Ex 3.5

Q2 $y = 2x^2 + x$

(i) $\frac{dy}{dx} = 4x + 1$ at $x = 4$

$4(4) + 1 = 17.$

(ii) $\frac{dy}{dx} = 9$

$4x + 1 = 9$

$4x = 8$

$x = 2.$

Q3 $A = \pi r^2$

(i) $\frac{dA}{dr} = 2\pi r$ at $r = 5$

$2\pi(5) = 10\pi$ cm² per cm

(ii) at $r = 10$ $2\pi(10)$

$= 20\pi$. cm² per cm.

Q4 $V = x^3$

(i) $\frac{dV}{dx} = 3x^2$

at $x = 10$

$3(10)^2 = 300$ cm³ per cm.

(ii) $V = 125$ cm³

$x^3 = 125$

$x = 5.$ $\frac{dV}{dx}$ at $x = 5$

$\Rightarrow 3(5)^2 = 75$ cm³ per cm.

Q5 $P = 100(5 + t - 0{\cdot}25t^2)$
$= 500 + 100t - 25t^2$

$\frac{dP}{dt} = 100 - 50t$

when $t = 3$ $100 - 50(3)$
$100 - 150$
$= -50$

$\Rightarrow$ Pop declines by 50 people after 3 yrs.

Q6 $M = 200{,}000 + 600t^2 - \frac{200}{3}t^3$

(i) $\frac{dM}{dt} = 1200t - 200t^2$

(ii) $t = 3$ $\frac{dM}{dt} = 1200(3) - 200(3)^2 = 1800$ per Month

(iii) $\frac{dM}{dt} = 0$ $1200t - 200t^2 = 0$
$200t(6 - t) = 0$
$t = 0$ $t = +6.$

$\Rightarrow$ growth will be zero at the start and after 6 months

Q7 $s = t^3 - 2t^2 + 3t - 4$

(i) Speed: $= ds/dt = 3t^2 - 4t + 3$

When $t = 4$ Speed $= 3(4)^2 - 4(4) + 3 = 35$ m/s

(ii) acc: $\frac{d^2s}{dt^2} = 6t - 4$
when $t = 4$ acc $= 6(4) - 4 = 20$ m/s

Q8 $s = t^3 - 4t^2 + 4t$

(i) Speed: $ds/dt = 3t^2 - 8t + 4$

$t = 3$ speed $= 3(3)^2 - 8(3) + 4 = 7 m/sec$

(ii) acc: $d^2s/dt^2 = 6t - 8$

$t = 1$ acc $= 6(1) - 8 = -2 m/sec^2$

(iii) at rest $\Rightarrow dy/dx = 0$

$3t^2 - 8t + 4 = 0.$

$(3t - 2)(t - 2) = 0$

$t = 2/3 sec$ $t = 2 sec.$

Q9

$h = 600t - 5t^2$

(i) at Rest $\Rightarrow \frac{dh}{dt} = 0$ $\frac{dh}{dt} = 600 - 10t$

$600 - 10t = 0$

$10t = 600$

$t = 60 sec$

(ii) greatest $H \Rightarrow \frac{dh}{dt}$ is a Max.

