

### Ex 4.5

Q1  $3x - y + 5 = 0$  and  $x^2 + y^2 = 5$   
 $y = 3x + 5 \Rightarrow x^2 + (3x + 5)^2 = 5$   
 $x^2 + 9x^2 + 30x + 25 = 5$   
 $10x^2 + 30x + 20 = 0 (\div 10)$   
 $x^2 + 3x + 2 = 0$   
 $(x + 2)(x + 1) = 0$   
 $x = -2 \quad x = -1$

find  $y$ :  $y = 3x + 5$   
at  $x = -2$ :  $y = -6 + 5$  at  $x = -1$ :  $y = -3 + 5$   
 $y = -1$   $y = 2$   
pts of  $\cap$  are  $(-2, -1)$  and  $(-1, 2)$

Q2  $x - 3y - 10 = 0$  and  $x^2 + y^2 = 10$   
 $x = 3y + 10 \Rightarrow (3y + 10)^2 + y^2 = 10$   
 $9y^2 + 60y + 100 + y^2 = 10$   
 $10y^2 + 60y + 90 = 0 (\div 10)$   
 $y^2 + 6y + 9 = 0$   
 $(y + 3)(y + 3) = 0$   
 $y = -3 \quad y = -3$  same

find  $x$ :  $x = 3y + 10$   
at  $y = -3$ :  $x = -9 + 10$   
 $x = 1$

$(1, -3)$  is pt of intersection  
 $\Rightarrow$  only 1 pt  $\therefore$  is a tangent.

Q3

$$2x - y - 5 = 0$$

$$y = 2x - 5 \Rightarrow$$

$$x^2 + y^2 = 5$$

$$x^2 + (2x - 5)^2 = 5$$

$$x^2 + 4x^2 - 20x + 25 = 5$$

$$5x^2 - 20x + 20 = 0 (\div 5)$$

$$x^2 - 4x + 4 = 0$$

$$(x - 2)(x - 2) = 0$$

$$x = 2$$

common pt.

find  $y$ :  $y = 2x - 5$

at  $x = 2$ :  $y = 4 - 5$

$$y = -1$$

pt of  $\cap$  is  $(2, -1)$

Q4

(i)  $x + y = 6$

$$x = 6 - y \Rightarrow$$

$$x^2 + y^2 + 2x - 4y - 20 = 0$$

$$(6 - y)^2 + y^2 + 2(6 - y) - 4y - 20 = 0$$

$$36 - 12y + y^2 + y^2 + 12 - 2y - 4y - 20 = 0$$

$$2y^2 - 18y + 28 = 0 (\div 2)$$

$$y^2 - 9y + 14 = 0$$

$$(y - 2)(y - 7) = 0$$

$$y = 2 \quad y = 7$$

find  $x$ :  $x = 6 - y$

at  $y = 2$ :  $x = 6 - 2$

$$x = 4$$

at  $y = 7$ :  $x = 6 - 7$

$$x = -1$$

pts of  $\cap$  are  $(4, 2)$  and  $(-1, 7)$

Q4 (ii)  $2x + y - 2 = 0$ ,  $x^2 + y^2 - 10x - 4y - 11 = 0$   
 $y = 2 - 2x \Rightarrow x^2 + (2 - 2x)^2 - 10x - 4(2 - 2x) - 11 = 0$   
 $x^2 + 4 - 8x + 4x^2 - 10x - 8 + 8x - 11 = 0$   
 $5x^2 - 10x - 15 = 0$  ( $\div 5$ )  
 $x^2 - 2x - 3 = 0$   
 $(x - 3)(x + 1) = 0$   
 $x = 3$   $x = -1$

Find  $y$ :  $y = 2 - 2x$   
 at  $x = 3$ :  $y = 2 - 6$       at  $x = -1$ :  $y = 2 + 2$   
 $y = -4$        $y = 4$

Pts of  $\cap$  are  $(3, -4)$  and  $(-1, 4)$

Q4 (iii)  $3x - y - 5 = 0$ ,  $x^2 + y^2 - 2x + 4y - 5 = 0$   
 $y = 3x - 5 \Rightarrow x^2 + (3x - 5)^2 - 2x + 4(3x - 5) - 5 = 0$   
 $x^2 + 9x^2 - 30x + 25 - 2x + 12x - 20 - 5 = 0$   
 $10x^2 - 20x = 0$  ( $\div 10$ )  
 $x^2 - 2x = 0$   
 $x(x - 2) = 0$   
 $x = 0$   $x = 2$

