

Ex 4.3

Q1 (iii) $1 + 1.1 + 1.2 + 1.3 + \dots$
 $a = 1 \quad d = 0.1$

$$S_n = \frac{n}{2} \{ 2a + (n-1)d \}$$

$$S_n = \frac{n}{2} \{ 2(1) + (n-1)(0.1) \}$$

$$S_n = \frac{n}{2} \{ 2 + 0.1n - 0.1 \}$$

$$S_n = \frac{n}{2} \{ 1.9 + 0.1n \}$$

$$\begin{aligned} S_{20} &= \frac{20}{2} \{ 1.9 + 0.1(20) \} \\ &= 10 \{ 1.9 + 2 \} \\ &= 10 (3.9) \\ &= 39 \end{aligned}$$

(iv) $-7, -3, 1, 5, \dots$ $a = -7 \quad d = 4$

$$S_n = \frac{n}{2} \{ 2(-7) + (n-1)(4) \}$$

$$= \frac{n}{2} (-14 + 4n - 4)$$

$$= \frac{n}{2} (4n - 18)$$

$$= 2n^2 - 9n$$

$$\begin{aligned} S_{20} &= 2(20)^2 - 9(20) \\ &= 800 - 180 \\ &= 620 \end{aligned}$$

Q2 (i) $6 + 10 + 14 + 18 + \dots + 50$

need to find out what $T_n = 50$ so can find S_n
 $a = 6$ $d = 4$

$$\begin{aligned}T_n &= a + (n-1)d \\50 &= 6 + (n-1)(4) \\50 &= 6 + 4n - 4 \\48 &= 4n \\12 &= n\end{aligned}$$

$$\begin{aligned}S_n &= \frac{n}{2} \{2a + (n-1)d\} \\S_{12} &= \frac{12}{2} \{2(6) + (11)(4)\} \\&= 6(12 + 44) \\&= 6(56) \\&= 336\end{aligned}$$

(iii) $80 + 74 + 68 + 62 \dots -34$
find what $T_n = -34$ $a = 80$ $d = -6$

$$\begin{aligned}T_n &= a + (n-1)d \\-34 &= 80 + (n-1)(-6) \\-34 &= 80 - 6n + 6 \\6n &= 120 \\n &= 20\end{aligned}$$

$$\begin{aligned}S_n &= \frac{n}{2} \{2a + (n-1)d\} \\S_{20} &= \frac{20}{2} \{2(80) + (19)(-6)\} \\&= 10(160 - 114) \\&= 10(46) \\&= 460\end{aligned}$$

Q3

$$5 + 8 + 11 + 14 \dots \quad \text{Sum to } 98.$$

$$a = 5 \quad d = 3$$

$$S_n = \frac{1}{2} \{ 2a + (n-1)d \}$$

$$98 = \frac{1}{2} \{ 2(5) + (n-1)(3) \}$$

$$98 = \frac{1}{2} (10 + 3n - 3)$$

$$196 = 3n^2 + 7n.$$

$$3n^2 + 7n - 196 = 0$$

$$(3n \quad)(n \quad) \quad ?? \quad \text{Use formula.}$$

$$n = \frac{-7 \pm \sqrt{7^2 - 4(3)(-196)}}{2(3)}$$

$$= \frac{-7 \pm \sqrt{2401}}{6} = \frac{-7 \pm 49}{6} = \begin{matrix} \rightarrow 7 \\ \downarrow -\frac{28}{3} \end{matrix}$$

