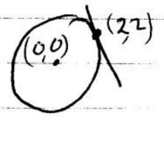
Ex 4.4

Q1 $x^2+y^2=8 \Rightarrow$ centre $(0,0)$ $r=\sqrt{8}$.



need pt $\Rightarrow (2,2)$
need slope $\Rightarrow \perp$ to radius.

$M_{radius} = \frac{2-0}{2-0} = \frac{2}{2} = 1 \Rightarrow \perp m = -1$

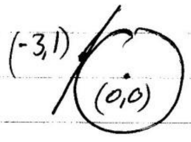
eqn: $M=-1$ pt $(2,2)$

$y-2 = -1(x-2)$

$y-2 = -x+2$

$x+y-4=0$ - Eqn of Tangent

Q2 $x^2+y^2=10$ at $(-3,1)$



require: pt $(-3,1)$
slope: opp to M of radius.

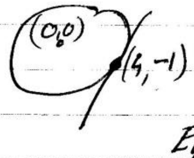
m $(-3,1)$ & $(0,0)$ $= \frac{0-1}{0+3} = -\frac{1}{3} \Rightarrow \perp m = 3$.

eqn: $y-1 = 3(x+3)$

$y-1 = 3x+9$

$3x-y+10=0$

Q3 $x^2+y^2=17$ at $(4,-1)$



$M = \frac{-1-0}{4-0} = -\frac{1}{4} \Rightarrow \perp m = 4$

Eqn: $y+1 = 4(x-4)$

$y+1 = 4x-16$

$4x-y-17=0$

Q4 $(x-1)^2 + (y+2)^2 = 20$

(i) $P(3,2)$ $\quad (3-1)^2 + (2+2)^2 = 20$

$4 + 16 = 20$

$20 = 20$ True $\therefore (3,2)$ is on circle.

(ii) Centre $(1,-2)$

(iii) Eqn of T at P. $\quad m = \frac{-2-2}{1-3} = \frac{-4}{-2} = 2 \Rightarrow \perp m = -\frac{1}{2}$

Eqn: $y - 2 = -\frac{1}{2}(x-3)$

$2y - 4 = -x + 3$

$x + 2y - 7 = 0$

Q5 $(x+4)^2 + (y-3)^2 = 17$ at $(0,2)$

Centre $(-4,3)$

$m = \frac{3-2}{-4-0} = \frac{1}{-4} \Rightarrow \perp m = 4.$

Eqn: $y - 2 = 4(x - 0)$

$y - 2 = 4x$

$4x - y + 2 = 0$

Q6 $x^2 + y^2 - 4x + 10y - 8 = 0$

Centre $(2, -5)$ radius $= \sqrt{2^2 + (-5)^2 + 8} = \sqrt{37}$

Tangent at $(3,1)$ $M = \frac{1+5}{3-2} = \frac{6}{1} = 6 \Rightarrow \perp M = -1/6$

Eqn: $y - 1 = -1/6(x - 3)$
$6y - 6 = -x + 3$
$x + 6y - 9 = 0$

Q7 $\perp$ dis $(0,0)$ to $3x - 4y - 25 = 0$

$$\frac{|3(0) - 4(0) - 25|}{\sqrt{3^2 + 4^2}} = \frac{|-25|}{\sqrt{25}} = \frac{25}{5} = 5$$

$x^2 + y^2 = 25 \Rightarrow$ centre $(0,0)$ radius $= 5$.

Since $\perp$ dis from $(0,0)$ [centre] to line $3x - 4y - 25 = 0$ is 5, which = radius $\Rightarrow$ Is a Tangent.

Q8
$x^2 + y^2 - 6x - 4y + 8 = 0$
centre $(3, 2)$ radius $= \sqrt{3^2 + 2^2 - 8} = \sqrt{5}$

$\perp$ dis $x + 2y - 12 = 0$ and $(3,2)$
$$\frac{|1(3) + 2(2) - 12|}{\sqrt{1^2 + 2^2}} = \frac{|-5|}{\sqrt{5}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$

Since $\sqrt{5}$ = length of radius $\Rightarrow$ is a Tangent.

