Ex 4.6

Q1 $x^2+y^2-2x-15=0$ centre $= (1,0)$

$r_1 = \sqrt{1^2+0^2+15} = \sqrt{16} = 4$

$x^2+y^2-14x-16y+77=0$ centre $= (7,8)$

$r_2 = \sqrt{7^2+8^2-77} = 6$

distance between centres $= \sqrt{(7-1)^2+(8-0)^2} = 10$

$r_1 + r_2 = 10 = d \Rightarrow$ touch externally

Q2 $x^2+y^2+4x-6y+12=0$ centre $(-2,3)$

$r_1 = \sqrt{2^2+3^2-12} = 1$

$x^2+y^2-12x+6y-76=0$ centre $(6,-3)$

$r_2 = \sqrt{6^2+3^2+76} = 11$

$d = \sqrt{(6+2)^2+(-3-3)^2} = 10$

$r_2 - r_1 = 10 = d \Rightarrow$ touch internally

Q3 $x^2+y^2-4x-2y-20=0$ centre $(2,1)$

$r_1 = \sqrt{2^2+1^2+20} = 5$

$x^2+y^2-16x-18y+120=0$ centre $(8,9)$

$r_2 = \sqrt{8^2+9^2-120} = 5$

$d = \sqrt{(8-2)^2+(9-1)^2} = 10$

$r_1 + r_2 = 10 = d \Rightarrow$ Touch Externally

Q4 $x^2+y^2-16y+32=0$ centre $=(0,8)$

$r_1 = \sqrt{0^2+8^2-32} = 4\sqrt{2}$

$x^2+y^2-18x+2y+32=0$ centre $=(9,-1)$

$r_2 = \sqrt{9^2+1^2-32} = 5\sqrt{2}$

$d = \sqrt{(9-0)^2+(-1-8)^2} = 9\sqrt{2}$

$r_1 + r_2 = 9\sqrt{2} = d \Rightarrow$ externally.

Q5 (i) $x^2+y^2-4x-6y+5=0$ centre $(2,3)$

$r_1 = \sqrt{2^2+3^2-5} = 2\sqrt{2}$

$x^2+y^2-6x-8y+23=0$ centre $(3,4)$

$r_2 = \sqrt{3^2+4^2-23} = \sqrt{2}$

$d = \sqrt{(3-2)^2+(4-3)^2} = \sqrt{2}$.

$r_1 - r_2 = \sqrt{2} = d \Rightarrow$ Internally

(ii) eqn common tangent

$$\begin{array}{r} x^2+y^2-4x-6y+5=0 \\ \ominus x^2 \ominus y^2 \oplus 6x \oplus 8y \ominus 23=0 \\ \hline 2x+2y-18=0 \quad (\div 2) \end{array}$$

eqn tangent is $x+y-9=0$.

(iii) $x+y-9=0$ and $x^2+y^2-4x-6y+5=0$

$x = 9-y \Rightarrow (9-y)^2+y^2-4(9-y)-6y+5=0$

$81-18y+y^2+y^2-36+4y-6y+5=0$

$2y^2-20y+50=0 \quad (\div 2)$

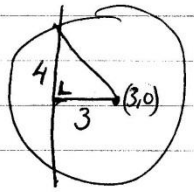
$y^2-10y+25=0$

$(y-5)(y-5)=0$

$y=5$

Solve for x: $x = 9-y \Rightarrow x = 9-5 = 4$ Pt of $\cap$ $(4,5)$

Q6

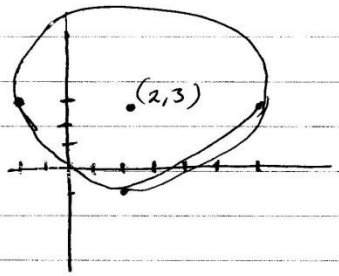


$\Rightarrow r = 5$ units

Eqn of circle $= (x-3)^2 + y^2 = 25$

Q7 centre $(2,3)$ $r = 4$.

