Exercise 5·2

Q1 (ii) $\sin 75 = \sin(30+45)$

$$\sin(30+45) = \sin 30 \cos 45 + \cos 30 \sin 45$$
$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right) + \left(\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$
$$= \frac{1+\sqrt{3}}{2\sqrt{2}}$$

$$\frac{1+\sqrt{3}}{2\sqrt{2}} \times \frac{2\sqrt{2}}{2\sqrt{2}}$$
$$= \frac{2\sqrt{2}+2\sqrt{6}}{8} = \frac{\sqrt{2}+\sqrt{6}}{4}.$$

Q2 (i) $\tan 15 = \tan(45-30)$

$$\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$$

$$= \frac{1-\frac{1}{\sqrt{3}}}{1+(1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}} \quad \left(\text{mult each by } \sqrt{3}\right)$$

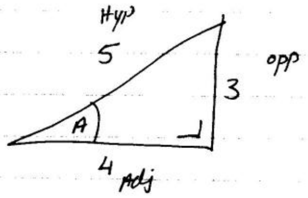
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

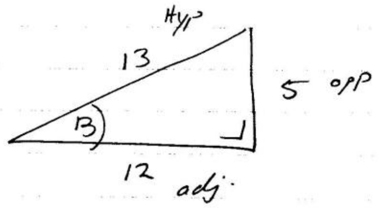
$$\frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

Q3





$\sin A = 3/5$

$\cos A = 4/5$

$\tan A = 3/4$

$\sin B = 5/13$

$\cos B = 12/13$

$\tan B = 5/12.$

(i) $\cos(A+B) = \cos A \cos B - \sin A \sin B.$

$$\left(\tfrac{4}{5}\right)\left(\tfrac{12}{13}\right) - \left(\tfrac{3}{5}\right)\left(\tfrac{5}{13}\right)$$

$$\frac{48}{65} - \frac{3}{13} = \frac{48-15}{65} = 33/65.$$

(ii) $$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{3/4 - 5/12}{1 + (3/4)(5/12)} = \frac{\frac{9-5}{12}}{1 + 5/16} = \frac{1/3}{21/16}$$

$$= \frac{1}{3} \times \frac{16}{21} = \frac{16}{63}.$$

Q4 (i) $\sin 45 \cos 15 + \cos 45 \sin 15$.

$= \sin(45+15) = \sin 60 = \sqrt{3}/2$.

(iv) $\dfrac{\tan 25 + \tan 20}{1 - \tan 25 \tan 20} = \tan(25+20)$

$= \tan 45 = 1$

Q5 $\dfrac{\tan 2A + \tan A}{1 - \tan 2A \tan A} = \tan(2A + A) = \tan 3A$.

Q6 (i) show $\sin(90^\circ - A) = \cos A$

$\sin(90 - A) = \sin 90 \cos A - \cos 90 \sin A$.

$= (1) \cos A - (0) \sin A$

$= \cos A$.

Q7 $\tan(A - B) = 2$ $\tan B = 1/4$ find $\tan A$

$\dfrac{\tan A - \tan B}{1 + \tan A \tan B} = 2$

$\dfrac{\tan A - 1/4}{1 + \tan A\left(\frac{1}{4}\right)} = 2$.

$\tan A - \frac{1}{4} = 2\left(1 + \frac{1}{4}\tan A\right)$

$\tan A - \frac{1}{4} = 2 + \frac{1}{2}\tan A$

$\tan A - \frac{1}{2}\tan A = 2 + \frac{1}{4}$

$\frac{1}{2}\tan A = \frac{9}{4}$

$\tan A = \frac{18}{4}$

$\tan A = \frac{9}{2}$

Q8 $\quad \tan A = \frac{1}{2} \quad \tan B = \frac{1}{3} \quad$ find $(A+B)$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

$$\tan(A+B) = \frac{(\frac{1}{2}) + (\frac{1}{3})}{1 - (\frac{1}{2})(\frac{1}{3})} = \frac{\frac{3+2}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