Q9 $x^2+y^2-6x-2y-15=0$

centre $(3, 1)$ radius $=\sqrt{3^2+1^2+15} = \sqrt{25}=5$.

$3x+4y+c=0$ is a Tangent $\Rightarrow \perp$ dis from $(3,1)$ to line $=5$

$$\frac{|3(3)+4(1)+c|}{\sqrt{3^2+4^2}} = 5$$

$$\frac{|13+c|}{5} = 5$$

$$|13+c| = 25$$

$13+c=25$ $\quad$ $13+c=-25$

$\underline{c=12}$ $\quad$ $\underline{c=-38}$

Q10 $2x-ky-3=0$ Tangent to $x^2+y^2+4x-4y-5=0$

centre $(-2,2)$ $r=\sqrt{(-2)^2+2^2+5}=\sqrt{13}$.

$2x-ky-3=0$ Tangent $\Rightarrow \perp$ dis to $(-2,2)$ is $\sqrt{13}$

$$\frac{|2(-2)-k(2)-3|}{\sqrt{2^2+(-k)^2}} = \sqrt{13}$$

$$\frac{|-2k-7|}{\sqrt{4+k^2}} = \sqrt{13}$$

$$\frac{4k^2+28k+49}{4+k^2} = 13$$

$$4k^2 + 28k + 49 = 52 + 13k^2$$

$$9k^2 - 28k + 3 = 0$$
$$(9k - 1)(k - 3) = 0$$

$$\underline{k = 1/9} \qquad \underline{k = 3}.$$

Q11 Tangent to $x^2 + y^2 = 13$ at $(-2,3)$
Centre $(0,0)$
Slope $= \frac{3-0}{-2-0} = 3/{-2} \Rightarrow \perp m = 2/3$

Eqn of Tangent is $y - 3 = 2/3(x+2)$
$$3y - 9 = 2x + 4$$
$$2x - 3y + 13 = 0.$$

Circle $x^2 + y^2 - 10x + 2y - 26 = 0$
centre $(5,-1)$ radius $= \sqrt{5^2 + (-1)^2 + 26} = \sqrt{52}$
$= 2\sqrt{13}.$

$\perp$ dis from $(5,-1)$ to $2x - 3y + 13 = 0$.

$$\frac{|2(5) - 3(-1) + 13|}{\sqrt{2^2 + 3^2}} = \frac{26}{\sqrt{13}} = 2\sqrt{13}$$

$=$ radius $\Rightarrow$ is a Tangent.

Q12 Eqn of circle: Centre $(2,-1)$, require radius

radius $= \perp$ dis from $(2,-1)$ to $3x+y=0$

$$\frac{|3(2)+1(-1)|}{\sqrt{3^2+1^2}} = \frac{|5|}{\sqrt{10}} = \frac{\sqrt{10}}{2}.$$

eqn of circle: $(x-2)^2+(y+1)^2 = \frac{10}{4}$

$$(x-2)^2+(y+1)^2 = 5/2$$

Q13 Eqn of line, $(0,0)$ and Slope $= m$

$$y-0 = m(x-0)$$
$$y = mx$$
$$mx-y=0$$

Circle: $x^2+y^2-4x-2y+4=0$ centre $=(2,1)$

radius $= \sqrt{2^2+1^2-4} = 1$

$\therefore \perp$ dis from $(2,1)$ to $mx-y=0$ is 1.

$$\frac{|m(2)-1(1)|}{\sqrt{m^2+(-1)^2}} = 1$$

$$\frac{|2m-1|}{\sqrt{m^2+1}} = 1$$

$$\frac{4m^2-4m+1}{m^2+1} = 1$$

$$4m^2-4m+1 = m^2+1$$
$$3m^2-4m = 0$$

$3m^2 - 4m = 0$

$m(3m - 4) = 0$

$m = 0 \qquad 3m = 4$

$m = 4/3$

Eqn of Tangents:

