Ex 2·2 (Pg 55)

Q1 (i) real roots = f
(ii) real + equal roots = h
(iii) imaginary roots = g

(iv)
$x = 1.5$ or $x = 4.4$
$x = 3$.

Q2. A & B are the roots

$$ax^2 + bx + c = 0 \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = \left(\frac{-b - \sqrt{b^2-4ac}}{2a}, 0\right) \qquad B = \left(\frac{-b + \sqrt{b^2-4ac}}{2a}, 0\right)$$

Q3

(iii) $3x^2 + 2x - 1 = 0$ $\quad a = 3 \quad b = 2 \quad c = -1$

$b^2 - 4ac$
$(2)^2 - 4(3)(-1)$
$4 + 12$
$16 > 0 \Rightarrow$ 2 real roots

Q3 (iv) $-3 + 2x - x^2 = 0$

$x^2 - 2x + 3 = 0$ $\quad a = 1 \quad b = -2 \quad c = 3$

$b^2 - 4ac$

$(-2)^2 - 4(1)(3)$

$4 - 12$

$-8 < 0 \quad \Rightarrow$ No real roots, imaginary roots

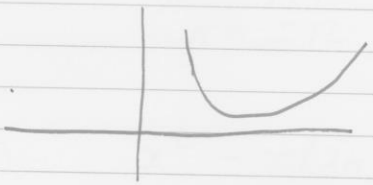
(v) $x^2 + 8x + 16 = 0$ $\quad a = 1 \quad b = 8 \quad c = 16$

$b^2 - 4ac$

$(8)^2 - 4(1)(16)$

$64 - 64 = 0 \quad \Rightarrow$ real + equal root.

Q4



Positive for all values of x.

