Ex 3.3

Q1 (ii) $z = 2-6i \qquad \bar{z} = 2+6i$

(iii) $z = -5-2i \qquad \bar{z} = -5+2i$

Q2 (ii) $z = -3-4i \qquad \bar{z} = -3+4i$

(iii) $z = 1+7i \qquad \bar{z} = 1-7i$

Q3 (i) $\dfrac{2+3i}{4-i} \times \dfrac{4+i}{4+i}$

$= \dfrac{8+2i+12i+3i^2}{17}$

$= \dfrac{5+14i}{17} = \dfrac{5}{17} + \dfrac{14}{17}i$

(ii) $\dfrac{4+3i}{5+i} \times \dfrac{5-i}{5-i}$

$= \dfrac{20-4i+15i-3i^2}{26}$

$= \dfrac{23+11i}{26} = \dfrac{23}{26} + \dfrac{11}{26}i$

(iii) $\dfrac{8-i}{2+3i} \times \dfrac{2-3i}{2-3i} = \dfrac{16-24i-2i+3i^2}{13}$

$\dfrac{13-26i}{13} = 1-2i$

(iv) $\dfrac{2+5i}{-3+2i} \times \dfrac{-3-2i}{-3-2i} = \dfrac{-6-4i-15i-10i^2}{13}$

$= \dfrac{4-19i}{13} = \dfrac{4}{13} - \dfrac{19}{13}i$

Q4 (iii) $z - \bar{z}$ $\qquad z = 2+6i$

$(2+6i)-(2-6i) = 2+6i-2+6i = 0+12i$

(iv) $z^2 = (2+6i)^2 = 4+24i+36i^2$

$= -32+24i$

Q5 (ii) $\dfrac{(2-6i)-(3+2i)}{2+2i}$

$\dfrac{-1-8i}{2+2i} \times \dfrac{2-2i}{2-2i} = \dfrac{-2+2i-16i+16i^2}{4+4}$

$\dfrac{-18-14i}{8} = \dfrac{-9}{4} - \dfrac{7}{4}i$

(iv) $\dfrac{(2+i)+(3-2i)}{(4+i)-(3+2i)} = \dfrac{5-i}{1-i}$

$\dfrac{5-i}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{5+5i-i-i^2}{1+1}$

$\dfrac{6+4i}{2} = 3+2i$

(vi) $\frac{(3+i)(2-i)}{(4+i)(2+i)} = \frac{6-3i+2i \overset{+1}{\cancel{-i^2}}}{8+4i+2i \overset{-1}{\cancel{+i^2}}} = \frac{7-i}{7+6i}$

$\frac{7-i}{7+6i} \times \frac{7-6i}{7-6i} = \frac{49-42i-7i \overset{-}{\cancel{+6i^2}}}{49+36}$

