Ex 3·6

Q1 $\quad -2+4i$ root of $z^2+4z+20=0$

$$(-2+4i)^2+4(-2+4i)+20$$
$$4\cancel{-16i}+16i^2-8\cancel{+16i}+20$$
$$-20+20$$
$$=0$$

$\therefore$ True is a root

other root is $-2-4i$

Q2 (i) Solve $\quad z^2-2z+17=0$

$$z=\frac{2\pm\sqrt{4-4(17)}}{2}=\frac{2\pm\sqrt{4-68}}{2}$$

$$=\frac{2\pm\sqrt{-64}}{2}=\frac{2\pm 8i}{2}=1\pm 4i$$

$z=1+4i$ and $1-4i$

(ii) $\quad z^2+4z+7=0$

$$z=\frac{-4\pm\sqrt{16-4(7)}}{2}=\frac{-4\pm\sqrt{16-28}}{2}$$

$$=\frac{-4\pm\sqrt{-12}}{2}=\frac{-4\pm 2\sqrt{3}\,i}{2}$$

$$=-2\pm\sqrt{3}\,i$$

Q3 (i) $1 \pm 3i$ Sum $= (1+3i)+(1-3i) = 2$
Product $= (1+3i)(1-3i) = 1+9 = 10$

Eqn is $z^2 - \text{Sum}\, x + \text{Prod} = 0$

$$z^2 - 2z + 10 = 0$$

(ii) $-2 \pm i$ Sum $= (-2+i) + (-2-i) = -4$
Prod $= (-2+i)(-2-i) = 4+1 = 5$

eqn : $z^2 + 4z + 5 = 0$

(iii) $4 \pm 2i$ Sum $= (4+2i) + (4-2i) = 8$
Prod $= (4+2i)(4-2i) = 16+4 = 20$

Eqn : $z^2 - 8z + 20 = 0$

(iv) $\pm 5i$ Sum : $(+5i) + (-5i) = 0$
prod $(5i)(-5i) = 25$.

Eqn : $z^2 - 0z + 25 = 0$
$z^2 + 25 = 0$

Q4 $z = 4 - i \Rightarrow \bar{z} = 4 + i$
show $\bar{z}\,(4+i)$ a root of $z^2 - 8z + 17 = 0$.

$(4+i)^2 - 8(4+i) + 17$
$16 + \cancel{8i} + \underset{}{\overset{-1}{i^2}} - 32 - \cancel{8i} + 17$
$17 - 32 + 17$
$= 0$ True $\therefore$ $\bar{z}$ is a root

Q5 Show $(-2+2i)$ root of $z^3+3z^2+4z-8=0$

$(-2+2i)^3+3(-2+2i)^2+4(-2+2i)-8$

$(-2+2i)(4-8i+4i^2)+3(4-8i+4i^2)-8+8i-8$

$(-2+2i)(-8i) \quad +3(-8i)$

$16i+16 \quad -24i \quad -8+8i-8$

$\cancel{16i}+\cancel{16} \quad -\cancel{24i} \quad \cancel{-8}+\cancel{8i}\cancel{-8}$

$=0 \quad \therefore \; -2+2i$ is a root

Since Coeffs are real $\Rightarrow$ $-2-2i$ is another Root

$\therefore$ Sum : $(-2+2i)+(-2-2i)=-4$

Prod : $(-2+2i)(-2-2i)=4+4=8$

Eqn : $z^2+4z+8=0$

Use long div to find 3^{rd} root.

$$
\begin{array}{r|l}
 & z-1 \\
z^2+4z+8 & z^3+3z^2+4z-8 \\
 & -(z^3+4z^2+8z) \\
\hline
 & -z^2-4z-8 \\
 & -(-z^2-4z-8) \\
\hline
 & 0
\end{array}
$$

$\Rightarrow z-1=0$ is a factor

$\Rightarrow z=1$ is 3^{rd} root

Q6 $2+3i$ a root of $2z^3 - 9z^2 + 30z - 13 = 0$

$\Rightarrow$ $2-3i$ is 2^{nd} root as coeff's $\in R$.

Sum : $(2+3i)+(2-3i) = 4$
Prod : $(2+3i)(2-3i) = 4+9 = 13$.

Eqn : $z^2 - 4z + 13 = 0$.

$$
\begin{array}{r}
2z - 1 \\
z^2 - 4z + 13 \overline{) \; 2z^3 - 9z^2 + 30z - 13} \\
- (2z^3 - 8z^2 + 26z) \\
\hline
-z^2 + 4z - 13 \\
- (-z^2 + 4z - 13) \\
\hline
0
\end{array}
$$

$\Rightarrow$ $2z - 1 = 0$ is a factor $\therefore$ $2z = 1$
$z = 1/2$ is 3^{rd} Root.

Roots are: $2+3i$, $2-3i$, $\frac{1}{2}$.

Q7 $1+2i$ root of $z^2 + (-1+5i)z + 14 - 7i = 0$

$(1+2i)^2 + (-1+5i)(1+2i) + 14 - 7i$
$1 + 4i + 4i^2 - 1 - 2i + 5i + 10i^2 + 14 - 7i$
$\cancel{1} + 4i - \cancel{4} - \cancel{1} - 2i + 5i - \cancel{10} + \cancel{14} - 7i$
$= 0$ $\therefore$ $1+2i$ is a Root

$1-2i$ $(1-2i)^2 + (-1+5i)(1-2i) + 14 - 7i = 0$
$1 - 4i + 4i^2 - 1 + 2i + 5i - 10i^2 + 14 - 7i$
$1 - 4i - 4 - 1 + 2i + 5i + 10 + 14 - 7i$
$20 - 4i \neq 0$ $\therefore$ $1-2i$ is Not a root

$1-2i$ is not a root as coeff's are not $\in R$.