from (i) This occurs at $t = 60$

$\Rightarrow H = 600(60) - 5(60)^2$

$= 18000m$

in km $= 18 km.$

Q10

$$s = t^3 - 2t^2 + 4t.$$

(i) $t = 2$ $\quad s = (2)^3 - 2(2)^2 + 4(2)$

$s = 8m.$

(ii) Velocity (Speed) $ds/dt = 3t^2 - 4t + 4$

t at $V = 4$? $\quad 3t^2 - 4t + 4 = 4$

$\Rightarrow 3t^2 - 4t = 0$

$t(3t - 4) = 0$

$t = 0 \quad t = 4/3.$

$\Rightarrow$ Speed is 4 m/s at $t=0$ and $t = 4/3$

Q11 $\quad s = 2t^3 - 5t^2 + 4t - 5$

(i) Velocity (Speed) $= 0$

$$\frac{ds}{dt} = 6t^2 - 10t + 4$$

$6t^2 - 10t + 4 = 0$

$3t^2 - 5t + 2 = 0$

$(3t - 2)(t - 1) = 0$

$t = 2/3 \quad t = 1$

acc $\div \frac{d^2s}{dt^2} = 12t - 10$

at $t = 2/3$ $\quad 12(2/3) - 10$

acc $= -2$

at $t = 1$ $\quad 12(1) - 10$

acc $= 2$

(ii) acc $= 0$

$12t - 10 = 0$

$12t = 10$

$t = 10/12 = 5/6$

speed at $t = 5/6$

$6(5/6)^2 - 10(5/6) + 4$

$= -1/6.$

Q12 $\quad x = t^3 - 11t^2 + 24t - 3$

(i) (initial) at $t = 0$ $\quad x = (0)^3 - 11(0)^2 + 24(0) - 3$

$$x = -3$$

$$dx/dt = 3t^2 - 22t + 24$$

at $t = 0$ $\quad dx/dt = 3(0)^2 - 22(0) + 24$

$$Vel = 24$$

$\Rightarrow$ 3cm to the left of 0, moving to the right at 24m/sec

(ii) Velocity : $ds/dt = 3t^2 - 22t + 24$

(iii) stationary $\Rightarrow ds/dt = 0$

$$3t^2 - 22t + 24 = 0$$

$$(3t - 4)(t - 6) = 0$$

$$t = 4/3 \quad t = 6$$

(iv) at $t = 4/3$ $\quad s = \left(\frac{4}{3}\right)^3 - 11\left(4/3\right)^2 + 24\left(4/3\right) - 3$

$$= \frac{319}{27} = 11 \cdot 815 \text{ or } 11\tfrac{22}{27}$$

at $t = 6$ $\quad s = (6)^3 - 11(6)^2 + 24(6) - 3$

$$= -39$$

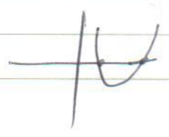
$\Rightarrow$ 11·815cm to the right of 0 and 39 cm to the left of 0.

(v) Velocity Neg $\Rightarrow ds/dt < 0$

$$3t^2 - 22t + 24 < 0$$

$$(3t - 4)(t - 6) = 0$$

$$t = 4/3 \quad t = 6$$



$< 0 \quad \Rightarrow \quad 4/3 < t < 6 \quad \Rightarrow$ for $6 - 4/3 = 4\tfrac{2}{3}$ sec.

(vi) acceleration : $\frac{d^2s}{dt^2} = 6t - 22$.

(vii) $6t - 22 = 0$

$6t = 22$

$t = 22/6 = 11/3$ sec

When $t = 11/3$

$$S = (11/3)^3 - 11(11/3)^2 + 24(11/3) - 3 = -\frac{367}{27}$$

$$= -13.593$$

$\Rightarrow$ at $t = 11/3$ the particle is 13.593cm to the left of O.

$$ds/dt = 3(11/3)^2 - 22(11/3) + 24 = -49/3$$

$$= -16.3$$

$\Rightarrow$ at $t = 11/3$ the particle is moving to the left at 16.3 cm/sec

$n = N_0 e^{0.2t}$ where $n_0 = 5$.

(i) at $t=0$ $\quad n = 5e^{0.2(0)}$

$= 5(1) = 5.$

at $t=10$ $\quad n = 5e^{0.2(10)}$

$= 5e^{2}$

$= 36.945 = 37$

(ii) $(0,5)$ $(10,37)$ Avg growth $= \frac{37-5}{10-0}$

$= {}^{32}/_{10} = 3.2$

(iii) $\frac{dn}{dt} = 5e^{0.2t}(0.2) = (1)e^{0.2t} = e^{0.2t}$

when $t=5$ $\quad e^{0.2(5)} = e^{(1)} = e.$