Ex 3.3

Q1 (i) fixed No of independent trials with 2 outcomes that have constant probabilities

(ii) $p = \frac{1}{2} \quad q = \frac{1}{2} \quad n = 8$

Q2 (i) $\binom{5}{1}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 = 5 \cdot \frac{1}{16} \cdot \frac{1}{2} = \frac{5}{32}$

(ii) $\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32}$

Q3 $P(\text{success of a 3}) = 1/6 \qquad P(\text{first Not a 3}) = 5/6$

(i) $P(\text{No occasion}) = \binom{5}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^5 = \frac{3125}{7776}$

(ii) $P(\text{once only}) = \binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 = \frac{3125}{7776}$

(iii) $P(\text{Twice}) = \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = 10 \cdot \frac{1}{36} \cdot \frac{125}{216} = \frac{625}{3888}$

Q4 $P(\text{success}) = 1/3 \qquad P(\text{failure}) = 2/3$

3 Times $= \binom{7}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^4 = 35 \cdot \frac{1}{27} \cdot \frac{16}{81} = \frac{560}{2187}$

Q5 $P(\text{Boy}) = \frac{1}{2}$ $P(\text{Girl}) = \frac{1}{2}$

$P(3 \text{ Boys}) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 = 10 \cdot \frac{1}{8} \cdot \frac{1}{4} = \frac{10}{32} = \frac{5}{16}$

$P(2 \text{ Girls}) = \binom{5}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3 = 10 \cdot \frac{1}{4} \cdot \frac{1}{8} = \frac{5}{16}$

Q6 $P(\text{Walk}) = 0.7$ $P(\text{not Walk}) = 0.3$

(i) Walks once $= \binom{5}{1}(0.7)^1(0.3)^4 = 0.028$

(ii) Walks 3 Times $= \binom{5}{3}(0.7)^3(0.3)^2 = 0.3087$

Q7 $P(\text{Vote X}) = \frac{3}{5}$ $P(\text{Not Vote X}) = \frac{2}{5}$

3 Vote X $= \binom{8}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^5 = 0.1238$

Q8 $P(\text{Complete}) = \frac{1}{3}$ $P(\text{Not Complete}) = \frac{2}{3}$

At Least $\Rightarrow$ 3 or 4

3 out of 4 complete $= \binom{4}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^1 = \frac{8}{81}$

4 out of 4 complete $= \binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = \frac{1}{81}$

$\Rightarrow$ At Least 3 $= \frac{8}{81} + \frac{1}{81} = \frac{9}{81}$

Q9 20% defective $= \frac{1}{5}$.

(i) $\binom{4}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625}$

(ii) Not more than 2 defective
$\Rightarrow$ 2 defective or 1 defective or None defective

$P(1 \text{ defective}) = \binom{4}{1}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3 = \frac{256}{625}$

$P(\text{None defective}) = \binom{4}{0}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$

Total $P(\text{Not more than 2}) = \frac{96}{625} + \frac{256}{625} + \frac{256}{625} = \frac{608}{625}$

Q10

$P(\text{Bus}) = \frac{2}{5}$ $P(\text{Not Bus}) = \frac{3}{5}$

4 children

(i) $P(\text{None Bus}) = \binom{4}{0}\left(\frac{2}{5}\right)^0\left(\frac{3}{5}\right)^4 = \frac{81}{625}$

(ii) $P(3 \text{ on Bus}) = \binom{4}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1 = \frac{96}{625}$

(iii) $P(\text{at least 1 on Bus}) = 1 - \text{Prob(None)}$

$= 1 - \frac{81}{625} = \frac{544}{625}$

Q11 $P(\text{Sink Putt}) = \frac{7}{10}$ $P(\text{Not Sink Putt}) = \frac{3}{10}$

(i) $P(\text{Sink 2 in 3}) = \binom{3}{2}\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^1 = \frac{441}{1000}$

(iii) $P(\text{Miss 3 in 4}) = P(\text{will sink 1 in 4})$

$\binom{4}{1}\left(\frac{7}{10}\right)^1\left(\frac{3}{10}\right)^3 = \frac{189}{2500}$

Q12 $P(\text{win}) = \frac{2}{5}$ $P(\text{Not win}) = \frac{3}{5}$

(i) $P(\text{win 3 in 5}) = \binom{5}{3}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2 = \frac{144}{625}$

(ii) $P(\text{win } 1^{st}\ 3^{rd}\ 5^{th} \text{ races})$

$P(\text{Win } 1^{st}) = \frac{2}{5}$

Q13 $P(\text{Boy}) = \frac{1}{2}$ $P(\text{Girl}) = \frac{1}{2}$

2000 families, 4 children.

