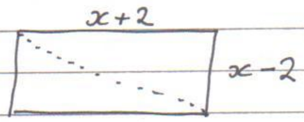
Ex 2·8

① 

Use Pythagoras.

$$(Hyp)^2 = (x+2)^2 + (x-2)^2$$

$$(Hyp)^2 = x^2 + \cancel{4x} + 4 + x^2 - \cancel{4x} + 4$$

$$(Hyp)^2 = 2x^2 + 8$$

$$(Hyp)^2 = 2(x^2+4)$$

$$Hyp = \sqrt{2(x^2+4)}$$

② (a) $(Hyp)^2 = (2+\sqrt{3})^2 + (2-\sqrt{3})^2$

$$(Hyp)^2 = 4 + 4\sqrt{3} + 3 + 4 - 4\sqrt{3} + 3$$

$$(Hyp)^2 = 8 + 6$$

$$(Hyp)^2 = 14$$

$$Hyp = \sqrt{14}$$

(b) (i) Runner 1: $2(2-\sqrt{3}) + 2(2+\sqrt{3})$

$= 4 - 2\sqrt{3} + 4 + 2\sqrt{3} = 8\,Km.$

Runner 2: $2(\sqrt{14}) = 2\sqrt{14}$

Difference: $8 - 2\sqrt{14} = 2(4-\sqrt{14})\ Km$

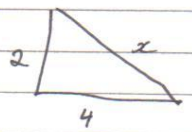
(ii) $T = \frac{D}{S}$ Runner 1: $T = \frac{8}{1\cdot5} = \frac{16}{3} = 5\frac{1}{3}$

Runner 2: $T = \frac{2\sqrt{14}}{1\cdot4} = \frac{20\sqrt{14}}{14} = \frac{10\sqrt{14}}{7}$

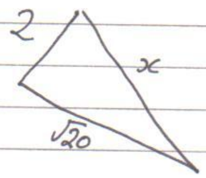
Difference: $\frac{10\sqrt{14}}{7} - 5\frac{1}{3} =$

(3)

$$\left.\begin{array}{l} G \to F = 4\,km \\ F \to E = 2\,km \\ E \to F = 2\,km \end{array}\right] \quad 8\,km$$



$$x^2 = 2^2 + 4^2$$
$$x^2 = 4 + 16$$
$$x^2 = 20$$
$$x = \sqrt{20}$$



$$x^2 = 2^2 + \sqrt{20}^2$$
$$x^2 = 4 + 20$$
$$x^2 = 24$$
$$x = \sqrt{24}$$

Total distance $= 8 + \sqrt{24}$

$8 + 2\sqrt{6}$

$2(4 + \sqrt{6})$

$\sqrt{24} = \sqrt{4 \times 6}$
$= 2\sqrt{6}$

(4)

$$\frac{-1 + \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$\frac{-1 + \sqrt{3} + \sqrt{3} - 3}{1 - 3} = \frac{-4 + 2\sqrt{3}}{-2} = 2 - \sqrt{3}$$

True.

Q5

$$\frac{\sqrt{3}}{1 - \sqrt{3}} - \frac{1}{\sqrt{3}} = \frac{3 - (1 - \sqrt{3})}{(1 - \sqrt{3})(\sqrt{3})}$$

$$= \frac{2 + \sqrt{3}}{\sqrt{3} - 3} \times \frac{-3 - \sqrt{3}}{-3 - \sqrt{3}} = \frac{-6 - 2\sqrt{3} - 3\sqrt{3} - 3}{9 - 3}$$

$-3 + \sqrt{3}$

$$= \frac{-9 - 5\sqrt{3}}{-6} = \frac{9 + 5\sqrt{3}}{6}$$

⑥ $x = \sqrt{a} + \frac{1}{\sqrt{a}}$ $\quad y = \sqrt{a} - \frac{1}{\sqrt{a}}$

(i) $x + y$ $\quad \sqrt{a} + \frac{1}{\sqrt{a}} + \sqrt{a} - \frac{1}{\sqrt{a}} = 2\sqrt{a}$

(ii) $x - y$ $\quad \left(\sqrt{a} + \frac{1}{\sqrt{a}}\right) - \left(\sqrt{a} - \frac{1}{\sqrt{a}}\right)$

$$\sqrt{a} + \frac{1}{\sqrt{a}} - \sqrt{a} + \frac{1}{\sqrt{a}} = \frac{2}{\sqrt{a}}$$

