Ex 1.9

Q1   (iii) $2x - 5y = 1$ (×2)
               $4x - 3y = 9$

       $2x - 10y = 2$ $2x - 5y = 1$
       $4x - 3y = 9$ $2x - 5(1) = 1$
       $ 7y = 7$ $2x = 6$
       $ y = 1$ $x = 3$

                                         $(3,1)$

Q2 (ii) $\dfrac{x}{2} - \dfrac{y}{6} = \dfrac{1}{6}$ (×6)

            $x - 2y = -8$

            $3x - y = 1$ (×2)
            $x - 2y = -8$

            $6x - 2y = 2$ $x - 2y = -8$
            $-x - 2y = -8$ $2 - 2y = -8$
            $5x = 10$ $-2y = -10$
            $x = 2$ $y = 5$

                                          $(2,5)$

(iii) $\dfrac{4x-2}{5} = \dfrac{8y}{10}$ (×10)

         $18x - 20y = 4$

         $8x - 4 = 8y$ $18x - 20y = 4$
         $18x - 20y = 4$ $18(3) - 20y = 4$
         $8x - 8y = 4$ (×5) $54 - 20y = 4$
         $18x - 20y = 4$ (×2)
         $40x - 40y = 20$ $-20y = -50$
         $-36x + 40y = -8$ $y = \dfrac{50}{20}$
         $4x = 12$ $x = 3$ $y = \dfrac{5}{2}$
                                 $\left(3, \tfrac{5}{2}\right)$

Q3

$$\frac{2x-5}{3} + \frac{y}{5} = 6 \qquad (\times 15)$$

$$5(2x-5) + 3y = 90$$
$$10x - 25 + 3y = 90$$
$$\boxed{10x + 3y = 115}$$

$$\frac{3x}{10} + 2 = \frac{3y-5}{2} \qquad (\times 10)$$
$$3x + 20 = 5(3y-5)$$
$$3x + 20 = 15y - 25$$
$$\boxed{3x - 15y = -45}$$

$$10x + 3y = 115 \qquad (\times 5)$$
$$3x - 15y = -45$$
$$50x + 15y = 575$$
$$3x - 15y = -45$$
$$53x = 530$$
$$x = 10$$

$$10x + 3y = 115$$
$$10(10) + 3y = 115$$
$$3y = 15$$
$$y = 5$$

$$(10, 5)$$

Q4 $y = 3x - 23$ $y = \frac{x}{2} + 2 \quad (\times 2)$

$2y = x + 4$

$y = 3x - 23$
$2y = x + 4 \quad (\times 3)$
$-\; y = 3x \pm 23$
$6y = 3x + 12$
$\overline{\quad 5y = 35 \quad}$
$y = 7$

$2y = x + 4$
$14 = x + 4$
$10 = x$

$(10, 7)$

Q5 A: $2x + y + z = 8$
(i) B: $5x - 3y + 2z = 3$
C: $7x + y + 3z = 20$

$2A: \; -4x - 2y - 2z = -16$ $3B: \; 15x - 9y + 6z = 9$
$B: \; 5x - 3y + 2z = 3$ $2C: \; -14x - 2y - 6z = -40$
$\overline{\quad x - 5y = -13 \quad}$ $\overline{\quad x - 11y = -31 \quad}$

$x - 5y = -13$
$-\; x - 11y = \pm 31$ $x - 5y = -13$
$6y = 18$ $x - 15 = -13$
$\boxed{y = 3}$ $\boxed{x = 2}$

$2x + y + z = 8$ Check: A: $2(2) + 3 + 1 = 8 \checkmark$
$4 + 3 + z = 8$ B: $5(2) - 3(3) + 2(1) = 3 \checkmark$
$\boxed{z = 1}$ C: $7(2) + (3) + 3(1) = 20 \checkmark$

(ii) A: $2x - y - z = 6$
B: $3x + 2y + 3z = 3$
C: $4x + y - 2z = 3$

$2A$: $4x - 2y - 2z = 12$
B: $3x + 2y + 3z = 3$
_____
$7x + z = 15$

B: $3x + 2y + 3z = 3$
$2C$: $8x + 2y - 4z = 6$
_____
$5x - 7z = 3$

$7x + z = 15 \quad (\times 7)$
$5x - 7z = 3$

$49x + 7z = 105$
$5x - 7z = 3$
_____
$54x = 108$
$\boxed{x = 2}$

$7x + z = 15$
$14 + z = 15$
$\boxed{z = 1}$

$2x - y - z = 6$
$4 - y - 1 = 6$
$-y = 3$
$\boxed{y = -3}$

$(2, -3, 1)$

Check:
A: $2(2) - (-3) - (1) = 6$ ✓
B: $3(2) + 2(-3) + 3(1) = 3$ ✓
C: $4(2) + (-3) - 2(1) = 3$ ✓

