Ex 4.6

Q1 $f(x) = -x^2 + 4x + 12.$

(i)

x	0	1	2	3	4	5	6
$f(x)$	12	15	16	15	12	7	0

$$\text{Average} = \frac{12 + 15 + 16 + 15 + 12 + 7 + 0}{7} = \frac{77}{7} = 11$$

(ii) $$\text{Average Value} = \frac{1}{6-0}\int_0^6 (-x^2 + 4x + 12)\,dx$$

$$= \frac{1}{6}\left[-\frac{x^3}{3} + \frac{{}^2\cancel{4}x^2}{\cancel{2}} + 12x\right]_0^6$$

$$= \frac{1}{6}\left[\left(-\frac{6^3}{3} + 2(6)^2 + 12(6)\right) - 0\right]$$

$$= \frac{1}{6}(72) = 12.$$

(iii) Integration gives the better estimate.

Q5 (i) $f(x) = \sin x \quad [0, \pi/2]$

$$\text{Average Value} = \frac{1}{\pi/2 - 0} \int_0^{\pi/2} \sin x \, dx$$

$$= \frac{2}{\pi} \Big[-\cos x \Big]_0^{\pi/2}$$

$$= \frac{2}{\pi} \Big[\big(-\cos(\pi/2)\big) - \big(-\cos(0)\big) \Big]$$

$$= \frac{2}{\pi} \Big[(-0) - (-1) \Big]$$

$$= \frac{2}{\pi}(1) = \frac{2}{\pi}$$

(ii)

$f(x) = \cos x \quad [0, 2\pi]$

$$\text{Average Value} = \frac{1}{2\pi - 0} \int_0^{2\pi} \cos x \, dx$$

$$= \frac{1}{2\pi} \Big[\sin x \Big]_0^{2\pi}$$

$$= \frac{1}{2\pi} \Big[\sin(2\pi) - \sin(0) \Big]$$

$$= \frac{1}{2\pi} (0 - 0) = 0.$$

(iii) $f(x) = e^x \quad [0,3]$

$$\text{Average Value} = \frac{1}{3-0}\int_0^3 e^x \, dx$$

$$= \frac{1}{3}\left[e^x\right]_0^3$$

$$= \frac{1}{3}\left(e^3 - e^0\right) = \frac{1}{3}\left(e^3 - 1\right)$$

(iv) $f(x) = e^{4x} \quad [0,2]$

$$\text{Average Value} = \frac{1}{2-0}\int_0^2 e^{4x} \, dx$$

$$= \frac{1}{2}\left[\frac{e^{4x}}{4}\right]_0^2$$

$$= \frac{1}{2}\left[\frac{e^{4(2)}}{4} - \frac{e^{4(0)}}{4}\right]$$

$$= \frac{1}{2}\left(\frac{e^8}{4} - \frac{1}{4}\right)$$

$$= \frac{1}{2}\left(\frac{e^8 - 1}{4}\right) = \frac{e^8 - 1}{8}$$

Q6 $f(x) = x+1 \quad 2 \le x \le k$ is 8

$$\frac{1}{k-2}\int_2^k (x+1)\,dx = 8$$

$$\frac{1}{k-2}\left[\frac{x^2}{2}+x\right]_2^k = 8$$

$$\frac{1}{k-2}\left[\left(\frac{k^2}{2}+k\right)-\left(\frac{2^2}{2}+2\right)\right] = 8$$

$$\frac{1}{k-2}\left[\frac{k^2+2k}{2}-4\right] = 8$$

$$\frac{k^2+2k}{2}-4 = 8k-16$$

$$k^2+2k-8 = 16k-32$$

$$k^2-14k+24 = 0$$

$$(k-2)(k-12) = 0$$

$$k = 2 \qquad k = 12$$

given already $\Rightarrow k = 12$

Q9 $V = \frac{\pi h^3}{12}$ 2cm to 8cm

$$\text{Average} = \frac{1}{8-2}\int_2^8 \frac{\pi h^3}{12}\,dx$$

$$= \frac{1}{6}\cdot\frac{\pi}{12}\int_2^8 h^3\,dx$$

$$= \frac{1}{6}\cdot\frac{\pi}{12}\left[\frac{h^4}{4}\right]_2^8$$

$$= \frac{\pi}{72}\left(\frac{8^4}{4} - \frac{2^4}{4}\right)$$

$$\pi/72\,(1024 - 4)$$

$$\frac{\pi}{72}(1020)$$

$$= \frac{85}{6}\pi \text{ cm}^3$$

Q11 $\quad v = 3t^2 - 4 \qquad t=1$ to $t=3$

(i) Average Vel $= \dfrac{1}{3-1}\displaystyle\int_1^3 (3t^2-4)\,dt$

$$= \tfrac{1}{2}\left[\frac{3t^3}{3} - 4t\right]_1^3$$

$$= \tfrac{1}{2}\left[(3^3-4(3)) - (1^3-4(1))\right]$$

$$= \tfrac{1}{2}(15+3)$$

$$= \tfrac{1}{2}(18) = 9 \text{ m/sec}$$

(ii) acc $= \dfrac{dv}{dt} = 6t$

Average acc $= \dfrac{1}{3-1}\displaystyle\int_1^3 6t\,dt$

$$= \tfrac{1}{2}\left[\frac{\cancel{6}3t^2}{\cancel{2}}\right]_1^3$$

$$= \tfrac{1}{2}\left(3(3)^2 - 3(1)^2\right)$$

$$= \tfrac{1}{2}(27-3)$$

$$= \tfrac{1}{2}(24) = 12 \text{ m/sec}^2$$

Q14 $\quad y = x^{-1/2}$

(i) $\quad y = \frac{1}{\sqrt{x}} \Rightarrow \frac{1}{y} = \sqrt{x} = x^{\frac{1}{2}}$

$$\text{Average Value} = \frac{1}{4-1}\int_1^4 x^{\frac{1}{2}}\,dx$$

$$= \tfrac{1}{3}\left[\frac{x^{3/2}}{3/2}\right]_1^4$$

$$= \frac{1}{3}\left[\frac{2x^{3/2}}{3}\right]_1^4$$

$$= \frac{1}{3}\left(\frac{2(4)^{3/2}}{3} - \frac{2(1)^{3/2}}{3}\right)$$

$$= \tfrac{1}{3}\left(\frac{16}{3} - \frac{2}{3}\right)$$

$$\frac{1}{3}\left(\frac{14}{3}\right) = \frac{14}{9}$$

(ii)

$$\text{Area} = \int_1^4 x^{-\frac{1}{2}}\,dx$$

$$= \left[\frac{x^{1/2}}{\frac{1}{2}}\right]_1^4 = \left[2x^{1/2}\right]_1^4$$

$$= \left(2(4)^{\frac{1}{2}} - 2(1)^{1/2}\right)$$

$$= 4 - 2 = 2 \text{ sq units}$$