Hence: $\sqrt{\cancel{x}^2 - y^2} = \sqrt{(x+y)(x-y)}$

$$= \sqrt{(2\sqrt{a})\left(\frac{2}{\sqrt{a}}\right)} = \sqrt{4} = 2.$$

⑦

(i) $\sqrt{2x+1} = 3$

$2x + 1 = 9$

$2x = 8$

$x = 4$

(ii) $\sqrt{3x+10} = x$

$3x + 10 = x^2$

$0 = x^2 - 3x - 10$

$0 = (x-5)(x+2)$

$x = 5 \quad x = -2$

$\Rightarrow$ 1 solution: $x = 5$.

check.:

$\sqrt{3(5)+10} = \sqrt{25} = 5$ True

$\sqrt{3(2)+10} = \sqrt{16} = 4$ Not True.

(iii) $\sqrt{2x-1} = \sqrt{x+8}$

$$2x-1 = x+8$$
$$x = +9$$

check

$$\sqrt{2(9)-1} = \sqrt{9+8}$$
$$\sqrt{17} = \sqrt{17} \quad \text{True.} \qquad \text{Sol: } x=9.$$

(iv) $\left(\sqrt{3x-5}\right)^2 = (x-1)^2$

$$3x-5 = x^2-2x+1$$
$$0 = x^2-5x+6$$
$$0 = (x-3)(x-2)$$
$$x=3 \qquad x=2$$

check.

$\sqrt{3(3)-5} = 3-1$	$\sqrt{3(2)-5} = 2-1$
$\sqrt{4} = 2$	$\sqrt{1} = 1$
$2 = 2$ True	$1 = 1$ True.

2 Solutions $x=3$ and $x=2$

(v) $\sqrt{2x+5} = x+1$

$$2x+5 = x^2+2x+1$$
$$0 = x^2-4$$
$$0 = (x+2)(x-2)$$
$$x=-2 \qquad x=2$$

check:

$\sqrt{2(-2)+5} = -2+1$	$\sqrt{2(2)+5} = 2+1$
$\sqrt{1} = -1$	$\sqrt{9} = 3$
$1 \neq -1$	$3 = 3$ True.

Only 1 Solution $x=2$.

(vi) $\sqrt{2x^2-7} = x+3$

$2x^2-7 = x^2+6x+9$

$x^2-6x-16=0$

$(x+2)(x-8)=0$

$x=-2 \quad x=8$

check: $\sqrt{2(-2)^2-7} = -2+3$ | $\sqrt{2(8)^2-7} = 8+3$

$\sqrt{1} = 1$ | $\sqrt{121} = 11$

$1 = 1$ True | $11 = 11$ True.

2 Solutions : $x=-2$ and $x=8$

8 (i) $\sqrt{x+5} = 5-\sqrt{x}$

$x+5 = 25-10\sqrt{x}+x$

$10\sqrt{x} = 20$

$100x = 400$

$x = 4$

check: $\sqrt{4+5} = 5-\sqrt{4}$

$\sqrt{9} = 5-2$

$3 = 3$ True.

(ii) $\sqrt{5x+6} = \sqrt{2x}+2$

$5x+6 = 2x+4\sqrt{2x}+4$

$3x+2 = 4\sqrt{2x}$

$9x^2+12x+4 = 16(2x) = 32x$

$9x^2-20x+4=0$

$(9x-2)(x-2)=0$

$9x=2$

$x=2/9 \quad x=2.$

check:

