Exercise 4·4

Q1 RHS

(ii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27} \ldots$ $\quad \frac{1/3}{1} = \frac{1}{3} \quad \frac{1/9}{1/3} = \frac{1}{9} \times \frac{3}{1} = \frac{1}{3}$

Is Geometric $a = 1 \quad r = \frac{1}{3}$, $\left[\frac{1}{81}, \frac{1}{243}\right]$

(iv) $1, -1, 1, -1 \ldots$ $\quad \frac{-1}{1} = -1 \quad \frac{1}{-1} = -1$

Is Geometric $a = 1 \quad r = -1$, $[1, -1]$

(vi) $a, a^2, a^3, a^4 \ldots$ $\quad \frac{a^2}{a} = a \quad \frac{a^3}{a^2} = a$

Is Geometric $a = a \quad r = a$. $[a^5, a^6]$

(viii) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{36}$ $\quad \frac{1/6}{1/2} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$

$\frac{1/12}{1/6} = \frac{1}{12} \times \frac{6}{1} = \frac{1}{2}$.

$\frac{1}{3} \neq \frac{1}{2} \Rightarrow$ Is <u>not</u> Geometric.

(x) $\frac{3}{4}, \frac{9}{2}, 27, 162 \ldots$ $\quad \frac{9/2}{3/4} = \frac{9}{2} \times \frac{4}{3} = 6$

$\frac{27}{9/2} = 27 \times \frac{2}{9} = 6$.

Is Geometric $a = \frac{3}{4} \quad r = 6 \quad [972, 5832]$

Q2 (i) $5, 10 \ldots \quad a = 5 \quad r = 2$.

$T_n = ar^{n-1}$

$T_{11} = 5(2)^{10} = 5120$

(ii) $10, 25, \ldots \quad a = 10 \quad r = \frac{25}{10} = 2.5$

$T_7 = 10(2.5)^6 = 2441.41$

(iii) $1\cdot1,\ 1\cdot21 \ldots\ldots \quad a = 1\cdot1 \quad r = \frac{1\cdot21}{1\cdot1} = \frac{11}{10} = 1\cdot1$

$T_8 = (1\cdot1)(1\cdot1)^7 = 2\cdot14$

(iv) $24,\ -12,\ 6 \ldots \qquad a = 24 \quad r = -\frac{1}{2}$

$T_{10} = 24\left(-\frac{1}{2}\right)^9 = -0\cdot046875,\ \left(-\frac{3}{64}\right)$

Q3 $\quad T_2 = 12 \qquad T_5 = 324.$

$ar^1 = 12 \qquad ar^4 = 324$

$$\frac{\cancel{a}r^4}{\cancel{a}r^1} = \frac{324}{12}$$

$r^3 = 27$

$r = \sqrt[3]{27}$

$r = 3$

Q4 $\quad T_3 = 6 \qquad T_8 = 1458$

$ar^2 = 6 \qquad ar^7 = 1458$

$$\frac{\cancel{a}r^7}{\cancel{a}r^2} = \frac{1458}{6}$$

$r^5 = 243$

$r = \sqrt[5]{243}$

$r = 3$

Q5 $T_2 = 4$ $T_5 = -\frac{1}{16}$

$ar = 4$ $ar^4 = -\frac{1}{16}$

$$\frac{\cancel{a}r^4}{\cancel{a}r} = \frac{-\frac{1}{16}}{4}$$

$$r^3 = -\frac{1}{64}$$

$$r = \sqrt[3]{-\frac{1}{64}}$$

$r = -\frac{1}{4}$ $a\left(-\frac{1}{4}\right) = 4$ $a = -16$

first 5 Terms are: $-16,\ 4,\ -1,\ \frac{1}{4},\ -\frac{1}{16}$

Q6 ~~A~~ is Geometric $r = 3$.

B, C, D are not Geometric.

Q7 $n-2,\ n,\ n+3$.

$$\frac{n}{n-2} = \frac{n+3}{n}$$

$$\cancel{n^2} = \cancel{n^2} + 3n - 2n - 6$$
$$6 = n.$$

first 4 Terms are: $4 \xrightarrow{\times 3/2} 6 \xrightarrow{\times 3/2} 9$. $\frac{27}{2}$

Q8 $T_3 = -63$ $T_4 = 189$

(i) $\frac{189}{-63} = -3$ $\underline{\underline{r = -3}}$

$T_3 = -63 \Rightarrow ar^2 = -63$

$a(-3)^2 = -63$

$a = -63/9$

$\underline{\underline{a = -7}}$.