at $m = 0$

$(0)x - y = 0$

$-y = 0$

$y = 0$

at $m = 4/3$

$(4/3)x - y = 0 \quad (\times 3)$

$4x - 3y = 0.$

Q14 Eqn through $(3,5)$, slope $= m$

$y - 5 = m(x - 3)$

$y - 5 = mx - 3m$

$mx - y - 3m + 5 = 0.$

2 Tangents from $(3,5)$ to $x^2 + y^2 + 2x - 4y - 4 = 0$

centre $(-1, 2)$ $r = \sqrt{(-1)^2 + (2)^2 + 4} = 3.$

$\Rightarrow \perp$ distance from $(-1,2)$ to $mx - y - 3m + 5 = 0$ is 3.

$$\frac{|m(-1) - 1(2) - 3m + 5|}{\sqrt{m^2 + (-1)^2}} = 3$$

$$\frac{|-m - 2 - 3m + 5|}{\sqrt{m^2 + 1}} = 3$$

$$\frac{|3 - 4m|}{\sqrt{m^2 + 1}} = 3$$

$$9 - 24m + 16m^2 = 9m^2 + 9.$$

$7m^2 - 24m =$

$m(7m-24) = 0$

$m = 0 \qquad 7m = 24$

$m = 24/7$

Eqns of 2 Tangents:

at $m=0$

$(0)x - y - 3(0) + 5 = 0$

$-y + 5 = 0$

$y - 5 = 0$

at $m = 24/7$

$\frac{24}{7}(x) - y - 3\left(\frac{24}{7}\right) + 5 = 0 \; (\times 7)$

$24x - 7y - 72 + 35 = 0$

$24x - 7y - 37 = 0$

Q15 line parallel to $3x + 4y - 6 = 0$

is $3x + 4y + k = 0$.

Circle: $x^2 + y^2 - 2x - 2y - 7 = 0$

centre $(1,1)$ $r = \sqrt{1^2 + 1^2 + 7} = 3$.

Parallel to $3x + 4y - 6 = 0 \Rightarrow m = -3/4$.

Tangents $\Rightarrow \perp$ dis from $(1,1)$ to $3x + 4y + k = 0$ is 3.

$$\frac{|3(1) + 4(1) + k|}{\sqrt{3^2 + 4^2}} = 3$$

$$\frac{|7 + k|}{5} = 3$$

$|7 + k| = 15$

$7 + k = 15 \qquad 7 + k = -15$

$k = 8 \qquad k = -22.$

$\therefore$ 2 Tangents are:

$3x + 4y + 8 = 0$ and $3x + 4y - 22 = 0$

Q16 circle centre $(3,5)$ touches $y = 2x + 4$.

$2x - y + 4 = 0$.

(i) radius $= \perp$ distance.

$$\frac{|2(3) - 1(5) + 4|}{\sqrt{2^2 + (-1)^2}} = \frac{|5|}{\sqrt{5}} = \sqrt{5}$$

(ii) eqn of circle centre $(3,5)$ and $r = \sqrt{5}$

$$(x-3)^2 + (y-5)^2 = 5$$

(iii) Eqn of Tangent at $(1,4)$

slope of $r = \frac{4-5}{1-3} = \frac{-1}{-2} = \frac{1}{2} \Rightarrow \perp m = -2$

Eqn: $y - 4 = -2(x-1)$

$y - 4 = -2x + 2$

$2x + y - 6 = 0$

Q17 $r = 7$

$x^2 + y^2 - 10kx + 6y + 60 = 0$, $k > 0$.

(i) centre = $(5k, -3)$

(ii) $r = \sqrt{(5k)^2 + (-3)^2 - 60} = 7$

$\sqrt{25k^2 + 9 - 60} = 7$

$\sqrt{25k^2 - 51} = 7$

$25k^2 - 51 = 49$

$25k^2 = 100$

$k^2 = 4$

$k = \pm 2$

But $k > 0 \Rightarrow k = 2$

(iii) $3x + 4y + d = 0$ is a Tangent.

$\Rightarrow \perp$ dis from centre $(5k, -3)$ to $3x + 4y + d = 0$ is 7.

$(5(2), -3)$

$(10, -3)$ is centre.