$3x^2 - kx + 12$ is pos for all values of x $\Rightarrow$ imaginary roots

$a = 3 \quad b = -k \quad c = 12 \quad \Rightarrow \quad b^2 - 4ac < 0$

$b^2 - 4ac$

$(-k)^2 - 4(3)(12) < 0$

$k^2 - 144 < 0$

$k^2 < 144$

$k < \pm 12$

Q5 equal roots $\Rightarrow b^2 - 4ac = 0$

(i) $x^2 - 10x + k = 0$ $a = 1$ $b = -10$ $c = k$

$b^2 - 4ac = 0$

$(-10)^2 - 4(1)(k) = 0$

$100 - 4k = 0$

$100 = 4k$

$25 = k$

(ii) $4x^2 + kx + 9 = 0$ $a = 4$ $b = k$ $c = 9$

$b^2 - 4ac$

$k^2 - 4(4)(9) = 0$

$k^2 - 144 = 0$

$k^2 = 144$

$k = \pm 12$

(iii) $x^2 - x(2k+2) + 5k + 1 = 0$ $a = 1$ $b = -(2k$

$c = 5k+1$

$b^2 - 4ac = 0$

$(-(2k+2))^2 - 4(1)(5k+1) = 0$

$4k^2 + 8k + 4 - 20k - 4 = 0$

$4k^2 - 12k = 0$

$4k(k-3) = 0$

$k = 0 \quad k = 3$

Q6

$k^2x^2 + 2(k+1)x + 4 = 0$ $\quad a = k^2 \quad b = 2k+2 \quad c = 4$

equal roots $\Rightarrow b^2 - 4ac = 0$

$$(2k+2)^2 - 4(k^2)(4) = 0$$

$$4k^2 + 8k + 4 - 16k^2 = 0$$

$$-12k^2 + 8k + 4 = 0$$

$$3k^2 - 2k - 1 = 0$$

$$(3k + 1)(k - 1) = 0$$

$$k = -1/3 \qquad k = 1$$

Q7 (i) $x^2 - 3kx - k^2 = 0$ $\quad a = 1 \quad b = -3k \quad c = -k^2$

real roots $\Rightarrow b^2 - 4ac > 0.$

$$(-3k)^2 - 4(1)(-k^2) > 0$$
$$9k^2 - 4k^2 > 0$$
$$5k^2 > 0 \quad \text{True}$$

(ii) $kx^2 + 2x + (2-k) = 0$ $\quad a = k \quad b = 2 \quad c = 2-k$

$$b^2 - 4ac > 0$$
$$(2)^2 - 4(k)(2-k) > 0$$
$$4 + 8k^2 > 0 \quad \text{True}$$

8/ $x^2 - 3x + 2 - c^2 = 0$ $a = 1$ $b = -3$ $c = 2 - c^2$

real roots $\Rightarrow$ $b^2 - 4ac \geq 0$

$$(-3)^2 - 4(1)(2 - c^2) \geq 0$$
$$9 - 8 + 4c^2 \geq 0$$
$$4c^2 + 1 \geq 0$$ True

as any value squared is positive.

9 $(k-2)x^2 + 2x - k = 0$ $a = k-2$ $b = 2$ $c = -k$

real roots $\Rightarrow$ $b^2 - 4ac \geq 0$

$$(2)^2 - 4(k-2)(-k) \geq 0$$
$$4 + 4k^2 - 8k \geq 0$$
$$4k^2 - 8k + 4 \geq 0$$
$$k^2 - 2k + 1 \geq 0$$
$$(k-1)(k-1) \geq 0$$
$$(k-1)^2 \geq 0$$ True.

10/ $(k-2)x^2 + x(2k+1) + k = 0$ $a = k-2$, $b = 2k+1$, $c = k$

equal roots $\Rightarrow$ $b^2 - 4ac = 0$

$$(2k+1)^2 - 4(k-2)(k) = 0$$
$$4k^2 + 4k + 1 - 4k^2 + 8k = 0$$
$$12k + 1 = 0$$
$$12k = -1$$
$$k = -1/12.$$

11 $(m+3)x^2 + (6-2m)x + m - 1 = 0 \quad m = 3/2$

$\left(\frac{3}{2}+3\right)x^2 + \left(6 - 2\left(\frac{3}{2}\right)\right)x + \frac{3}{2} - 1 = 0$

$\frac{9}{2}x^2 + 3x + \frac{1}{2} = 0 \quad (\times 2)$

$9x^2 + 6x + 1 = 0 \qquad a = 9 \quad b = 6 \quad c = 1$

equal roots $\Rightarrow b^2 - 4ac = 0$

$(6)^2 - 4(9)(1) = 0$

$36 - 36 = 0$ True $\Rightarrow$ equal roots.

12 $ax^2 + bx + 1 = 0$

$ax^2 = -bx - 1$

$a = \frac{-bx - 1}{x^2}$

~~$\left(\frac{-bx-1}{x^2}\right) x^2 + bx + 1 = 0$~~

$-bx - 1 + bx + 1 = 0$

equal Roots $\Rightarrow$

$b^2 - 4ac = 0$

$b^2 - 4a(1) = 0$

$b^2 - 4a = 0$

$b^2 = 4a$

$\frac{b^2}{4} = a.$

Sub into original eqn.

$\frac{b^2}{4}x^2 + bx + 1 = 0 \quad (\times 4)$

$b^2x^2 + 4bx + 4 = 0$

$(bx + 2)(bx + 2) = 0$

$bx + 2 = 0$

$bx = -2$

$x = -2/b$

(13) $x^2 - 2px + 3p^2 + q^2 = 0$. $a = 1$ $b = -2p$ $c = 3p^2 + q^2$

$b^2 - 4ac$

$(-2p)^2 - 4(1)(3p^2 + q^2)$

$4p^2 - 12p^2 - 4q^2$

$-8p^2 - 4q^2$

$-4(2p^2 + q^2)$. $\longleftrightarrow$ Any value mult by -4 is negative $\therefore$ No real roots

$b^2 - 4ac < 0$.