(ii) $T_n = ar^{n-1}$

$T_n = -7(-3)^{n-1}$

Q9 $T_1 = 16$ $T_5 = 9$

$\Rightarrow a = 16$

$ar^4 = 9$

$16r^4 = 9$

$r^4 = 9/16$

$r = \sqrt[4]{9/16}$

$r = 0\cdot866$

$T_7 = 16(0\cdot866)^6$

$= 6\cdot75$

Q10 first 3 terms are $\frac{a}{r}$, a, ar

$$\frac{a}{\cancel{r}} \times a \times a\cancel{r} = 27$$

$$a^3 = 27$$
$$a = 3$$

$$\frac{a}{r} + a + ar = 13$$

$$\frac{3}{r} + 3 + 3r = 13$$

$$\frac{3}{r} + 3r - 10 = 0$$

$$3r^2 - 10r + 3 = 0$$
$$(3r - 1)(r - 3) = 0$$
$$3r = 1$$
$$r = 1/3 \qquad r = 3$$

first 4 terms: $\frac{a}{r}$, a, ar

$$\frac{3}{1/3}, \ 3, \ 3\left(\tfrac{1}{3}\right), \ 3\left(\tfrac{1}{3}\right)^2$$

$$9, \ 3, \ 1, \ \tfrac{1}{3}.$$

OR $\quad 3/3, \ 3, \ 3(3), \ 3(3)^2$

$$1, \ 3, \ 9, \ 27.$$

Q11 $T_n = 3 \times 2^{n-1}$

$$T_1 = 3 \times 2^{1-1} = 3$$
$$T_2 = 3 \times 2^{2-1} = 6$$
$$T_3 = 3 \times 2^{3-1} = 12$$
$$T_4 = 3 \times 2^{4-1} = 24$$
$$T_5 = 3 \times 2^{5-1} = 48$$

Q12 $T_n = 8\left(\frac{3}{4}\right)^n$

$$T_1 = 8\left(\frac{3}{4}\right)^1 = 6$$
$$T_2 = 8\left(3/4\right)^2 = 9/2$$
$$T_3 = 8\left(3/4\right)^3 = 27/8$$
$$T_4 = 8\left(3/4\right)^4 = 81/32$$

Q13 $T_n = (-1)^{n+1} \times \frac{5}{2^{n-4}}$

$$T_1 = (-1)^{1+1} \times \frac{5}{2^{1-4}} = (-1)^2 \times \frac{5}{2^{-3}} = 1 \times 40 = 40$$
$$T_2 = (-1)^{2+1} \times \frac{5}{2^{2-4}} = (-1)^3 \times \frac{5}{2^{-2}} = -1 \times 20 = -20$$
$$T_3 = (-1)^{3+1} \times \frac{5}{2^{3-4}} = (-1)^4 \times 5/2^{-1} = 1 \times 10 = 10$$
$$T_4 = (-1)^{4+1} \times \frac{5}{2^{4-4}} = (-1)^5 \times 5/2^0 = -1 \times 5 = -5$$

Q14 (ii) $x+1, \quad x+4, \quad 3x+2.$

$$\frac{x+4}{x+1} = \frac{3x+2}{x+4}$$

$$(x+4)^2 = (3x+2)(x+1)$$

$$x^2+8x+16 = 3x^2+5x+2$$

$$2x^2-3x-14=0$$

$$(2x-7)(x+2)=0$$

$$2x=7$$

$$x = 7/2 \qquad x=-2.$$

Seq is: $7/2+1, \quad 7/2+4, \quad 3(7/2)+2$

$= 4.5, \quad 7.5, \quad 12.5$

OR

$-2+1, \quad -2+4, \quad 3(-2)+2$

$-1, \quad +2, \quad -4,$

(iv) $x-6, \quad 2x, \quad x^2$

$$\frac{2x}{x-6} = \frac{x^2}{2x}$$

$$4x^2 = x^3-6x^2 \qquad (\div x^2)$$

$$4 = x-6$$

$$10 = x$$

Seq is: $4, \quad 20, \quad 100$

Q15 $T_n = 2 \times 3^n$

$T_{n+1} = 2 \times 3^{n+1}$

$\frac{T_{n+1}}{T_n} = \frac{\cancel{2} \times 3^n}{\cancel{2} \times 3^{n+1}} = \frac{\cancel{3^n}}{\cancel{3^n} . 3^1} = \frac{1}{3}.$

