Ex 3.9

Q1 (ii) $\left(\cos \pi/6 + i \sin \pi/6\right)^7 = \cos \frac{7\pi}{6} + i \sin 7\pi/6$

$-\sqrt{3}/2 + -\frac{1}{2} i$

$-\sqrt{3}/2 - 1/2 i$

(iv) $\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)^3 = \cos 2\pi + i \sin 2\pi$

$= 1 + 0i$

(vi) $\left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}\right)^{10} = \cos 4\pi + i \sin 4\pi$

$= 1 + 0i$

(vii) $\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)^{-3} = \cos \frac{-\pi}{2} + i \sin \frac{-\pi}{2}$

$0 - i$

Q2 $\left[\sqrt{2}\left(\cos \pi/3 + i \sin \pi/3\right)\right]^4$

$= (\sqrt{2})^4 \left(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}\right)$

$= 4\left(-\frac{1}{2} + -\sqrt{3}/2\, i\right)$

$= -2 - 2\sqrt{3} i$

Q3 $\left[3\left(\cos \pi/10 + i \sin \pi/10\right)\right]^5$

$= 3^5 \left(\cos \pi/2 + i \sin \frac{\pi}{2}\right)$

$= 243(0 + 1i)$

$= 0 + 243i$

Q4 (i) $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(ii) $\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^4 = \left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right)$

$$\Rightarrow \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^4$$
$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\left(\cos\frac{8\pi}{3} + i\sin\frac{8\pi}{3}\right)$$
$$= \left(\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right)$$
$$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

Q5 (i) $z_1 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(ii) $z_2 = 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(iii) $\overline{z_1} = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$
$= 2\left(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right)$ reflection in x axis.

(iv) $\overline{z_2} = 3\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right)$
$= 3\left(\cos -\frac{2\pi}{3} + i\sin -\frac{2\pi}{3}\right)$

(v) $z_1 \cdot z_2 = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \times 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

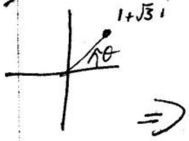
Mult the r's & add the θ's $= 6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

(vi) $\frac{z_1}{z_2} = \frac{2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)}{3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)}$

Div the r's
Sub the θ's $= \frac{2}{3}\left(\cos -\frac{\pi}{2} + i\sin -\frac{\pi}{2}\right)$

$\frac{\pi}{6} - \frac{2\pi}{3}$
$\cdot \frac{\pi}{6} - \frac{4\pi}{6}$
$= \frac{-3\pi}{6} = \frac{-\pi}{2}$

Q6 (ii) $1+i\sqrt{3} \Rightarrow r=\sqrt{1^2+\sqrt{3}^2}=2.$



$\theta = \operatorname{Tan}^{-1}\left(\frac{\sqrt{3}}{1}\right) = \pi/3.$

$\Rightarrow 1+i\sqrt{3} = 2\left(\operatorname{Cos}\pi/3 + i\operatorname{Sin}\pi/3\right).$

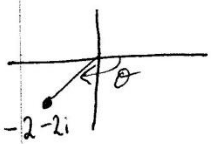
$\therefore \left(1+i\sqrt{3}\right)^3 = \left[2\left(\operatorname{Cos}\frac{\pi}{3} + i\operatorname{Sin}\pi/3\right)\right]^3$

$= 2^3\left(\operatorname{Cos}\pi + i\operatorname{Sin}\pi\right)$

$= 8\left(-1+0i\right)$

$= -8+0i$

(iii) $-2-2i \Rightarrow r=\sqrt{(-2)^2+(-2)^2}=\sqrt{8}$



$\theta = \operatorname{Tan}^{-1}\left(\frac{-2}{-2}\right) = \operatorname{Tan}^{-1}(1) = \frac{\pi}{4}$

$\therefore \theta = -3\pi/4$

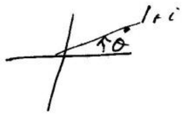
$\Rightarrow -2-2i = \sqrt{8}\left(\operatorname{Cos}-3\pi/4 + i\operatorname{Sin}-3\pi/4\right)$

$\therefore (-2-2i)^4 = \sqrt{8}^4\left(\operatorname{Cos}-3\pi + i\operatorname{Sin}-3\pi\right)$

