


Ex 1

$$\cos \theta = -\frac{\sqrt{3}}{2} \quad (\text{radians})$$

cos is Neg  ref angle =  $\frac{\pi}{6}$

2 sols are ①  $\pi - \frac{\pi}{6} = \frac{5}{6}\pi$  ②  $\pi + \frac{\pi}{6} = \frac{7}{6}\pi$

General Sol ①  $\theta = \frac{5}{6}\pi + 2n\pi$  ②  $\theta = \frac{7}{6}\pi + 2n\pi$

## Example 2

$$\sin 3\theta = \frac{1}{2} \quad (\text{in } \underline{\text{Radians}})$$



ref angle is  $\frac{\pi}{6}$

Sols: ①  $3\theta = \frac{\pi}{6}$

②  $3\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

N.B. write gen sol before find  $\theta$


General Sols ①  $3\theta = \frac{\pi}{6} + 2\pi n$       ②  $3\theta = \frac{5\pi}{6} + 2\pi n$

$$\Rightarrow \theta = \frac{\pi}{18} + \frac{2\pi n}{3}$$

$$\Rightarrow \theta = \frac{5\pi}{18} + \frac{2\pi n}{3}$$

### Example 3

$$\cos 2\theta = -\frac{1}{2} \quad \text{For } \underline{0 \leq \theta \leq 360}.$$

cos neg  ref angle is  $60^\circ$

Sols: ①  $2\theta = 120^\circ$       ②  $2\theta = 240$

General Sols ①  $2\theta = 120^\circ + 360n$       ②  $2\theta = 240 + 360n$   
 $\Rightarrow \theta = 60^\circ + 180n$        $\Rightarrow \theta = 120 + 180n$

for  $0 \leq \theta \leq 360$

Ans  $\theta = \underbrace{60, 120}_{n=0}, \underbrace{240, 300}_{n=1}$

Ex 2.5

Ex 2.5

$\sin x = \frac{1}{2}$  find 2 values

$0 \leq x \leq 360$



ref angle =  $30^\circ$

Sols ①  $\theta = 30^\circ$

②  $\theta = 150^\circ$

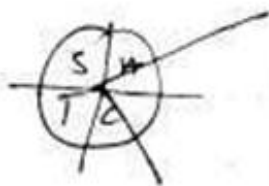
Ex 2.6

2 sols

to

$\cos x = \frac{\sqrt{3}}{2}$

$0 \leq x \leq 2\pi$



Ref angle =  $\frac{\pi}{6}$  rad. or  $30^\circ$

Sol ①  $\theta = \frac{\pi}{6}$  or  $30^\circ$

②  $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$  or  $330^\circ$

Ex 2.7

$\tan \theta = 1$

2 sols for  $0 \leq \theta \leq 2\pi$



Ref angle =  $45^\circ$  or  $\frac{\pi}{4}$  rad.

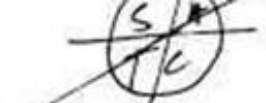
Sol

①

$\theta = 45^\circ$  or  $\frac{\pi}{4}$  rad

②

$\theta = 180 + 45 = 225^\circ$  or  $\frac{5\pi}{4}$  rad

 Ref Angle =  $45^\circ$  or  $\frac{1}{4}$  rad.

Sol

①  $\theta = 45^\circ$  or  $\frac{\pi}{4}$  rad

②  $\theta = 180 + 45 = 225^\circ$  or  $\frac{5}{4}\pi$  rad

Q4

all sols  $\sin 2\theta = \frac{1}{2}$   $\theta \in \mathbb{R}$  & in radians.



ref Angle =  ~~$30^\circ$~~   $\frac{\pi}{6}$

Sol ①  $2\theta = \frac{\pi}{6}$

②  $2\theta = \frac{5\pi}{6}$

General Sol ①  $2\theta = \frac{\pi}{6} + 2n\pi$  ②  $2\theta = \frac{5\pi}{6} + 2n\pi$

$\Rightarrow$  ①  $\theta = \frac{\pi}{12} + n\pi$  ②  $\theta = \frac{5\pi}{12} + n\pi$



Ex 2.8 (Text + Test 4)