$$n = 7.$$

Q4

$$T_n = 5 - 3n, \quad 2, -1, -4, \dots$$

$$a = 2 \quad d = -3$$

$$S_{10} = \frac{10}{2} \{ 2(2) + 9(-3) \}$$

$$= 5(4 - 27)$$

$$= 5(-23)$$

$$= -115$$

Q5

$$10, 12, 14, \dots \quad a = 10 \quad d = 2$$

$$S_n = 190$$

$$190 = \frac{n}{2} \{ 2(10) + (n-1)(2) \}$$

$$190 = \frac{n}{2} (20 + 2n - 2)$$

$$380 = 2n^2 + 18n$$

$$2n^2 + 18n - 380 = 0$$

$$n^2 + 9n - 190 = 0$$

$$(n + 19)(n - 10) = 0$$

$$n = -19 \quad n = 10$$

$$\Rightarrow n = 10 \quad \text{as } n \in \mathbb{N}.$$

Q6 (i) $\sum_{r=1}^6 (3r+1)$

$$4 + 7 + 10 + 13 + 16 + 19 = 69$$

(ii) $\sum_{r=0}^5 (4r-1)$

$$-1 + 3 + 7 + 11 + 15 + 19 = 54$$

(iii) $\sum_{r=1}^{100} r$, $1 + 2 + 3 + 4 + \dots + 100$

$$a = 1 \quad d = 1$$

$$\begin{aligned} S_{100} &= \frac{100}{2} (2(1) + 99(1)) \\ &= 50(101) \\ &= 5050 \end{aligned}$$

Q7 (i) $4 + 8 + 12 + 16 + \dots + 124$
need to find what $T_n = 124$

$$\begin{aligned} a &= 4 \quad d = 4 \\ T_n &= 4 + (n-1)(4) \\ &= 4 + 4n - 4 \\ &= 4n \end{aligned}$$

$$\begin{aligned} T_n &= a + (n-1)d \\ 124 &= 4 + (n-1)4 \\ 124 &= 4 + 4n - 4 \\ 124 &= 4n \\ 31 &= n \end{aligned}$$

$$\sum_{n=1}^{31} 4n$$

Q7(ii) $-10 - 9\frac{1}{2} - 8 - 7\frac{1}{2} + \dots + 4$.
 Find T_n and What $T_n = 4$
 $a = -10$ $d = \frac{1}{2}$

$$T_n = -10 + (n-1)\left(\frac{1}{2}\right)$$

$$= -10 + \frac{n}{2} - \frac{1}{2}$$

$$= -10\frac{1}{2} + \frac{n}{2} = \frac{-21}{2} + \frac{n}{2} = \frac{n-21}{2}$$

$$A = \frac{n-21}{2}$$

$$8 = n-21$$

$$29 = n$$

$$\sum_{n=1}^{29} \left(\frac{n-21}{2}\right)$$

(iii) $10 + 10 \cdot 1 + 10 \cdot 2 + 10 \cdot 3 + \dots + 50$
 Find T_n and What $T_n = 50$
 $a = 10$ $d = 0 \cdot 1$

$$T_n = 10 + (n-1)(0 \cdot 1)$$

$$= 10 + 0 \cdot 1n - 0 \cdot 1$$

$$= 9 \cdot 9 + 0 \cdot 1n$$

$$50 = 9 \cdot 9 + 0 \cdot 1n \quad (\times 10)$$

$$500 = 99 + n$$

$$401 = n$$

$$\sum_{n=1}^{401} (9 \cdot 9 + 0 \cdot 1n)$$

Q8

$$T_n = 15$$

$$S_5 = 55$$

$$a + (5-1)d = 15$$

$$\textcircled{A} \quad a + 4d = 15$$

$$\frac{5}{2}(2a + (5-1)d) = 55$$

$$\frac{5}{2}(2a + 4d) = 55$$

$$5(2a + 4d) = 110$$

$$10a + 20d = 110$$

$$\textcircled{B} \quad a + 2d = 11$$

$$\textcircled{A} \quad a + 3d = 15$$

$$\textcircled{B} \quad \begin{array}{r} a + 3d = 15 \\ a + 2d = 11 \\ \hline d = 4 \end{array}$$

$$\textcircled{A} \quad a + 3(4) = 15$$
$$\underline{a = 3}$$

first 5 Terms are: 3, 7, 11, 15, 19

Q9

$$T_3 = 18$$

$$T_7 = 30$$

find S_{33}

$$a + 2d = 18$$

$$a + 6d = 30$$

$$\begin{array}{r} a + 2d = 18 \\ a + 6d = 30 \\ \hline 4d = 12 \\ \underline{d = 3} \end{array}$$

$$a + 6 = 18$$

$$\underline{a = 12}$$

$$S_{33} = \frac{33}{2}(2(12) + 32(3))$$

$$= \frac{33}{2}(24 + 96)$$

$$= \frac{33}{2}(120)$$

$$= 1980$$

Q10 Rings: 6, 11, 16 ... $a = 6$ $d = 5$

$$T_{10} = 6 + 9(5) = 51 \text{ rings}$$

$$T_{20} = 6 + 19(5) = 101 \text{ rings}$$

$$\begin{aligned} S_{20} &= \frac{20}{2} \{ 2(6) + 19(5) \} \\ &= 10(12 + 95) \\ &= 10(107) \\ &= 1070 \text{ rings in total} \end{aligned}$$

