Exercise 5.1

1/ $\cos A \tan A = \sin A$

$\Rightarrow \cos A \cdot \frac{\sin A}{\cos A}$

$\Rightarrow \sin A = RHS$

2/ $\sin\theta \sec\theta = \tan\theta$

$\Rightarrow \sin\theta \frac{1}{\cos\theta}$

$\Rightarrow \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta = RHS$

3/ $\sin\theta \tan\theta + \cos\theta = \sec\theta$

$\Rightarrow \sin\theta \frac{\sin\theta}{\cos\theta} + \cos\theta$

$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$

$\Rightarrow \frac{1}{\cos\theta}$

$\Rightarrow \sec\theta = RHS.$

4/ $\frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \tan\theta$

$\Rightarrow \frac{\sin\theta}{\sqrt{\cos^2\theta}}$

$\Rightarrow \frac{\sin\theta}{\cos\theta}$

$\Rightarrow \tan\theta = RHS$

5/ $\text{Sec}A - \text{Sin}A\,\text{Tan}A = \text{Cos}A$

$\Rightarrow \frac{1}{\text{Cos}A} - \text{Sin}A\,\frac{\text{Sin}A}{\text{Cos}A}$

$\Rightarrow \frac{1 - \text{Sin}^2A}{\text{Cos}A}$

$\Rightarrow \frac{\text{Cos}^2A}{\text{Cos}A}$

$\Rightarrow \text{Cos}A = RHS$

6/ $1 - \text{Tan}^2\theta\,\text{Cos}^2\theta = \text{Cos}^2\theta$

$1 - \frac{\text{Sin}^2\theta}{\text{Cos}^2\theta} \times \text{Cos}^2\theta$

$\text{Cos}^2\theta = R.H.S.$

7/ $\frac{(1+\text{Cos}\theta)(1-\text{Cos}\theta)}{\text{Cos}^2\theta} = \text{Tan}^2\theta$

$\frac{1 - \text{Cos}\theta + \text{Cos}\theta - \text{Cos}^2\theta}{\text{Cos}^2\theta}$

$\frac{1-\text{Cos}^2\theta}{\text{Cos}^2\theta}$

$\frac{\text{Sin}^2\theta}{\text{Cos}^2\theta}$

$\text{Tan}^2\theta = R.H.S.$

8/ $\sec^2 A - \tan^2 A = 1$

$$\frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$\frac{1-\sin^2 A}{\cos^2 A}$$

$$\frac{\cos^2 A}{\cos^2 A}$$

$$1 = RHS$$

9/ $$\frac{\sqrt{1-\cos^2\theta}}{\tan\theta} = \cos\theta$$

$$\frac{\sqrt{\sin^2\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{\sin\theta}{} \times \frac{\cos\theta}{\sin\theta}$$

$$\cos\theta = RHS$$

10/

$$(1+\tan^2\theta)\cos^2\theta = 1$$

$$\cos^2\theta + \tan^2\theta\cos^2\theta$$

$$\cos^2\theta + \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta$$

$$1 = RHS$$

11/ $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2$

$\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta$

$2\cos^2\theta + 2\sin^2\theta$

$2(\cos^2\theta + \sin^2\theta)$

$2(1)$

$= 2$ (RHS)

12/ $(1 + \tan^2 A)(1 - \sin^2 A) = 1$

$(\sec^2 A)(\cos^2 A)$

$\frac{1}{\cos^2 A} \times \cos^2 A$

$= 1$ (RHS)

13/ $(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = 1$

$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 2\sin\theta\cos\theta$

$\sin^2\theta + \cos^2\theta$

$= 1$ (RHS)

14/

$\frac{1 - \cos^2 A}{\sin A \cos A} = \tan A$

$\frac{\sin^2 A}{\sin A \cos A}$

$\frac{\sin A}{\cos A}$

$= \tan A$ (RHS)

15/ $\frac{1}{1-\sin A} + \frac{1}{1+\sin A} = 2\sec^2 A$

$\frac{1+\sin A + 1-\sin A}{(1-\sin A)(1+\sin A)}$

$\frac{2}{1+\sin A - \sin A - \sin^2 A}$

$\frac{2}{1-\sin^2 A}$

$\frac{2}{\cos^2 A}$

$\frac{1}{\cos^2 A} \times 2$

$\sec^2 A \times 2$

$= 2\sec^2 A.$ (RHS)

16/ $(1-\sin^2 A)\tan^2 A + \cos^2 A = 1$

$(\cos^2 A)\frac{\sin^2 A}{\cos^2 A} + \cos^2 A$

$\sin^2 A + \cos^2 A$

$= 1$ (RHS)