Q5  $\cos 3\theta = \frac{\sqrt{3}}{2}$  (Sols in degrees)



ref angle is  $30^\circ$

Sol ①  $3\theta = 30^\circ$

②  $3\theta = 360 - 30 = 330^\circ$

General Sols

①  $3\theta = 30^\circ + n360$

②  $3\theta = 330^\circ + n360$

$\Rightarrow \theta = 10^\circ + 120^\circ n$

$\Rightarrow \theta = 110^\circ + 120^\circ n$

Q6  $\sin 3\theta = -\frac{\sqrt{3}}{2}$  (radians)



ref angle is  $\frac{\pi}{3}$

Sols ①  $3\theta = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

②  $3\theta = 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$

$$\textcircled{1} \quad 3\theta = 30^\circ + n360$$
$$\Rightarrow \theta = 10^\circ + 120^\circ n$$

$$\textcircled{2} \quad 3\theta = 330^\circ + n360$$
$$\Rightarrow \theta = 110^\circ + 120^\circ n$$

Q6  $\sin 3\theta = -\frac{\sqrt{3}}{2}$  (radians)



ref angle is  $\frac{\pi}{3}$

$$\text{Sols } \textcircled{1} \quad 3\theta = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$$

$$\textcircled{2} \quad 3\theta = 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$$

General Sols:

$$\textcircled{1} \quad 3\theta = \frac{4}{3}\pi + 2n\pi$$
$$\Rightarrow \theta = \frac{4}{9}\pi + \frac{2}{3}n\pi$$

$$\textcircled{2} \quad 3\theta = \frac{5}{3}\pi + 2n\pi$$

$$\Rightarrow \theta = \frac{5}{9}\pi + \frac{2}{3}n\pi$$

Q7  $2 \cos 4\theta = 1$  (radians)  
 $\Rightarrow \cos 4\theta = \frac{1}{2}$



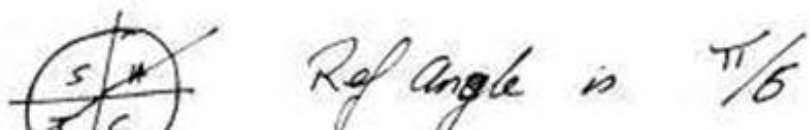
Sols: ①  $4\theta = \frac{\pi}{3}$       ②  $4\theta = 2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$

General Sols! ①  $4\theta = \frac{\pi}{3} + 2n\pi$       ②  $4\theta = \frac{5}{3}\pi + 2n\pi$

$\Rightarrow \theta = \frac{\pi}{12} + \frac{1}{2}n\pi$        $\Rightarrow \theta = \frac{5}{12}\pi + \frac{1}{2}n\pi$

Q8

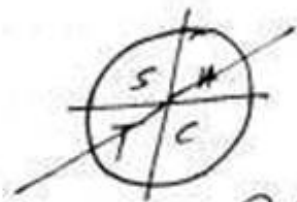
$\tan x = \frac{1}{\sqrt{3}}$  in radians.





Q8

$$\tan x = \frac{1}{\sqrt{3}} \quad \text{in radians.}$$



Ref Angle is  $\frac{\pi}{6}$

Sols: ①  $x = \frac{\pi}{6}$

②  ~~$x = \frac{\pi}{6} + \pi = \frac{7}{6}\pi$~~

General Sols: ①  $x = \frac{\pi}{6} + \pi n$

②  ~~$x = \frac{7}{6}\pi + \pi n$~~

Q9

$$\sin 3x = -\frac{1}{2}$$

$$0 < x \leq 360^\circ$$



Ref Angle is  $30^\circ$

Sols: ①  $3x = 30^\circ + 180^\circ = 210^\circ$

②  $3x = 360 - 30^\circ = 330^\circ$

General Sol:  $3x = 210 + n360 \quad || \quad 3x = 330 + n360$   
 $\Rightarrow x = 70 + n120 \quad || \quad \Rightarrow x = 110^\circ + n120.$

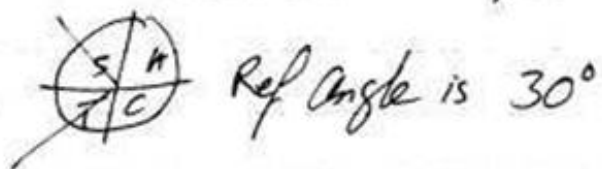
$\Rightarrow x = 70, 110, 190, 230, 310, 350.$

