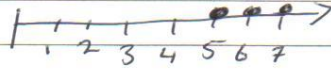


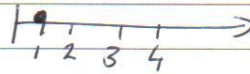
### Exercise 7.1

$$x \in \mathbb{N}$$

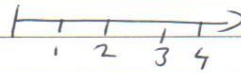
Q1 (i)  $3x - 5 > x + 3$   
 $2x > 8$   
 $x > 4$



(ii)  $6x - 5 \leq 2x - 1$   
 $4x \leq 4$   
 $x \leq 1$



(iii)  $1 - 3x > 10$   
 $-3x > 9$   
 $3x < -9$   
 $x < -3$



Q2

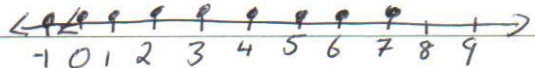
(ii)  $\frac{1}{6}(x-1) \geq \frac{1}{3}(x-4) \quad x \in \mathbb{Z}$   
(x6)

$$x-1 \geq 2(x-4)$$

$$x-1 \geq 2x-8$$

$$-x \geq -7$$

$$x \leq 7$$



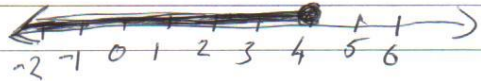
Q3

(i)  $12x - 3(x-3) < 45 \quad x \in \mathbb{R}$

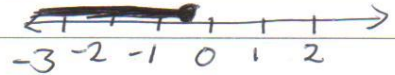
$$12x - 3x + 9 < 45$$

$$9x < 36$$

$$x < 4$$



Q3 (ii)  $x(x-4) \geq x^2+2 \quad x \in \mathbb{R}$   
 $x^2-4x \geq x^2+2$   
 $-4x \geq 2$   
 $4x \leq -2$   
 $x \leq -\frac{1}{2}$



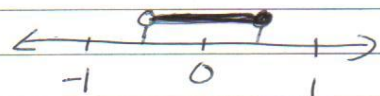
Q4 (ii)  $-1 > 1-3x \geq 7 \quad x \in \mathbb{R}$   
 $-1 > 1-3x \quad 1-3x \geq 7$   
 $-2 > -3x \quad 1-7 \geq 3x$   
 $3x > 2 \quad -6 \geq 3x$   
 $x > \frac{2}{3} \quad -2 \geq x$

$-2 \geq x > \frac{2}{3}$



(iii)  $3 \geq 4x+1 > -1 \quad x \in \mathbb{R}$   
 $3 \geq 4x+1 \quad 4x+1 > -1$   
 $2 \geq 4x \quad 4x > -2$   
 $\frac{1}{2} \geq x \quad x > -\frac{1}{2}$

$\frac{1}{2} \geq x > -\frac{1}{2}$   
 $-\frac{1}{2} < x \leq \frac{1}{2}$



Q5

(ii)  $-4 \leq \frac{2}{5}(1-3x) \leq 1$   $x \in \mathbb{R}$

(x5)  $-4 \leq \frac{2}{5}(1-3x)$   $\frac{2}{5}(1-3x) \leq 1$  (x5)

$-20 \leq 2(1-3x)$

$-20 \leq 2-6x$

$6x \leq 22$

$x \leq \frac{22}{6}$

$x \leq 3\frac{2}{3}$

$2(1-3x) \leq 5$

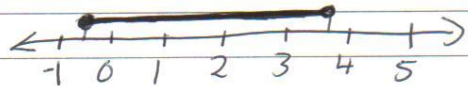
$2-6x \leq 5$

$2-5 \leq 6x$

$-3 \leq 6x$

$-\frac{1}{2} \leq x$

$-\frac{1}{2} \leq x \leq 3\frac{2}{3}$



(iii)  $3 \leq 2 - \frac{x}{7} < 4$

(x7)  $3 \leq 2 - \frac{x}{7}$

$21 \leq 14 - x$

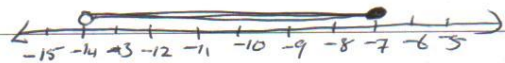
$x \leq -7$

$2 - \frac{x}{7} < 4$  (x7)

$14 - x < 28$

$-14 < x$

$-14 < x \leq -7$



Q8 (i)  $2x - 3 > 2$

$2x > 5$

C:  $x > 2\frac{1}{2}$

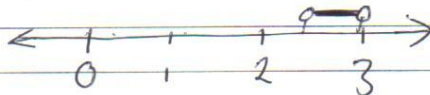
(ii)  $3/(x+2) < 12+x$

$3x+6 < 12+x$

$2x < 6$

D:  $x < 3$

$C \cap D$ :  $2\frac{1}{2} < x < 3$



Q10

(i)  $3x + 8 \leq 20$

$$3x \leq 12$$

$$G: x \leq 4$$

(ii)  $2(3x-7) \geq x+6$

$$6x - 14 \geq x + 6$$

$$5x \geq 20$$

$$H: x \geq 4$$

$$(iii) G \cap H: x = \{4\}$$

Q9

(i)  $15 - 2x < 2(11 - 2x)$

$$15 - 2x < 22 - 2x$$

$$E: x < 7$$

(ii)  $5(3x-1) > 12x+19$

$$15x - 5 > 12x + 19$$

$$3x > 24$$

$$F: x > 8$$

$$(iii) E \cap F: \{ \}$$

Q11

$$\text{width} = x$$

$$\text{length} = x+1$$

$$\text{Perimeter} = 2(x) + 2(x+1) \leq 38$$

$$2x + 2x + 2 \leq 38$$

$$4x \leq 36$$

$$x \leq 9$$

$$\Rightarrow \text{width} = 9 \text{ and length} = 10$$

Q12

$$100 < 2^n < 200$$

$$2^a = 100$$

$$2^6 = 64$$

$$2^b = 200$$

$$2^7 = 128, \quad 2^8 = 256$$

$$6 < n < 8$$

Q13  $a > b > 0$  and  $n > 0 \Rightarrow a^n > b^n$

let  $a=5$ ,  $b=3$  and  $n=2$ .

$$\Rightarrow 5^2 > 3^2$$
$$25 > 9 \quad \text{True.}$$

$a > b > 0$        $n < 0 \Rightarrow a^n < b^n$

let  $a=5$ ,  $b=3$  and  $n=-2$

$$\Rightarrow 5^{-2} < 3^{-2}$$
$$\frac{1}{5^2} < \frac{1}{3^2}$$
$$\frac{1}{25} < \frac{1}{9} \quad \text{True.}$$

If  $a < b < 0$  and  $n > 0 \Rightarrow$   
let  $a=-4$ ,  $b=-2$ ,  $n=2$

~~$-4^2 \square -2^2$~~   
 $-4^3 < -2^3$        $16 > 4$        $\Rightarrow a^n > b^n$   
 $-216 < -27$

If  $a < b < 0$  and  $n < 0$

let  $a=-4$ ,  $b=-2$ ,  $n=-2$

$$(-4)^{-2} \square (-2)^{-2}$$
$$\frac{1}{(-4)^2} \square \frac{1}{(-2)^2}$$
$$\frac{1}{16} < \frac{1}{4}$$

But  $-4^{-3} > -2^{-3}$   
 $-\frac{1}{64} > -\frac{1}{8}$

Q14

$$5 - 3x < -10$$

$\cap$

$$4x + 6 < 32$$

$$-3x < -15$$

$$4x < 26$$

$$3x > 15$$

$$x < 6\frac{1}{2}$$

$$x > 5$$

$$\Rightarrow x = 6.$$