17 $\text{Cosec}^2\theta(\tan^2\theta - \sin^2\theta) = \tan^2\theta$

$$\frac{1}{\sin^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right)$$

$$\frac{1}{\cos^2\theta} - 1$$

$$\sec^2\theta - 1$$

$$= \tan^2\theta \quad (RHS)$$

18 $(1-\sin A)(\sec A + \tan A) = \cos A$

$$(1-\sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$\frac{1}{\cos A} + \frac{\sin A}{\cos A} - \frac{\sin A}{\cos A} - \frac{\sin^2 A}{\cos A}$$

$$\frac{1}{\cos A} - \frac{\sin^2 A}{\cos A}$$

$$\frac{1-\sin^2 A}{\cos A}$$

$$\frac{\cos^2 A}{\cos A}$$

$$= \cos A \quad (RHS)$$

Q19 $b\cos C + c\cos B = a$.

$$\cos C = \frac{a^2+b^2-c^2}{2ab} \qquad \cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$b\left(\frac{a^2+b^2-c^2}{2ab}\right) + c\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$\frac{a^2+b^2-c^2+a^2+c^2-b^2}{2a}$$

$$\frac{2a^2}{2a} = a \qquad \text{RHS.}$$

Q20 $bc\cos A + ca\cos B = c^2$

$$bc\left(\frac{b^2+c^2-a^2}{2bc}\right) + ca\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$\frac{b^2+c^2-a^2+a^2+c^2-b^2}{2}$$

$$\frac{2c^2}{2} = c^2 \qquad (\text{RHS})$$

21/ $c = b\cos A + a\cos B$

$$\cancel{b}\left(\frac{b^2+c^2-a^2}{2\cancel{b}c}\right) + \cancel{a}\left(\frac{a^2+c^2-b^2}{2\cancel{a}c}\right)$$

$$\frac{b^2+c^2-a^2+a^2+c^2-b^2}{2c}$$

$$\frac{2c^2}{2c} = c \quad (LHS)$$

22/ $a\cos B - b\cos A = \frac{a^2-b^2}{c}$

$$\cancel{a}\left(\frac{a^2+c^2-b^2}{2\cancel{a}c}\right) - \cancel{b}\left(\frac{b^2+c^2-a^2}{2\cancel{b}c}\right)$$

$$\frac{a^2+c^2-b^2-b^2-c^2+a^2}{2c}$$

$$\frac{2a^2-2b^2}{2c} = \frac{\cancel{2}(a^2-b^2)}{\cancel{2}c} = \frac{a^2-b^2}{c} \quad RHS$$

23 $ab\cos C - ac\cos B = b^2 - c^2$

$$ab\left(\frac{a^2+b^2-c^2}{2ab}\right) - ac\left(\frac{a^2+c^2-b^2}{2ac}\right)$$

$$\frac{a^2+b^2-c^2-a^2-c^2+b^2}{2}$$

$$\frac{2b^2-2c^2}{2} = \frac{2(b^2-c^2)}{2} = b^2-c^2 \quad (RHS$$

24 $c\cos B - b\cos C = \frac{c^2-b^2}{a}$

$$\cancel{a}\left(\frac{a^2+c^2-b^2}{2a\cancel{c}}\right) - \cancel{b}\left(\frac{a^2+b^2-c^2}{2a\cancel{b}}\right)$$

$$\frac{a^2+c^2-b^2-a^2-b^2+c^2}{2a}$$

$$\frac{2c^2-2b^2}{2a}$$

$$\frac{\cancel{2}(c^2-b^2)}{\cancel{2}a} = \frac{c^2-b^2}{a} \quad (RHS)$$

Q25

$$\frac{\sin A - \sin B}{\sin B} = \frac{a-b}{b}$$

$$\frac{\frac{a\sin B}{b} - \sin B}{\sin B}$$

$$\frac{\frac{a\sin B - b\sin B}{b}}{\sin B}$$

$$\frac{\sin B(a-b)}{b} \times \frac{1}{\sin B}$$

$$= \frac{a-b}{b}$$

Q26 $\quad \frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$

$$\frac{b^2+c^2-a^2}{2bc} \times \frac{1}{a} + \frac{a^2+c^2-b^2}{2ac} \times \frac{1}{b} + \frac{a^2+b^2-c^2}{2ab} \times \frac{1}{c}$$

$$\frac{b^2+c^2-a^2}{2abc} + \frac{a^2+c^2-b^2}{2abc} + \frac{a^2+b^2-c^2}{2abc}$$

$$\frac{b^2+c^2-a^2+a^2+c^2-b^2+a^2+b^2-c^2}{2abc}$$

$$\frac{a^2+b^2+c^2}{2abc} \quad (\text{RHS})$$