$\frac{43-49i}{85} = \frac{43}{85} - \frac{49}{85}i$

Q6 (i) $x + iy = 4 - 2i$

RE $x = 4$ IM $y = -2$

(ii) $x + yi = (2+i)(3-2i)$

$x + yi = 6 - 4i + 3i \overset{+}{\cancel{-2i^2}}$

$x + yi = 8 - i$

RE $x = 8$ IM $y = -1$

(iii) $x + yi = \frac{7+i}{2-i} \times \frac{2+i}{2+i}$

$x + yi = \frac{14 + 7i + 2i \overset{-1}{\cancel{+i^2}}}{4+1}$

$x + yi = \frac{13}{5} + \frac{9}{5}i$

RE $x = 13/5$ IM $y = 9/5$

(iv) $x + yi = (2-3i)^2$

$x + yi = 4 - 12i + 9i^2$

$x + yi = -5 - 12i$

RE $\quad$ IM

$x = -5 \qquad y = -12$

Q7 (i) $a + bi + 3 - 2i = 4(-2+5i)$

$a + bi + 3 - 2i = -8 + 20i$

RE $\quad$ IM

$a + 3 = -8 \qquad b - 2 = 20$

$a = -11 \qquad b = 22$

(ii) $a(1+2i) - b(3+4i) = 5$

$a + 2ai - 3b - 4bi = 5$

RE $\quad$ IM

(x2) $a - 3b = 5 \qquad 2a - 4b = 0$

$\ominus 2a \oplus 6b = \ominus 10$

$\cancel{2a} - 4b = 0$

$2b = -10$

$b = -5$

$2a - 4(-5) = 0$

$2a + 20 = 0$

$2a = -20$

$a = -10$

Q8 $z = x + yi$ $\quad 3(z-1) = i(3+i)$

$\Rightarrow 3(x + yi - 1) = i(3+i)$

$3x + 3yi - 3 = 3i + {}^{-1}\cancel{i^2}$

RE $\qquad$ IM

$3x - 3 = -1 \qquad 3y = 3$

$3x = 2 \qquad y = 1$

$x = 2/3$

Q9 $z_1 = -3 + 4i$ $\quad z_2 = 1 + 2i$

$z_1 + (p + iq) z_2 = 0$

$-3 + 4i + (p + iq)(1 + 2i) = 0$

$-3 + 4i + p + 2pi + iq + {}^{-}2q\cancel{i^2} = 0$

RE $\qquad$ IM

$-3 + p - 2q = 0 \qquad 4 + 2p + q = 0$

$p - 2q = 3 \qquad 2p + q = -4 \quad (\times 2)$

$p - 2q = 3$

$4p + 2q = -8$

$5p = -5$

$p = -1$

$2(-1) + q = -4$

$-2 + q = -4$

$q = -2$

Q10 $\quad z = \sqrt{3+4i}$

$$a + bi = \sqrt{3+4i}$$

$$(a+bi)^2 = 3+4i$$

$$a^2 + 2abi + b^2 i^2 = 3+4i$$

$$a^2 - b^2 + 2abi = 3+4i$$

RE

$$a^2 - b^2 = 3$$

IM

$$2ab = 4$$

$$a = \frac{2}{b}$$

$$\left(\tfrac{2}{b}\right)^2 - b^2 = 3$$

$$\frac{4}{b^2} - b = 3 \quad (\times b^2)$$

$$4 - b^4 = 3b^2$$

$$b^4 + 3b^2 - 4 = 0$$

$$(b^2 + 4)(b^2 - 1) = 0$$

$$b^2 = -4 \qquad b^2 = 1$$

$$b = \pm 2i \qquad b = \pm 1$$

not poss

as $b \in R$. $\quad \Rightarrow b = 1$ and $b = -1$

Find a: $\quad a = 2/b$

at $b = 1 \Rightarrow a = 2$. $\qquad$ at $b = -1 \Rightarrow a = -2$

$$\Rightarrow \sqrt{3+4i} = 2+i \quad \text{or} \quad -2-i$$

Q11 $(x+iy)^2 = 8-6i$

$x^2 + 2xyi - y^2 \cancel{i^2} = 8-6i$

RE

$x^2 - y^2 = 8$

IM

$2xy = -6$

$x = -3/y$

$\left(-3/y\right)^2 - y^2 = 8$

$\frac{9}{y^2} - y^2 = 8 \quad (\times y^2)$

$9 - y^4 = 8y^2$

$y^4 + 8y^2 - 9 = 0$

$(y^2 + 9)(y^2 - 1) = 0$

$y^2 = -9 \qquad y^2 = 1$

Not Poss $\cancel{y = \pm 3i} \qquad y = \pm 1$

find x: $x = -3/y$

at $y = 1$, $x = -3$

$-3 + i$

at $y = -1$, $x = 3$

$3 - i$

Q12

(i) $\sqrt{-12-16i} = a + bi$

$-12-16i = (a+bi)^2$

$-12-16i = a^2 + 2abi - b^2 \cancel{i^2}$

RE

$-12 = a^2 - b^2$

IM

$-16 = 2ab$

$\frac{-8}{b} = a$

$-12 = \left(\frac{-8}{b}\right)^2 - b^2$

$-12 = \frac{64}{b^2} - b^2 \quad (\times b^2)$

$-12b^2 = 64 - b^4$

$b^4 - 12b^2 - 64 = 0$

$(b^2 - 16)(b^2 + 4) = 0$

$b^2 = 16 \qquad b^2 = -4$

$b^2 = \pm 4 \qquad \cancel{b = \pm 2i}$

find a: $a = -8/b$

at $b = 4$, $a = -2$

$-2 + 4i$

at $b = -4$, $a = 2$

$2 - 4i$

$\Rightarrow \sqrt{-12-16i} = -2+4i$ or $2-4i$

(ii) $\left(\sqrt{-15+8i}\right)^2 = (a+bi)^2$

$-15+8i = a^2 + 2abi \overset{\ominus}{+} b^2 \cancel{i^2}$

RE

$-15 = a^2 - b^2$

Im

$8 = 2ab$

$4/b = a$

$-15 = \left(4/b\right)^2 - b^2$

$-15 = 16/b^2 - b^2 \quad (\times b^2)$

$-15b^2 = 16 - b^4$

$b^4 - 15b^2 - 16 = 0$

$(b^2 - 16)(b^2 + 1) = 0$

$b^2 = 16 \qquad b^2 = -1$

$b = \pm 4 \qquad \cancel{b = \pm i}$

find a $\quad a = 4/b$

at $b = 4 \quad a = 1$

$1 + 4i$

at $b = -4 \quad a = -1$

$-1 - 4i$

$\Rightarrow \sqrt{-15+8i} = 1+4i$ or $-1-4i$

(iii) $\left(\sqrt{9-40i}\right)^2 = (a+bi)^2$

$9 - 40i = a^2 + 2abi \overset{\ominus}{+} b^2 \cancel{i^2}$

RE

$9 = a^2 - b^2$

Im

$-40 = 2ab$

$-20/b = a$

$9 = \left(\frac{-20}{b}\right)^2 - b^2$

$9 = 400/b^2 - b^2 \quad (\times b^2)$

$9b^2 = 400 - b^4$

$b^4 + 9b^2 - 400 = 0$

$(b^2 + 25)(b^2 - 16) = 0$

$b^2 = -25 \qquad b^2 = 16$

$\cancel{b = \pm 5i} \qquad b = \pm 4$

find a: $\quad a = -20/b$

at $b = 4$, $a = -5$

$-5 + 4i$

at $b = -4$, $a = 5$

$5 - 4i$

$\therefore \sqrt{9-40i} = 5-4i$ or $-5+4i$

Q13 (i) $z_1 = 2+3i \qquad z_2 = -1-5i$

$$\overline{z_1+z_2} = \overline{(2+3i)+(-1-5i)} = \overline{1-2i} = 1+2i$$

(ii) $$\overline{z_1 z_2} = \overline{(2+3i)(-1-5i)} = \overline{-2-10i-3i \overset{+}{-} 15\cancel{i^2}}$$

$$= \overline{13-13i} = 13+13i$$