$P(2 \text{ Boys}) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 = \frac{3}{8}$

$2000 \times \frac{3}{8} = 750$ families.

(ii) $P(\text{No Girls}) = P(4 \text{ Boys})$

$= \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = \frac{1}{16}$

$2000 \times \frac{1}{16} = 125$ families.

(iii) $P(\text{At least 1 boy}) = 1 - P(\text{No Boy})$

$P(\text{No Boy}) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$1 - \frac{1}{16} = \frac{15}{16}$

$2000 \times \frac{15}{16} = 1,875$ families.

Q14 $P(\text{correct}) = 1/3$ $P(\text{not correct}) = 2/3$

(i) fixed N° of trials
2 possible outcomes, success or failure
Trials are independent
Probabilities constant.

(ii) $\binom{4}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^0 = \frac{1}{81}$

(iii) $P(1^{st} \text{ ans correct}) = 1/3$

$P(\text{one ans correct}) = \binom{4}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2 = \frac{32}{81}$

Q15

Toss coin ⇒ only 2 outcomes
$P(H) = \text{success}\ (p)$ $P(T) = \text{failure}\ (q)$
$P(r \text{ successes}) = \binom{n}{r} p^r q^{n-r}$

Q16

$P(5) = 1/6$ $P(\text{not } 5) = 5/6$

(i) $P(2\ 5\text{'s in 10 Throws}) = \binom{10}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 = 0.29071$

(ii) $P(3^{rd}\ 5 \text{ on } 11^{th} \text{ Throw}) = P(2\ 5\text{'s in } 10) \times P(5)$

$= 0.29071 \times \frac{1}{6} = 0.048451$

Q17

(i) $P(\text{Picture}) = \frac{12}{52} = \frac{3}{13}$

(ii) $P(\text{Picture}) = 3/13 \quad P(\text{not Picture}) = 10/13$

$P(3^{rd} \text{ Pic on } 13^{th}) \Rightarrow$ 2 Picture cards in 1^{st} 12 draws

$P(2 \text{ in } 12) = \binom{12}{2}\left(\frac{3}{13}\right)^2\left(\frac{10}{13}\right)^{10} = 0.2548$

$P(3^{rd} \text{ on } 13^{th}) = 0.2548 \times 3/13 = 0.0588$

Q18

$P(\text{red}) = 0.3 \quad P(\text{not red}) = 0.7$

$P(4^{th} \text{ red on } 10^{th} \text{ Spin}) \Rightarrow$ 3 reds in 9 spins.

$P(3 \text{ red in } 9) = \binom{9}{3}(0.3)^3(0.7)^6 = 0.2668$

$P(\text{red on } 10^{th}) = 0.2668 \times 0.3 = 0.08$

Q19 $P(\text{red}) = 0.4$ $P(\text{not red}) = 0.6$

(i) $P(3 \text{ red from } 8) = \binom{8}{3}(0.4)^3(0.6)^5$
$= 0.27869$

(ii) $P(4^{\text{th}} \text{ on } 9^{\text{th}} \text{ draw}) = 0.27869 \times 0.4$
$= 0.11148$

Q20 $P(\text{correct}) = 1/4$ $P(\text{not correct}) = 3/4$

(i) $P(\text{none correct out of } 10) = \binom{10}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$
$= 0.0563$

(ii) $P(7 \text{ correct out of } 10) = \binom{10}{7}\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3$
$= 0.003089$

$P(3^{\text{rd}} \text{ correct on the } 10^{\text{th}} \text{ Q})$

$\Rightarrow$ 2 correct in 9 Q's

$= \binom{9}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^7 = 0.3003387$

$P(3^{\text{rd}} \text{ correct on } 10^{\text{th}}) = 0.3003387 \times \frac{1}{4}$
$= 0.07508$