$\sqrt{5(2/9)+6} = \sqrt{2(2/9)}+2$ | $\sqrt{5(2)+6} = \sqrt{2(2)}+2$

$\frac{8}{3} = 8/3$ True | $4 = 4$ True

(iii) $\sqrt{x+7} + \sqrt{x} = 7$

$\sqrt{x+7} = 7 - \sqrt{x}$

$x+7 = 49 - 14\sqrt{x} + x$

$+42 = +14\sqrt{x}$

$1764 = 196x$

$9 = x$

check: $\sqrt{9+7} + \sqrt{9} = 7$

$4 + 3 = 7$

$7 = 7$ True

(iv) $\sqrt{3x-2} = \sqrt{x-2} + 2$

$3x-2 = x-2 + 4\sqrt{x-2} + 4$

$2x-4 = 4\sqrt{x-2}$

$4x^2 - 16x + 16 = 16(x-2)$

$4x^2 - 16x + 16 = 16x - 32$

$4x^2 - 32x + 48 = 0$

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$x = 2 \quad x = 6$

Check: $\sqrt{3(2)-2} = \sqrt{2-2} + 2$ | $\sqrt{3(6)-2} = \sqrt{6-2} + 2$

$2 = 2$ True | $4 = 4$ True

Q9 $x = \sqrt{a} + \frac{1}{\sqrt{a}} + 1$

$x^2 - 2x$

$$\left(\sqrt{a} + \frac{1}{\sqrt{a}} + 1\right)\left(\sqrt{a} + \frac{1}{\sqrt{a}} + 1\right) - 2\left(\sqrt{a} + \frac{1}{\sqrt{a}} + 1\right)$$

$$a + 1 + \sqrt{a} + 1 + \frac{1}{a} + \cancel{\frac{1}{\sqrt{a}}} + \cancel{\sqrt{a}} + \cancel{\frac{1}{\sqrt{a}}} + 1 - \cancel{2\sqrt{a}} - \cancel{\frac{2}{\sqrt{a}}} - 2$$

$$1 + a + \frac{1}{a}$$

Q10 $(a + \sqrt{3})(b - \sqrt{3}) = 7 + 3\sqrt{3}$

$ab - a\sqrt{3} + b\sqrt{3} - 3 = 7 + 3\sqrt{3}$

$ab - 3 + (b - a)\sqrt{3} = 7 + 3\sqrt{3}$

Equate Coefficients:

$ab - 3 = 7$ $\qquad$ $b - a = 3$

$ab = 10$ $\qquad$ $b = 3 + a$

$a(3 + a) = 10$

$3a + a^2 = 10$

$a^2 + 3a - 10 = 0$

$(a + 5)(a - 2) = 0$

$a = -5 \qquad a = 2$

find b: $\quad b = 3 - 5 \qquad b = 3 + 2$

$b = -2 \qquad b = 5$

check $\begin{pmatrix} a & b \\ -5, & -2 \end{pmatrix}$ $(-5 + \sqrt{3})(-2 - \sqrt{3}) = 7 + 3\sqrt{3}$

$10 + 5\sqrt{3} - 2\sqrt{3} - 9$

$1 + 3\sqrt{3} \neq 7 + 3\sqrt{3}$

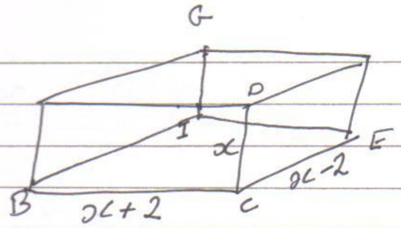
$\begin{pmatrix} a, & b \\ 2, & 5 \end{pmatrix}$ $(2 + \sqrt{3})(5 - \sqrt{3}) = 7 + 3\sqrt{3}$

$10 - 2\sqrt{3} + 5\sqrt{3} - 3$

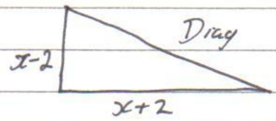
$7 + 3\sqrt{3} = 7 + 3\sqrt{3}$

TRUE.

(ii)

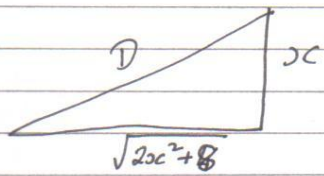


(i) $|IC|$



$$(Diag)^2 = (x-2)^2 + (x+2)^2$$
$$= x^2 - 4x + 4 + x^2 + 4x + 4$$
$$= 2x^2 + 8$$
$$= 2(x^2+4)$$
$$Diag = \sqrt{2(x^2+4)}$$

(ii) $|ID|$



$$D^2 = (x)^2 + \left(\sqrt{2x^2+4}\right)^2$$
$$D^2 = x^2 + 2x^2 + 8$$
$$D^2 = 3x^2 + 8$$
$$D = \sqrt{3x^2+8}$$

(iii) $|ID| = \sqrt{56}$ $\Rightarrow$ $\sqrt{3x^2+8} = \sqrt{56}$

$$3x^2 + 8 = 56$$
$$3x^2 = 48$$
$$x^2 = 16$$
$$x = \pm 4$$

Length cannot be negative

$\Rightarrow x = 4$.

Check: $\sqrt{3(4)^2+8} = \sqrt{56}$

$\sqrt{56} = \sqrt{56}$ True.