Q6 (i) A: $2a + b + c = 8$

B: $5a - 3b + 2c = -3$

C: $7a - 3b + 3c = 1$

$2A: \quad 4a + 2b + 2c = 16$

$B: \quad 5a - 3b + 2c = -3$

$\overline{\qquad a - 5b = -19}$

$3B: 15a - 9b + 6c = -9$

$2C: \quad 14a - 6b + 6c = 2$

$\overline{\qquad a - 3b = -11}$

$-a \pm 5b = \pm 19$

$a - 3b = -11$

$\overline{\qquad 2b = 8}$

$\boxed{b = 4}$

$a - 3b = -11$

$a - 12 = -11$

$\boxed{a = 1}$

$2a + b + c = 8$

$2 + 4 + c = 8$

$\boxed{c = 2}$

check A: $2(1) + (4) + (2) = 8$ ✓

B: $5(1) - 3(4) + 2(2) = -3$ ✓

C: $7(1) - 3(4) + 3(2) = 1$ ✓

(iii) $\quad x + y + z = 2$

$2x + 3y + z = 7$

$\frac{x}{2} - \frac{y}{6} + \frac{z}{3} = \frac{2}{3} \quad (\times 6)$

A: $x + y + z = 2$

B: $2x + 3y + z = 7$

C: $3x - y + 2z = 4$

A: $-x - y + z = -2$

B: $2x + 3y + z = 7$

$\overline{\qquad x + 2y = 5}$

$2B: 4x + 6y + 2z = 14$

$C: -3x + y + 2z = -4$

$\overline{\qquad x + 7y = 10}$

$x + 2y = 5$

$x + 7y = 10$

$\overline{\qquad 5y = 5}$

$\boxed{y = 1}$

$x + 2y = 5$

$x + 2 = 5$

$\boxed{x = 3}$

$x + y + z = 2$

$3 + 1 + z = 2$

$\boxed{z = -2}$

check A: $(3) + (1) + (-2) = 2$ ✓

B: $2(3) + 3(1) + (-2) = 7$ ✓

C: $\frac{3}{2} - \frac{1}{6} + \frac{-2}{3} = \frac{2}{3}$

$\frac{9}{6} - \frac{1}{6} - \frac{4}{6} = \frac{4}{6}$ ✓

$(-1, 2.5, -0.5)$

Q7 A: $6x + 4y - 2z = 5$
    B: $3x - 2y + 4z = -10$
    C: $5x - 2y + 6z = -13$

A: $6x + 4y - 2z = 5$
2B: $6x - 4y + 8z = -20$
    $12x + 6z = -15$

B: $3x - 2y + 4z = -10$
C: $5x - 2y + 6z = -13$
    $2x + 2z = -3$

$12x + 6z = -15$
$2x + 2z = -3$ $(\times 3)$
$12x + 6z = -15$
$-6x + 6z = +9$
$6x = -6$
$\boxed{x = -1}$

$2x + 2z = -3$
$-2 + 2z = -3$
$2z = -1$
$\boxed{z = -\frac{1}{2}}$

$6x + 4y - 2z = 5$
$6(-1) + 4y - 2(-\frac{1}{2}) = 5$
$-6 + 4y + 1 = 5$
$4y = 10$
$y = \frac{10}{4} = \frac{5}{2} = 2.5$
$\boxed{y = 2.5}$

Q9 $\qquad y = ax^2 + bx + c$

$(-3, -3) \qquad -3 = a(-3)^2 + b(-3) + c$
$\qquad\qquad\qquad -3 = 9a - 3b + c$

$(-2, -8) \qquad -8 = a(-2)^2 + b(-2) + c$
$\qquad\qquad\qquad -8 = 4a - 2b + c$

$(1, 1) \qquad\quad 1 = a(1)^2 + b(1) + c$
$\qquad\qquad\qquad 1 = a + b + c.$

A: $a + b + c = 1$
B: $9a - 3b + c = -3$
C: $4a - 2b + c = -8$

A: $a + b + c = 1$ $\qquad\qquad$ B: $9a - 3b + c = -3$
B: $9a - 3b + c = -3$ $\qquad$ C: $-4a \pm 2b \pm c = \pm 8$
$\qquad\overline{\quad 8a - 4b = -4 \quad}$ $\qquad\qquad\overline{\quad 5a - b = 5 \quad}$