Q10

$$2 \cos 2\theta = -\sqrt{3}$$

$$0 \leq \theta \leq 360^\circ$$

$$\cos 2\theta = -\frac{\sqrt{3}}{2}$$



Sols: ①  $180 - 30 = 150^\circ$

②  $180 + 30 = 210^\circ$

General Sol: ①  $2\theta = 150^\circ + n360$

②  $2\theta = 210 + n360$

$\Rightarrow \theta = 75^\circ + n180^\circ$

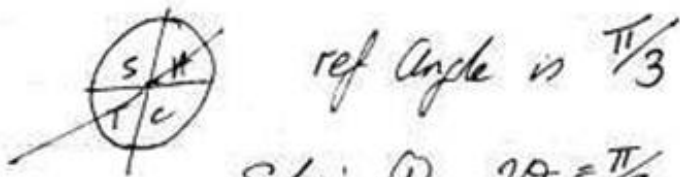
②  $\Rightarrow \theta = 105^\circ + n180^\circ$

$\therefore \theta = 75^\circ, 255^\circ, 105^\circ, 285^\circ$

Q11

$$\tan 2\theta = \sqrt{3}$$

radians.



Sols: ①  $2\theta = \frac{\pi}{3}$

②  $2\theta = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

General Sol: ①  $2\theta = \frac{\pi}{3} + n\pi$

②  $2\theta = \frac{4}{3}\pi + n\pi$

$\Rightarrow \theta = \frac{\pi}{6} + \frac{1}{2}n\pi$

$\theta = \frac{4}{6}\pi + \frac{1}{2}n\pi$   
 $= \frac{2}{3}\pi + \frac{1}{2}n\pi$



Sols: ①  $2\theta = \frac{\pi}{3}$

②  $2\theta = \pi + \frac{\pi}{3} = \frac{4}{3}\pi$

General Sols: ①  $2\theta = \frac{\pi}{3} + n\pi$

②  $2\theta = \frac{4}{3}\pi + n\pi$

$\Rightarrow \theta = \frac{\pi}{6} + \frac{1}{2}n\pi$

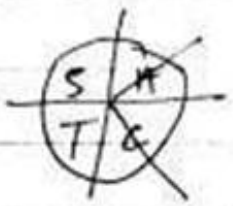
$\theta = \frac{4}{6}\pi + \frac{1}{2}n\pi$   
 $= \frac{2}{3}\pi + \frac{1}{2}n\pi$

Q12

$2 \cos 4\theta = \sqrt{3}$

radians.

$\cos 4\theta = \frac{\sqrt{3}}{2}$



ref angle is  $\frac{\pi}{6}$ .

Sols: ①  $4\theta = \frac{\pi}{6}$

②  $4\theta = 2\pi - \frac{\pi}{6} = \frac{11}{6}\pi$

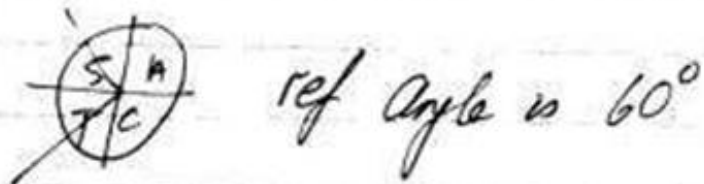
General Sol:  $4\theta = \frac{\pi}{6} + 2n\pi$

$4\theta = \frac{11}{6}\pi + 2n\pi$

$\Rightarrow \theta = \frac{\pi}{24} + \frac{1}{2}n\pi$

$\Rightarrow \theta = \frac{11}{24}\pi + \frac{1}{2}n\pi$

Q13  $\cos 3\theta = -\frac{1}{2}$   $0 \leq \theta \leq 360$ .



Sols: ①  $3\theta = 120^\circ$       ②  $3\theta = 240^\circ$

General Sols: ①  $3\theta = 120^\circ + n360$       ②  $3\theta = 240 + n360$   
 $\Rightarrow \theta = 40^\circ + n120$        $\Rightarrow \theta = 80 + n120$

$\theta = 40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ, 320^\circ$

Q14  $\sin 3\theta = 0.78$   $0 < \theta \leq 360$ .

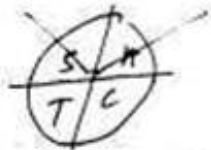


Ref angle is  $\sin^{-1}(0.78) = 51^\circ$

Q14

$$\sin 3\theta = 0.78$$

$$0 < \theta \leq 360$$



Ref angle is  $\sin^{-1}(.78) = 51^\circ$

Sols! ①  $3\theta = 51^\circ$

②  $129^\circ = 3\theta$

General Sols: ①  $3\theta = 51^\circ + n360$

②  $3\theta = 129^\circ + n360$

$\Rightarrow \theta = 17^\circ + n120$

②  $\Rightarrow \theta = 43^\circ + n120$

$\theta = 17^\circ, 43^\circ, 137^\circ, 163^\circ, 257^\circ, 283^\circ$