$$\tan(A+B) = 1$$

$$(A+B) = \tan^{-1}(1)$$

$$A+B = 45^\circ$$

Q4 $\quad \tan(A+B) = 1 \quad \tan A = \frac{1}{3} \quad$ find $\tan B$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\frac{1}{3} + \tan B = 1\left(1 - \frac{1}{3}\tan B\right)$$

$$\tan B + \frac{1}{3}\tan B = 1 - \frac{1}{3}$$

$$\frac{4}{3}\tan B = \frac{2}{3}$$

$$\tan B = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

$$\tan B = \frac{1}{2}$$

$$x = \sin^{-1}\left(\tfrac{1}{2}\right)$$
$$x = \tfrac{\pi}{6}.$$

Q10 $\quad \sin x = \tfrac{1}{2} \longrightarrow \quad$ find $\sin\left(x + \frac{\pi}{4}\right)$

$$\sin\left(x+\tfrac{\pi}{4}\right) = \sin x \cos\tfrac{\pi}{4} + \cos x \sin\tfrac{\pi}{4}$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right) + \cos x\left(\tfrac{1}{\sqrt{2}}\right)$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right) + \cos\left(\tfrac{\pi}{6}\right)\tfrac{1}{\sqrt{2}}$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right) + \left(\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}}.$$

Q 11

$$\tan 15 = \tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$$

$$= \frac{1-\frac{1}{\sqrt{3}}}{1+(1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$ mult each by $\sqrt{3}$.

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

$$\tan^2(15) = \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)^2 = \left(2-\sqrt{3}\right)^2$$

$$= 4 - 4\sqrt{3} + 3$$

$$= 7 - 4\sqrt{3}.$$

Q12 $\quad \text{Tan}\left(\frac{\pi}{4}+A\right) = \frac{\text{Cos}A + \text{Sin}A}{\text{Cos}A - \text{Sin}A}$

$$\text{Tan}\left(\frac{\pi}{4}+A\right) = \frac{\text{Tan}\frac{\pi}{4} + \text{Tan}A}{1 - \text{Tan}\frac{\pi}{4}\,\text{Tan}A.} = \frac{1+\text{Tan}A}{1-(1)(\text{Tan}A)}$$

$$= \frac{1+\frac{\text{Sin}A}{\text{Cos}A}}{1-\frac{\text{Sin}A}{\text{Cos}A}}$$ mult Each by Cos A.

$$= \frac{\text{Cos}A + \text{Sin}A}{\text{Cos}A - \text{Sin}A} \quad (RHS).$$

Q13 $\quad \text{Cos}(A+B)\,\text{Cos}B + \text{Sin}(A+B)\,\text{Sin}B = \text{Cos}A.$

$$(\text{Cos}A\text{Cos}B - \text{Sin}A\text{Sin}B)\text{Cos}B + (\text{Sin}A\text{Cos}B + \text{Cos}A\text{Sin}B)\text{Sin}$$

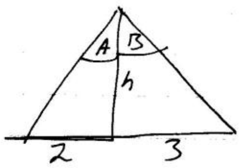
$$\text{Cos}A\text{Cos}^2B - \underline{\text{Sin}A\text{Sin}B\text{Cos}B} + \underline{\text{Sin}A\text{Cos}B\text{Sin}B} + \text{Cos}A\text{Sin}^2B$$

$$\text{Cos}A\text{Cos}^2B + \text{Cos}A\,\text{Sin}^2B.$$

$$\text{Cos}A\left(\text{Cos}^2B + \text{Sin}^2B\right)$$

$$\text{Cos}A\,(1) = \text{Cos}A \quad (RHS)$$

Q14



$\tan A = 2/h \qquad \tan B = 3/h.$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

$$\tan(45) = 1 = \frac{2/h + 3/h}{1 - (2/h)(3/h)}$$

$$1 = \frac{5/h}{1 - 6/h^2}$$

$$1 - 6/h^2 = 5/h \qquad (\times h^2)$$

$$h^2 - 6 = 5h$$
$$h^2 - 5h - 6 = 0$$
$$(h - 6)(h + 1) = 0$$
$$h = 6 \qquad \cancel{h = -1}$$