$= 64\left(-1+0i\right)$

$= -64+0i$

Q7 $(1+i)^4$



$1+i \Rightarrow r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1} 1 = \frac{\pi}{4}$

$(1+i)^4 = \left[\sqrt{2}\left(\cos \pi/4 + i \sin \pi/4\right)\right]^4$

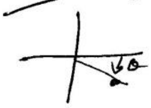
$= 4\left(\cos \pi + i \sin \pi\right)$

$= 4\left(-1 + 0i\right)$

$= -4 + 0i$

$= -4.$

Q8 $4-4i$



$r = \sqrt{4^2+(-4)^2} = \sqrt{32} = 4\sqrt{2}.$

$\theta = \tan^{-1}\left(\frac{-4}{4}\right) = \tan^{-1}(-1) = -\pi/4.$

$4\sqrt{2}\left(\cos -\frac{\pi}{4} + i \sin -\pi/4\right)$

$\frac{1}{(4-4i)^3} = (4-4i)^{-3} = \left[4\sqrt{2}\left(\cos -\frac{\pi}{4} + i \sin -\frac{\pi}{4}\right)\right]^{-3}$

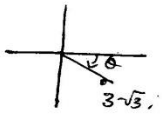
$= (4\sqrt{2})^{-3}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$

$= \frac{1}{(64)(2\sqrt{2})}\left(\frac{-\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$

$= \frac{1}{128}\left(-\frac{1}{2} + \frac{1}{2} i\right)$

$= \frac{-1}{256} + \frac{1}{256} i$

Q1 (i) $(3-\sqrt{3}\,i)^6$



$$r = \sqrt{3^2+(-\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) = -\frac{\pi}{6}$$

$$2\sqrt{3}\left(\cos -\tfrac{\pi}{6} + i\sin -\tfrac{\pi}{6}\right)$$

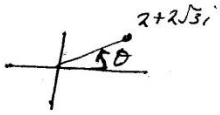
$$\therefore (3-\sqrt{3}i)^6 = \left[2\sqrt{3}\left(\cos -\tfrac{\pi}{6} + i\sin -\tfrac{\pi}{6}\right)\right]^6$$

$$= 1728\left(\cos -\pi + i\sin -\pi\right)$$

$$= 1728(-1+0i)$$

$$= -1728 + 0i$$

(ii) $(2+2i\sqrt{3})^6$



$$2+2\sqrt{3}i \Rightarrow r = \sqrt{2^2+(2\sqrt{3})^2} = 4$$

$$\theta = \tan^{-1}\left(\frac{2\sqrt{3}}{2}\right) = \pi/3$$

$$(2+2i\sqrt{3})^6 = \left[4\left(\cos \tfrac{\pi}{3} + i\sin \tfrac{\pi}{3}\right)\right]^6$$

$$= 4096\left(\cos 2\pi + i\sin 2\pi\right)$$

$$= 4096(1+0i)$$

$$= 4096 + 0i$$

Q10 $\dfrac{\sqrt{3}+i}{1+i\sqrt{3}} \times \dfrac{1-\sqrt{3}i}{1-\sqrt{3}i} = \dfrac{\sqrt{3}-3i+i-\sqrt{3}\cancel{i^2}}{1+3}$

$$\frac{2\sqrt{3}-2i}{4} = \boxed{\frac{\sqrt{3}}{2}-\frac{1}{2}i}$$

$\dfrac{\sqrt{3}}{2}-\dfrac{1}{2}i \Rightarrow r = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{-1}{2}\right)^2} = 1$

$\theta = \mathrm{Tan}^{-1}\left(\dfrac{-1/2}{\sqrt{3}/2}\right) = \mathrm{Tan}^{-1}\left(\dfrac{-1}{\sqrt{3}}\right) = \dfrac{-\pi}{6}$ $\quad \frac{-1}{2}\times\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)$

$\therefore \left(\dfrac{\sqrt{3}+i}{1+i\sqrt{3}}\right)^6 = \left(\cos{-\pi/6} + i\sin{-\pi/6}\right)^6$

$= \cos -\pi + i\sin -\pi$

$= -1 + 0i$