$8a - 4b = -4$
$5a - b = 5 \quad (\times 4)$
$-8a \pm 4b = \pm 4$ $\qquad\qquad 5a - b = 5$
$\underline{20a - 4b = 20}$ $\qquad\qquad 5(2) - b = 5$
$\qquad\overline{\quad 12a = 24 \quad}$ $\qquad\qquad 10 - b = 5$
$\qquad\boxed{a = 2}$ $\qquad\qquad\qquad -b = -5$
$\qquad\qquad\qquad\qquad\qquad\qquad \boxed{b = 5}$

$a + b + c = 1$
$2 + 5 + c = 1$
$\boxed{c = -6}$

10

$$x + y = 44000 \quad (\times 20)$$
$$30x + 20y = 1200000$$

$$-20x + 20y = -880000$$
$$\underline{30x + 20y = 1200000}$$
$$10x = 320000$$
$$x = 32000$$

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Callum   Lydia

$$x + 3 = 2(y - 5)$$
$$x + 3 = 2y - 10$$
$$x - 2y = -13$$

$$\frac{x+y}{2} = 16$$
$$x + y = 32$$

$$-x \pm 2y = \pm 13$$
$$\underline{x + y = 32}$$
$$3y = 45$$
$$y = 15$$

$$x + y = 32$$
$$x + 15 = 32$$
$$x = 17$$

(14)

$$\frac{a}{x-2} + \frac{b}{x+2} = \frac{4}{(x-2)(x+2)}$$

$$a(x+2) + b(x-2) = 4$$
$$ax + 2a + bx - 2b = 4$$
$$(a+b)x + 2a - 2b = 4$$
$$a + b = 0 \qquad\qquad 2a - 2b = 4$$
$$a - b = 2$$

$$a + b = 0$$
$$\underline{a - b = 2}$$
$$2a = 2$$
$$a = 1$$

$$a - b = 2$$
$$1 - b = 2$$
$$-b = 1$$
$$b = -1$$

$$\frac{1}{x-2} + \frac{-1}{x+2}$$

$$\frac{x+2 - x + 2}{(x-1)(x+2)}$$

$$= \frac{4}{(x-1)(x+2)} \quad \text{True.}$$

(4,26) (8,13)

⑲

$$60 = 2x + 2y$$
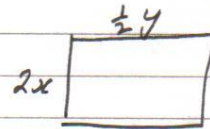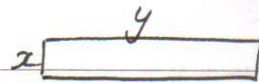
$$42 = 4x + y \quad (\times 2)$$

$$-60 = -2x + 2y$$
$$84 = 8x + 2y$$
$$24 = 6x$$
$$4 = x$$

$$2x + 2y = 60$$
$$8 + 2y = 60$$
$$2y = 52$$
$$y = 26.$$

$$(4, 26)$$
$$\downarrow$$
$$(8, 13)$$

㉑ (i) A: $y - z = 3$ $\longrightarrow$ A: $-z = 3 - y$
B: $x - 2y + z = -4$ $\quad z = (y-3)$
C: $x + 2y = 11$

sub into B: $x - 2y + (y-3) = -4$
$$x - y = -1$$

$$x + 2y = 11$$
$$-x + y = +1$$
$$3y = 12$$
$$\boxed{y = 4}$$

$$x + 2y = 11$$
$$x + 8 = 11$$
$$\boxed{x = 3}$$

$$y - z = 3$$
$$4 - z = 3$$
$$\boxed{z = 1}$$

(ii) $\quad \frac{x}{3} + \frac{y}{2} - z = 7 \quad (\times 6) \to A: 2x + 3y - 6z = 42$

$\frac{x}{4} - \frac{3y}{2} + \frac{z}{2} = -6 \quad (\times 4) \to B: x - 6y + 2z = -24$

$\frac{x}{6} - \frac{y}{4} - \frac{z}{3} = 1 \quad (\times 12) \to C: 2x - 3y - 4z = 12$

A: $2x + 3y - 6z = 42$	$2B: 2x - 12y + 4z = -48$
$3B: 3x - 18y + 6z = -72$	C: $2x - 3y - 4z = 12$
$\boxed{5x - 15y = -30}$	$\boxed{4x - 15y = -36}$

$\begin{array}{rr} 5x - 15y & = -30 \\ -4x \pm 15y & = \pm 36 \\ \hline \boxed{x \quad = \quad 6} \end{array}$

$\begin{array}{rr} 4x - 15y & = -36 \\ 24 - 15y & = -36 \\ -15y & = -60 \\ \hline \boxed{y = 4} \end{array}$

$2x - 3y - 4z = 12$
$2(6) - 3(4) - 4z = 12$
$12 - 12 - 4z = 12$
$-4z = 12$
$\boxed{z = -3}$