(i)



(ii) eqn is $(x-2)^2 + (y-3)^2 = 16$

cuts y axis $\Rightarrow x = 0$.

$$(0-2)^2 + (y-3)^2 = 16$$
$$4 + y^2 - 6y + 9 = 16$$
$$y^2 - 6y - 3 = 0$$
$$y = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm 4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}.$$

Pts of intersection are $(0, 3+2\sqrt{3})$ and $(0, 3-2\sqrt{3})$

Dis between pts. $\sqrt{(0-0)^2 + (3-2\sqrt{3} - (3+2\sqrt{3}))^2}$

$= \sqrt{(3-2\sqrt{3} - 3 - 2\sqrt{3})^2}$

$= \sqrt{(-4\sqrt{3})^2}$

$= \sqrt{48} = 4\sqrt{3}$ units.

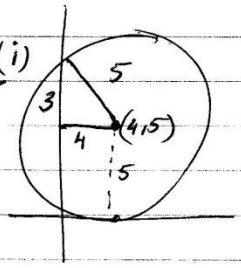
(iii) cuts $x \Rightarrow y = 0$

$$(x-2)^2 + (0-3)^2 = 16$$
$$x^2 - 4x + 4 + 9 = 16$$
$$x^2 - 4x - 3 = 0$$
$$x = \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$$

2 pts of n are $(2+\sqrt{7}, 0)$ and $(2-\sqrt{7}, 0)$

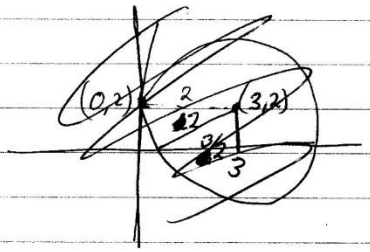
Dis between pt $= \sqrt{(2+\sqrt{7} - (2-\sqrt{7}))^2 + (0-0)^2}$
$= \sqrt{(2-\sqrt{7}-2-\sqrt{7})^2}$
$= \sqrt{(-2\sqrt{7})^2}$
$= 2\sqrt{7}$ units.

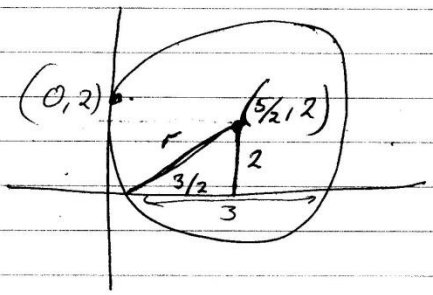
Q8 (i)



(ii) $(x-4)^2 + (y-5)^2 = 25$

Q9

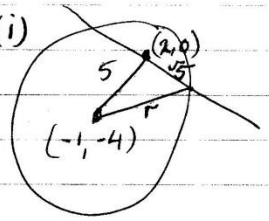




$r^2 = 2^2 + (3/2)^2$
$r^2 = 4 + 9/4$
$r^2 = 25/4$
$r = 5/2$

Eqn: $(x-5/2)^2 + (y-2)^2 = 25/4$

Q10 (i)

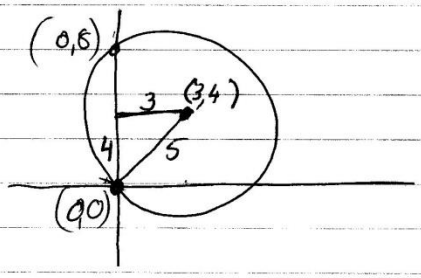


dis (2,0) to (-1 -4)
$= \sqrt{(-1-2)^2+(-4-0)^2} = \sqrt{25} = 5.$

$r^2 = \sqrt{5}^2 + 5^2$
$r^2 = 5 + 25$
$r^2 = 30$
$r = \sqrt{30}$

(ii) $(x+1)^2 + (y+4)^2 = 30$

Q11



centre (3,4)

eqn: $(x-3)^2 + (y-4)^2 = 25$

Q12 $x^2 + y^2 - 6x + 4y - 12 = 0$ centre $(3, -2)$

$r_1 = \sqrt{3^2+2^2+12} = 5$

$x^2 + y^2 + 12x - 20y + k = 0$ centre $(-6, 10)$

$r_2 = \sqrt{6^2+10^2-k} = \sqrt{136-k}.$

dis 2 centres $= \sqrt{(-6-3)^2+(10+2)^2} = 15$

externally $\Rightarrow r_1 + r_2 = 15$

$5 + \sqrt{136-k} = 15$

$\sqrt{136-k} = 10$ (sq both sides)

$136 - k = 100$

$36 = k$