Q11

$$\begin{aligned} T_1 &= -12 \\ \underline{a = -12} \\ T_n &= 40 \\ a + (n-1)d &= 40 \\ -12 + (n-1)d &= 40 \\ \underline{(n-1)d = 52} \end{aligned}$$

$$-12 \dots \dots 40$$

$$S_n = 196$$

$$\frac{n}{2} \{ 2(-12) + (n-1)d \} = 196$$

$$\frac{n}{2} \{ -24 + (n-1)d \} = 196$$

$$\frac{n}{2} \{ -24 + 52 \} = 196$$

$$n(28) = 392$$

$$\underline{\underline{n = 14}}$$

Q12 Sum of Natural N^{os} = 1+2+3+4 ... n
 $a = 1$ $d = 1$

$$S_n = \frac{n}{2} \{ 2a + (n-1)d \}$$

$$S_n = \frac{n}{2} \{ 2 + (n-1)1 \}$$

$$S_n = \frac{n}{2} (2 + n - 1)$$

$$S_n = \frac{n}{2} (1 + n)$$

$$S_n = \frac{n}{2} (n+1) \text{ Q.E.D.}$$

$$S_{99} = \frac{99}{2} (100) = 4950$$

Q13 $T_{21} = 5\frac{1}{2}$

$$a + (n-1)d = 5\frac{1}{2}$$

$$a + 20d = 5\frac{1}{2}$$

$$S_{21} = 94\frac{1}{2}$$

$$\frac{n}{2} (2a + (n-1)d) = 94\frac{1}{2}$$

$$\frac{21}{2} (2a + 20d) = 94\frac{1}{2}$$

$$21(a + 10d) = 94\frac{1}{2}$$

$$a + 10d = 4\frac{1}{2}$$

$$\begin{array}{r} a + 20d = 5\frac{1}{2} \\ \ominus a + 10d = 4\frac{1}{2} \\ \hline 10d = 1 \\ \underline{d = 0.1} \end{array}$$

$$a + 10(0.1) = 4\frac{1}{2}$$

$$a + 1 = 4\frac{1}{2}$$

$$\underline{a = 3\frac{1}{2}}$$

$$S_{30} = \frac{30}{2} \{ 2(3\frac{1}{2}) + (30-1)(0.1) \}$$

$$= 15 (7 + 2.9)$$

$$= 15 (9.9)$$

$$= 148.5$$

Q14

$$T_{21} = 37$$

$$S_{20} = 320$$

$$a + 20d = 37$$

$$\frac{20}{2}(2a + 19d) = 320$$

$$10(2a + 19d) = 320$$

$$2a + 19d = 32$$

$$\begin{array}{r} a + 20d = 37 \quad (\times 2) \\ 2a + 19d = 32 \end{array}$$

$$\begin{array}{r} 2a + 40d = 74 \\ \ominus 2a + 19d = 32 \\ \hline 21d = 42 \\ \underline{d = 2} \end{array}$$

$$a + 20(2) = 37$$

$$a + 40 = 37$$

$$\underline{\underline{a = -3}}$$

$$\begin{aligned} S_{10} &= \frac{10}{2} \{ 2(-3) + 9(2) \} \\ &= 5 \{ -6 + 18 \} \\ &= 5(12) \\ &= 60 \end{aligned}$$

Q15

$$T_1 + T_2 + T_3 + \dots + L$$

$$T_n = a + (n-1)d = L$$

$$\Rightarrow (n-1)d = L - a$$

$$S_n = \frac{n}{2} \{ 2a + \underbrace{(n-1)d}_{L-a} \}$$

$$S_n = \frac{n}{2} \{ 2a + L - a \}$$

$$= \frac{n}{2} (a + L)$$

$$= \frac{n(a+L)}{2}$$

Q16

$$S_{\infty} = \frac{n}{2} \{ 2a + (n-1)d \}$$

to infinity \Rightarrow No last term
 \therefore Sum cannot be calculated