Find  $y$ :  $y = 3x - 5$   
 at  $x = 0$ :  $y = -5$       at  $x = 2$ :  $y = 6 - 5$   
 $y = 1$

Pts of  $\cap$  are  $(0, -5)$  and  $(2, 1)$

Q5  $x - 2y + 12 = 0$ ,  $x^2 + y^2 - x - 31 = 0$

$$x = 2y - 12 \Rightarrow (2y - 12)^2 + y^2 - (2y - 12) - 31 = 0$$

$$4y^2 - 48y + 144 + y^2 - 2y + 12 - 31 = 0$$

$$5y^2 - 50y + 125 = 0 \quad (\div 5)$$

$$y^2 - 10y + 25 = 0$$

$$\sqrt{(y - 5)(y - 5)} = 0$$

$$y = 5 \quad y = 5$$

Find  $x$ :  $x = 2y - 12$   
 at  $y = 5$   $x = 10 - 12$   
 $x = -2$

Pt of  $\cap$  is  $(-2, 5)$  As only one pt  $\Rightarrow$  Tangent.

Q6  $x - 2y - 1 = 0$ ,  $x^2 + y^2 + 2x - 8y - 8 = 0$

(i)  $x = 2y + 1 \Rightarrow (2y + 1)^2 + y^2 + 2(2y + 1) - 8y - 8 = 0$

$$4y^2 + 4y + 1 + y^2 + 4y + 2 - 8y - 8 = 0$$

$$5y^2 - 5 = 0 \quad (\div 5)$$

$$y^2 - 1 = 0$$

$$\sqrt{(y + 1)(y - 1)} = 0$$

$$y = -1 \quad y = 1$$

Find  $x$ :  $x = 2y + 1$   
 at  $y = -1$ :  $x = -2 + 1$  at  $y = 1$   $x = 2 + 1$   
 $x = -1$   $x = 3$

Pts of  $\cap$  are  $(-1, -1)$  and  $M(3, 1)$

(ii) Mid pt  $LM = \left( \frac{-1+3}{2}, \frac{-1+1}{2} \right)$   
 $(1, 0)$

● (iii) dis LM =  $\sqrt{(3+1)^2 + (1+1)^2}$   
 $= \sqrt{16+4} = \sqrt{20} = 2\sqrt{5} = \text{diameter}$   
 $\Rightarrow \text{radius} = \sqrt{5}$

eqn of circle: Centre (1,0) and  $r = \sqrt{5}$   
 $(x-1)^2 + y^2 = 5$   
 $x^2 - 2x + 1 + y^2 = 5$   
 $x^2 + y^2 - 2x - 4 = 0$

● Q7  $x^2 + y^2 - 4x - 6y - 12 = 0$  intersects  $x$  at  $y=0$   
 $\Rightarrow x^2 - 4x - 12 = 0$   
 $(x+2)(x-6) = 0$   
 $x = -2 \quad x = 6$   
pts of  $\cap$  are  $(-2, 0)$  and  $(6, 0)$   
length = 8 units

● Q8  $x^2 + y^2 - 4x + 6y - 7 = 0$  intercepts  $y$  at  $x=0$   
 $\Rightarrow y^2 + 6y - 7 = 0$   
 $(y+7)(y-1) = 0$   
 $y = -7 \quad y = 1$   
 $\Rightarrow$  pts of  $\cap$  are  $(0, -7)$  and  $(0, 1)$   
length = 8 units

Q9  $x^2 + y^2 - 4x + 11y - 12 = 0$

cuts  $x$  axis at  $y=0$

$$\Rightarrow x^2 - 4x - 12 = 0$$

$$(x + 2)(x - 6) = 0$$

$$x = -2 \quad x = 6$$

Pts of  $\cap$  are  $(2, 0)$  and  $(6, 0)$

Pos  $x$  axis =  $A(6, 0)$

cuts  $y$  axis at  $x=0$

$$\Rightarrow y^2 + 11y - 12 = 0$$

$$(y + 12)(y - 1) = 0$$

$$y = -12 \quad y = 1$$

Pts of  $\cap$  are  $(0, -12)$  and  $(0, 1)$

pos  $y$  axis =  $B(0, 1)$

Q10  $x^2 + y^2 - 4x - 8y - 5 = 0$  cuts  $x$  axis  $\Rightarrow y=0$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

$$x = 5 \quad x = -1$$

pts of  $\cap$  are  $(5, 0)$  and  $(-1, 0)$

Length of chord = 6 units.