Q8 $\frac{1+2i}{1-2i} \times \frac{1+2i}{1+2i} = \frac{1+2i+2i+4i^2}{1+4} = \frac{-3+4i}{5}$

$\Rightarrow -\frac{3}{5} + \frac{4}{5}i$ is a root.

$\Rightarrow -3/5 - 4/5\, i$ is other Root (Coeffs $\in \mathbb{R}$)

Sum: $\left(-\frac{3}{5} + \frac{4}{5}i\right) + \left(-3/5 - 4/5\, i\right) = -\frac{6}{5}$

Prod: $\left(-3/5 + 4/5\, i\right)\left(-3/5 - 4/5\, i\right) = \left(\frac{-3}{5}\right)^2 + \left(\frac{-4}{5}\right)^2$

$= \frac{9}{25} + \frac{16}{25} = 25/25 = 1.$

Eqn: $z^2 + \frac{6}{5}z + 1 = 0 \quad (\times 5)$

$5z^2 + 6z + 5 = 0$

$\Rightarrow a = 5 \quad b = 6.$

Q4 $z^3 - 1 = (z-1)(z^2 + az + b)$ find a & b.

$$\frac{z^3-1}{z-1} = z^2 + az + b$$

$$
\begin{array}{r|l}
 & z^2 + z + 1 \\
\hline
z-1 & \cancel{z^3} - 1 \\
 & -\cancel{z^3} \oplus z^2 \\
\hline
 & \cancel{z^2} - 1 \\
 & \ominus\cancel{z^2} \oplus - z \\
\hline
 & \cancel{z - 1} \\
 & \cancel{z - 1} \\
\hline
\end{array}
$$

$$\frac{z^3-1}{z-1} = z^2 + z + 1$$

$$\Rightarrow a = 1 \quad \text{and} \quad b = 1$$

Solve $z^3 - 1 = 0$

$$z^3 - 1 = (z-1)(z^2 + z + 1) = 0$$

$z - 1 = 0$
$z = 1$

$z^2 + z + 1 = 0$

$$z = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= \frac{-1 \pm \sqrt{3}\, i}{2}$$

$\Rightarrow$ Roots are: 1, $\dfrac{-1+\sqrt{3}i}{2}$, $\dfrac{-1-\sqrt{3}i}{2}$

Q10 $\quad -2 \pm i$

Sum: $(-2+i)+(-2-i) = -4$
Prod: $(-2+i)(-2-i) = 4+1 = 5$

eqn: $z^2+4z+5=0$

find 3 Roots of $z^3+z^2-7z-15=0$ given $-2+i$

all coeffs $\in R \Rightarrow 2^{nd}$ root is $-2-i$.

$$
\begin{array}{r|l}
 & z-3 \\
\hline
z^2+4z+5 & z^3+z^2-7z-15 \\
 & -(z^3+4z^2+5z) \\
\hline
 & -3z^2-12z-15 \\
 & -3z^2-12z-15 \\
\hline
\end{array}
$$

3^{rd} factor is $z-3=0$

$\Rightarrow 3^{rd}$ Root is $z=3$

3 Roots are: $-2+i$, $-2-i$, 3.

Q11 $-3 \pm 2i$ Roots.

Sum: $(-3+2i)+(-3-2i) = -6$
Prod: $(-3+2i)(-3-2i) = 9+4 = 13$.

eqn: $z^2+6z+13=0$.

$-3 \pm 2i$ and 2 are roots form Quadratic.

$z=2$ is a root $\Rightarrow (z-2)$ is a factor

$\therefore \quad (z-2)(z^2+6z+13)$
$z^3+6z^2+13z-2z^2-12z-26$
$z^3+4z^2+z-26=0$ is the cubic

Q12
Roots 2, and $-1+i$, $\Rightarrow$ 3^{rd} Root $-1-i$

$-1 \pm i$: Sum: $(-1+i)+(-1-i) = -2$
Prod: $(-1+i)(-1-i) = 1+1 = 2$

eqn: $z^2+2z+2=0$.

3^{rd} root is $z=2 \Rightarrow (z-2)$ is a factor

$\therefore \quad (z-2)(z^2+2z+2)$
$z^3+\cancel{2z^2}+2z-\cancel{2z^2}-4z-4=0$
$z^3-2z-4=0$ is the cubic

Q13 $z = 1^{\frac{1}{3}}$

cube roots of Unity $\Rightarrow z^3 = 1$

$\therefore z^3 - 1 = 0$

from Q9 $(z-1)(z^2+z+1) = 0$

Roots are $z_1 = 1$, $z_2 = \frac{-1+\sqrt{3}i}{2}$, $z_3 = \frac{-1-\sqrt{3}i}{2}$.

$\therefore \alpha = \frac{-1+\sqrt{3}i}{2}$ and $\beta = \frac{-1-\sqrt{3}i}{2}$

(i) $\alpha^2 = \beta$

$\left(\frac{-1+\sqrt{3}i}{2}\right)^2 = \left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} + \frac{-2\sqrt{3}}{4}i + \frac{3}{4}i^2$

$= -\frac{2}{4} - \frac{2\sqrt{3}i}{4} = \frac{-2-2\sqrt{3}i}{4} = \frac{-1-\sqrt{3}i}{2} = \beta$.

True.

(ii) $1 + \alpha + \beta = 0$

$1 + \frac{-1}{2} + \cancel{\frac{\sqrt{3}}{2}i} + \frac{-1}{2} - \cancel{\frac{\sqrt{3}}{2}i}$

$1 - 1 = 0$. True