$\perp$ Dis $(10, -3)$ to $3x + 4y + d = 0$ is 7.

$$\frac{|3(10) + 4(-3) + d|}{\sqrt{3^2 + 4^2}} = 7$$

$$\frac{|18 + d|}{5} = 7$$

$|18 + d| = 35$

$18 + d = 35$ $\qquad$ $18 + d = -35$

$\underline{\underline{d = 17}}$ $\qquad$ $\underline{\underline{d = -53}}$.

Q18 $x^2 + y^2 + 4x - 2y - 4 = 0$.

centre $(-2, 1)$

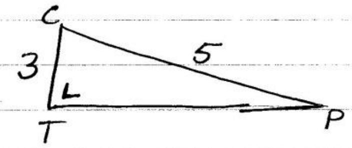
(i) radius $= \sqrt{(-2)^2 + (1)^2 + 4} = \sqrt{9} = 3$.

(ii) Property of circle: A ~~line~~ Tangent is perpendicular to the radius that goes to the point of contact.

(iii) Dis $(-2,1)$ to $(3,1)$

$\sqrt{(3+2)^2 + (1-1)^2} = 5$.

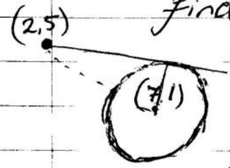
Right Angled Triangle:



$\Rightarrow |TP| = 4$.

Q19 $x^2 + y^2 - 14x - 2y + 34 = 0$

find length of Tangent from pt $(2,5)$ to circle.



centre $= (7, 1)$ radius $= \sqrt{7^2 + 1^2 - 34} = \sqrt{16} = 4$.

Distance $(2,5)$ to $(7,1)$

$= \sqrt{(7-2)^2 + (1-5)^2} = \sqrt{25+16} = \sqrt{41}$

$(\sqrt{41})^2 = (4)^2 + x^2$

$41 = 16 + x^2$

$25 = x^2$

$5 = x$

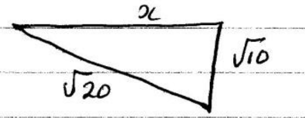
Q20 length of tangent from $(0,0)$ to circle.

$$x^2 + y^2 - 8x - 4y + 10 = 0$$

centre $= (4,2)$ $\quad r = \sqrt{4^2+2^2-10} = \sqrt{10}$

Distance $(4,2)$ to $(0,0)$

$$\sqrt{(4-0)^2+(2-0)^2} = \sqrt{20}$$



$$(\sqrt{20})^2 = (\sqrt{10})^2 + x^2$$

$$20 = 10 + x^2$$

$$10 = x^2$$

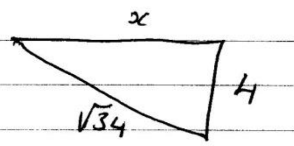
$$\sqrt{10} = x$$

Q21 length of Tangent from $(7,8)$ to $(x-2)^2+(y-5)^2 = 16$.

centre $(2,5)$ $\quad r = 4$.

Dis $(2,5)$ to $(7,8)$

$$\sqrt{(7-2)^2+(8-5)^2} = \sqrt{34}.$$



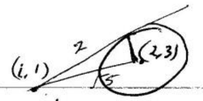
$$(\sqrt{34})^2 = (4)^2 + x^2$$

$$34 = 16 + x^2$$

$$18 = x^2$$

$$\sqrt{18} = x$$

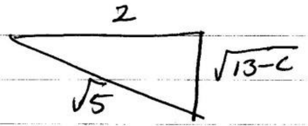
$$3\sqrt{2} = x$$



Q22 Length of Tangent from (1,1) to circle
$x^2+y^2-4x-6y+c=0$ is 2.

Centre $=(2,3)$ $\quad r=\sqrt{2^2+3^2-c}=\sqrt{13-c}$

Dis (2,3) to (1,1)
$\sqrt{(2-1)^2+(3-1)^2}=\sqrt{5}$



$(\sqrt{5})^2=(2)^2+(\sqrt{13-c})^2$

$5=4+13-c$

$c=12$