Q11

$$\begin{array}{r} \ominus x^2 + y^2 - 3x + 5y - 4 = 0 \\ \oplus x^2 + y^2 - 2x + 4y - 7 = 0 \\ \hline 2x - y - 3 = 0 = \text{common chord.} \end{array}$$

Pts of  $\Pi$  :  $2x - y - 3 = 0$  and  $x^2 + y^2 - 3x + 5y - 4 = 0$

$$y = 2x - 3 \Rightarrow x^2 + (2x - 3)^2 - 3x + 5(2x - 3) - 4 = 0$$

$$x^2 + 4x^2 - 12x + 9 - 3x + 10x - 15 - 4 = 0$$

$$5x^2 - 5x - 10 = 0 \quad (\div 5)$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \quad x = -1$$

Find  $y$  :  $y = 2x - 3$

at  $x = 2$  :  $y = 4 - 3 = 1$

at  $x = -1$  :  $y = -2 - 3 = -5$

Pts of  $\Pi$  are  $(2, 1)$  and  $(-1, -5)$

Q12

$$\begin{array}{r} \ominus x^2 + y^2 + 14x - 10y - 26 = 0 \\ \oplus x^2 + y^2 + 4x + 14y + 28 = 0 \\ \hline 18x - 24y - 54 = 0 \quad (\div 6) \\ 3x - 4y - 9 = 0 \end{array}$$

Pt of  $\Pi$  :  $3x - 4y - 9 = 0$  and  $x^2 + y^2 + 14x - 10y - 26 = 0$

$$3x = 4y + 9$$

$$x = \frac{4}{3}y + 3 \Rightarrow \left(\frac{4}{3}y + 3\right)^2 + y^2 + 14\left(\frac{4}{3}y + 3\right) - 26 = 0$$

$$\frac{16}{9}y^2 + 8y + 9 + y^2 + \frac{56}{3}y + 42 - 10y - 26 = 0$$

$$\frac{25}{9}y^2 + \frac{50}{3}y + 25 = 0 \quad (\times 9)$$

$$25y^2 + 150y + 225 = 0 \quad (\div 25)$$

$$y^2 + 6y + 9 = 0$$

$$(y + 3)(y + 3) = 0$$

$$y = -3$$

Find  $x$  :  $x = \frac{4}{3}y + 3 \Rightarrow x = -4 + 3 = -1$  Pt of  $\Pi$  is  $(-1, -3)$

Q13

$$\ominus \sqrt{x} \oplus \ominus \sqrt{y} \oplus \oplus \oplus \oplus$$
$$x^2 + y^2 + 4x - 2y - 5 = 0$$

$$x^2 + y^2 + 14x - 12y + 65 = 0$$

$$10x - 10y + 70 = 0 \quad (\div 10)$$

$$x - y + 7 = 0 \quad = \text{Egn of Chord.}$$

$$x - y + 7 = 0$$

$$x = y - 7$$

$$, \quad x^2 + y^2 + 4x - 2y - 5 = 0$$

$$\Rightarrow (y-7)^2 + y^2 + 4(y-7) - 2y - 5 = 0$$

$$y^2 - 14y + 49 + y^2 + 4y - 28 - 2y - 5 = 0$$

$$2y^2 - 12y + 16 = 0 \quad (\div 2)$$

$$y^2 - 6y + 8 = 0$$

$$(y-4)(y-2) = 0$$

$$y = 4 \quad y = 2$$

find  $x$  :  $x = y - 7$

at  $y = 4$  :  $x = 4 - 7$

$$x = -3$$

at  $y = 2$  :  $x = 2 - 7$

$$x = -5$$

Pts of  $n$  are  $(-3, 4)$  and  $(-5, 2)$

Q14

$$x^2 + y^2 + 2x - 8y + 4 = 0$$

(i) centre  $(-1, 4)$



(ii)  $y = 1$  :  $x^2 + (1)^2 + 2x - 8(1) + 4 = 0$

$$x^2 + 1 + 2x - 8 + 4 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x = -3 \quad x = 1$$

Pts of  $n$  are  $(-3, 1)$  and  $(1, 1)$

↑  
rising

↑  
setting