Exercise 3.2

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Q1 (iii) $\sqrt{-27} = \sqrt{9 \times -3} = 3\sqrt{-3} = 3\sqrt{3 \times -1} = 3\sqrt{3}\,i$

(iv) $\sqrt{-20} = \sqrt{4 \times 5 \times -1} = 2\sqrt{5}\,i$

Q2 (ii) $x^2 + 12 = 0$

$x^2 = -12$

$x = \sqrt{-12}$ NB ±

$x = \sqrt{4 \times 3 \times -1} = \pm 2\sqrt{3}\,i$

Q3 (iii) $(-3 + 4i) + (6 - 4i) = 3 + 0i$ ← NB

(vi) $(1 + i) + (2 - 3i) = 3 - 2i$

Q4 (iii) $(4 - 7i) - (-1 + 3i) = 5 - 10i$

(vi) $(-3 - 2i) - (4 - 7i) = -7 + 5i$

Q5 (iii) $(5 - 2i)(3 - 5i) = 15 - 25i - 6i \oplus 10i^2$

$= 5 - 31i$

(vi) $(3 - 2i)^2 = 9 - 12i \oplus 4i^2$

$= 5 - 12i$

Q6 $z_1 = 2+4i \quad z_2 = 3-i \quad z_3 = 4-2i$

(i) $3z_1 = 3(2+4i) = 6+12i$

(ii) $z_2 + z_3 = (3-i)+(4-2i) = 7-3i$

(iii) $2z_1 + z_2 = 2(2+4i) + (3-i)$
$= 4+8i+3-i = 7+7i$

(iv) $-3z_2 = -3(3-i) = -9+3i$

(v) $z_1 \cdot z_2 = (2+4i)(3-i) = 6-2i+12i \oplus 4i^2$
$= 10+10i$

(vi) $z_2 z_3 = (3-i)(4-2i) = 12-6i-4i \oplus 2i^2$
$= 10-10i$

(vii) $i(z_3) = i(4-2i) = 4i \oplus 2i^2$
$= 2+4i$

(viii) $z_2(z_1 - z_2) = (3-i)((2+4i)-(3-i))$
$= (3-i)(-1+5i)$
$= -3+15i+i \oplus 5i^2$
$= 2+16i$

Q7 (iv) $x^2 - 8x + 52 = 0$

$$x = \frac{8 \pm \sqrt{64 - 4(52)}}{2} = \frac{8 \pm \sqrt{64 - 208}}{2}$$

$$= \frac{8 \pm \sqrt{-144}}{2} = \frac{8 \pm 12i}{2} = 4 \pm 6i$$

Q8 $2z^2 - 8z + 9 = 0$

$$z = \frac{8 \pm \sqrt{64 - 4(2)(9)}}{2(2)} = \frac{8 \pm \sqrt{64 - 72}}{4}$$

$$= \frac{8 \pm \sqrt{-8}}{4} = \frac{8 \pm 2\sqrt{2}i}{4} = 2 \pm \frac{\sqrt{2}}{2} i$$

NB
Q9

$i = i^1 = i$
$i \times i = i^2 = -1$
$i \times i \times i = i^3 = -i$
$i \times i \times i \times i = i^4 = 1$
$i \times i \times i \times i \times i = i^5 = i$
$i \times i \times i \times i \times i \times i = i^6 = -1$

Pattern is $(i, -1, -i, 1)$ repeated

NB ✱
Divide by 4 and find remainder.
R = 0 ⇒ 1
R = 1 ⇒ i
R = 2 ⇒ -1
R = 3 ⇒ $-i$

$i^{32} = i^{(4\times8)+0} = i^0 = 1$

Q10 $i^{30} = i^{(4\times7)+2} = i^2 = -1$

$i^{11} = i^{(4\times2)+3} = i^3 = -i$

$i^{19} = i^{(4\times4)+3} = i^3 = -i$

$i^{21} = i^{(4\times5)+1} = i^1 = i$

(*) $i^{-4} = \frac{1}{i^4} = \frac{1}{1} = 1$

Q11 (i) $i^{16} + i^{10} + i^6 - i^{12}$

$1 + i^2 + i^2 - 1$

$1 - 1 - 1 - 1$

$= -2.$

(ii) $i^3 - i^{11} + i^{17} - i^{29}$

$i^3 - i^3 + i - i$

$= 0$

Q12 (i) $i^2 \cdot i^6 \cdot i^5$

$= i^{13} = i^1 = i$

(ii) $3i^3 \cdot 2i^5 \cdot 4i^2 = 24i^{10} = 24i^2 = -24$

(iii) $(2i^7)^3 = 8i^{21} = 8i$

Q13 $4i^3 + 7i^9 = 4(-i) + 7i = -4i + 7i = 3i$