$$h = 6.$$

Q15 $\quad \sin A = \sin(A+30) \quad$ Show $\tan A = 2+\sqrt{3}$

$$\sin(A+30) = \sin A \cos 30 + \cos A \sin 30 \qquad \tan A = \frac{\sin A}{\cos A}$$

$$\sin A = \sin A \left(\frac{\sqrt{3}}{2}\right) + \cos A \left(\frac{1}{2}\right)$$

$$\sin A - \frac{\sqrt{3}}{2}\sin A = \frac{1}{2}\cos A \qquad (\times 2)$$

$$2\sin A - \sqrt{3}\sin A = \cos A.$$

$$\sin A\left(2-\sqrt{3}\right) = \cos A$$

$$2-\sqrt{3} = \frac{\cos A}{\sin A}.$$

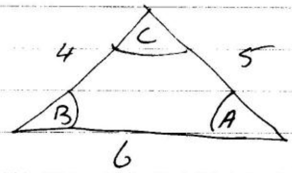
$$\frac{1}{2-\sqrt{3}} = \frac{\sin A}{\cos A}$$

$$\frac{1}{2-\sqrt{3}} = \tan A$$

$$\frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$$

$$\tan A = 2+\sqrt{3}.$$

Q.4 (i)



$$\cos A = \frac{5^2+6^2-4^2}{2(5)(6)} = \tfrac{3}{4}$$

$$\cos C = \frac{5^2+4^2-6^2}{2(5)(4)} = \frac{1}{8}.$$

Show $\cos A + \cos C = \tfrac{7}{8}$

$$\tfrac{3}{4} + \tfrac{1}{8} = \frac{6+1}{8} = \tfrac{7}{8}$$ QED.

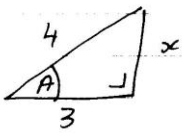
(ii)

show $\cos(A+C) = -\tfrac{9}{16}$

$$\cos(A+C) = \cos A \cos B - \sin A \sin B.$$

Require: $\sin A$ and $\sin B$.

$\cos A = \tfrac{3}{4}$

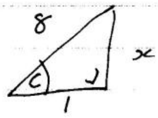


$4^2 = 3^2 + x^2$

$\sqrt{7} = x.$

$\Rightarrow \sin A = \tfrac{\sqrt{7}}{4}$

$\cos C = \tfrac{1}{8}$



$8^2 = 1^2 + x^2$

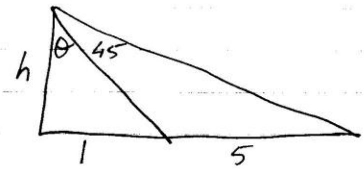
$\sqrt{63} = x$

$\Rightarrow \sin C = \tfrac{\sqrt{63}}{8}$

$$\cos(A+C) = \left(\tfrac{3}{4}\right)\left(\tfrac{1}{8}\right) - \left(\tfrac{\sqrt{7}}{4}\right)\left(\tfrac{\sqrt{63}}{8}\right)$$

$$= \frac{3}{32} - \frac{21}{32} = -\frac{18}{32} = \frac{-9}{16}$$ QED.

Q17



$\tan\theta = \frac{1}{h}$ $\quad\quad$ $\tan(\theta+45) = \frac{6}{h}$.

$$\tan(\theta+45) = \frac{\tan\theta + \tan 45}{1 - \tan\theta \tan 45}$$

$$\frac{6}{h} = \frac{\frac{1}{h}+1}{1-(\frac{1}{h})(1)}$$

$$\frac{6}{h} = \frac{\frac{1}{h}+1}{1-\frac{1}{h}} \quad \text{(mult each by } h\text{)}$$

$$\frac{6h}{h^2} = \frac{1+h}{h-1}$$

$$6h(h-1) = h^2(1+h)$$
$$6h^2-6h = h^2+h^3 \quad (\div h)$$
$$6h-6 = h+h^2$$
$$0 = h^2-5h+6$$
$$0 = (h-2)(h-3)$$
$$h = 2m \quad h = 3m.$$