$\frac{T_{n+1}}{T_n}$ is a constant $\Rightarrow$ Is Geometric

Q16 $T_n = 3 \times n^2$

$T_{n+1} = 3 \times (n+1)^2$

$\frac{T_{n+1}}{T_n} = \frac{\cancel{3} \times (n+1)^2}{\cancel{3} \times n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n}{n^2} + \frac{1}{n^2}$

$= 1 + \frac{2}{n} + \frac{1}{n^2}$

$\Rightarrow$ NO is not Geometric $\frac{T_{n+1}}{T_n} \neq$ constant.

Q17 (i) 5, 15, 45 3645

$a = 5 \quad r = 3. \quad T_n = 5 \times 3^{n-1}$

$3645 = 5 \times 3^{n-1}$

$729 = 3^{n-1}$

$729 = 3^n / 3^1$

$\overset{2187}{\cancel{3645}} = 3^n$

$3^7 = 3^n$

$7 = n$

or $\log 2187 = n \log 3$

$\frac{\log 2187}{\log 3} = n$

$7 = n.$

Q17 (ii) $48, 6, \frac{3}{4}, \ldots \frac{3}{2048}$

$a = 48 \quad r = \frac{6}{48} = \frac{1}{8}$

$T_n = (48)\left(\frac{1}{8}\right)^{n-1}$

$\frac{3}{2048} = (48)\left(\frac{1}{8}\right)^{n-1}$

$\frac{3}{98304} = \left(\frac{1}{8}\right)^{n-1} \longrightarrow \frac{1}{32768} = \left(\frac{1}{8}\right)^{n-1}$

$\frac{3}{98304} = 8^{1-n}$

$\frac{1}{8^5} = 8^{1-n}$

$8^{-5} = 8^{1-n}$

$\frac{3}{98304} = \frac{8^1}{8^n}$

$-5 = 1-n$

$n = 6.$

$\frac{3}{786432} = 8^{-n}$

$\log \frac{3}{786432} = -n \log 8$

$\frac{\log \frac{3}{786432}}{\log 8} = -n$

$-6 = -n$

$6 = n.$

Q18 height 27m each bounces 2/3 of height

(i) 1st bounce = $27 \times \frac{2}{3} = 18$
2nd " = $27 \times \frac{2}{3} \times \frac{2}{3} = 12$
3rd " = $27 \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = 8$
4th " = $27 \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{16}{3}$

(ii) $T_n = ar^{n-1}$
$= 27\left(\frac{2}{3}\right)^{n}$

(iii) $T_{12} = 27\left(\frac{2}{3}\right)^{12} = \frac{4096}{19683} = 0.21\text{m}.$

Q19 $A = 4000(1.03)^t$

(i) €4000

(ii) Yr 1 : $4000(1.03)^1 = 4120$
Yr 2 : $4000(1.03)^2 = 4243.60$
Yr 3 : $4000(1.03)^3 = 4370.91$
Yr 4 : $4000(1.03)^4 = 4502.04$

(iii) Yr 10 : $4000(1.03)^{10} = 5375.67$

(iv) To Double ⇒ to get to 8000

$4000(1.03)^t = 8000$
$(1.03)^t = 2$

$t \log(1.03) = \log 2$
$t = \frac{\log 2}{\log 1.03}$ $t = 23$ yrs

<u>Q20</u> $A = P(1+i)^t$

Over 10 yrs $2500 = P$ $A = 3047.$

$$3047 = 2500(1+i)^{10}$$
$$1.2188 = (1+i)^{10}$$
$$\sqrt[10]{1.2188} = 1+i$$
$$1.01998 = 1+i$$
$$0.01998 = i$$

$$\Rightarrow 1.998 = i$$
$$\Rightarrow 2\% = i$$