = \frac{|4(-1) + 3(1) + k|}{\sqrt{4^2 + 3^2}}$$

$$2 = \frac{|-4+3+k|}{\sqrt{25}}$$

$$10 = |-1+k| \quad \text{sq. both sides}$$

$$100 = 1 - 2k + k^2$$

$$k^2 - 2k - 99 = 0$$

$$(k + 9)(k - 11) = 0$$

$$k = -9 \qquad k = 11$$

The Eqns are: $4x + 3y - 9 = 0$

and $4x + 3y + 11 = 0$

Q14 $3x - 4y + 5 = 0$

$\perp$ Line $4x + 3y + k = 0$

2 Lines $d = 4$ from $(1,1)$

$$4 = \frac{|4(1) + 3(1) + k|}{\sqrt{4^2+3^2}}$$

$$4 = \frac{|7+k|}{5}$$

$$20 = |7+k|$$

$$400 = 49 + 14k + k^2$$

$$k^2 + 14k - 351 = 0$$

$$(k - 13)(k + 27) = 0$$

$$k = 13 \qquad k = -27$$

Eqns are: $4x + 3y + 13 = 0$

and $4x + 3y - 27 = 0$.

Q15 Eqn through $(-4, 2)$

$$y - 2 = m(x+4)$$
$$y - 2 = mx + 4m$$
$$mx - y + 4m + 2 = 0$$

eqn of 2 lines through $(-4,2)$ & $\perp$ dis from $(0,0)$ is 2

$$2 = \frac{|m(0) - (0) + 4m + 2|}{\sqrt{m^2 + 1^2}}$$

$$2 = \frac{|4m+2|}{\sqrt{m^2+1}}$$

$2\sqrt{m^2+1} = |4m+2|$. sq both sides.

$$4(m^2+1) = 16m^2 + 16m + 4$$
$$4m^2 + 4 = 16m^2 + 16m + 4$$
$$12m^2 + 16m = 0 \quad (\div 4)$$
$$3m^2 + 4m = 0$$
$$m(3m+4) = 0$$
$$m = 0 \qquad m = -4/3$$

$\Rightarrow$ The 2 eqns are:
$$y - 2 = m(x+4)$$
at $m = 0$ $\quad \boxed{y - 2 = 0}$

at $m = -4/3$ $\quad y - 2 = -4/3(x+4)$
$$3y - 6 = -4x - 16$$
$$\boxed{4x + 3y + 10 = 0}$$

Q16 Eqn through $(3,5)$

$$y-5=m(x-3)$$
$$y-5=mx-3m$$
$$mx-y-3m+5=0$$

d from $(0,0)$ is 5.

$$5=\frac{|m(0)-(0)-3m+5|}{\sqrt{m^2+(-1)^2}}$$

$$5=\frac{|-3m+5|}{\sqrt{m^2+1}}$$

$$5\sqrt{m^2+1}=|-3m+5| \quad \text{sq both sides}$$
$$25(m^2+1)=9m^2-30m+25$$
$$25m^2+25=9m^2-30m+25$$
$$16m^2+30m=0 \quad (\div 2)$$
$$8m^2+15m=0$$
$$m(8m+15)=0$$
$$m=0 \quad m=-15/8$$

The 2 eqns are:

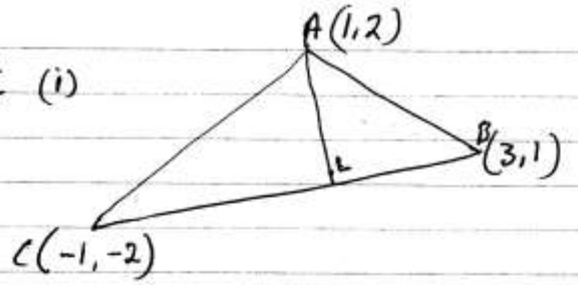
$$y-5=m(x-3)$$

at $m=0$ $\boxed{y-5=0}$

at $m=-15/8$ $y-5=-15/8(x-3)$

$$8y-40=-15x+45$$
$$15x+8y-85=0.$$

Q17 (i)



Eqn $(-1,-2)\ (3,1)$ $\qquad m = \frac{1+2}{3+1} = 3/4$

$$y + 2 = 3/4(x+1)$$
$$4y + 8 = 3x + 3$$
$$3x - 4y - 5 = 0$$

$\perp$ dis from $(1,2)$ to $3x - 4y - 5 = 0$

$$d = \frac{|3(1) - 4(2) - 5|}{\sqrt{3^2 + (-4)^2}} = \frac{|3-8-5|}{\sqrt{25}} = \frac{10}{5} = 2.$$

(ii) Area $= \frac{1}{2}$ base $\times h$.

Length of CB $= \sqrt{(3+1)^2 + (1+2)^2}$
$= \sqrt{16+9} = \sqrt{25} + 5.$

Area $= \frac{1}{2}(5)(2)$
$= 5$ sq units.